Thermometry for trapped fermionic atoms in the BCS limit

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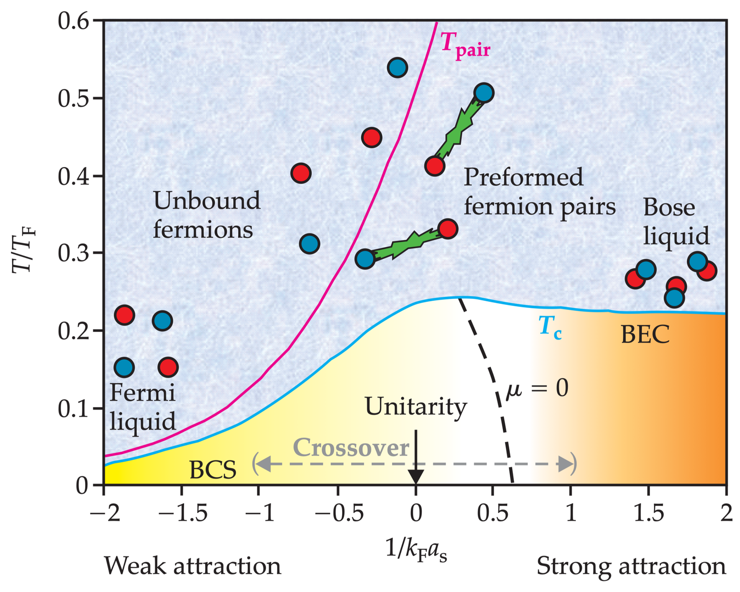
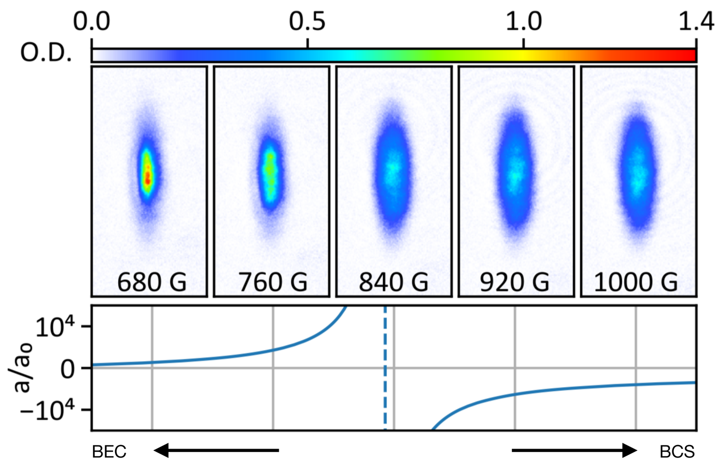
Introduction

- BCS-BEC crossover: Crossover of fermionic and bosonic superfluidities along attractive and repulsive interactions
- Preforming of fermionic pairs below T_{pair}.
- Condensation of Cooper pairs (BCS) or Cooper dimers (BEC) below T_C obtained by Gaussian fluctuations.
- Established MF-theory yields non-interacting ideal Fermi gas above T_{pair}.
- Thermometry and determination of superfluidity of interacting Fermi gas on BCS-side are challenging tasks.
- Indirect thermometry relies on rapid isentropic sweep from BCS- to BEC-side [3].
- New thermometry directly on BCS-side is desirable.



Phase diagram of BCS-BEC crossover [1]



Density distribution of 6Li atoms along the BCS-BEC crossover [2]

Theory

- Grand canonical many-body Hamiltonian operator

$$\hat{H} = \int d^3x \left\{ \sum_{\sigma=\uparrow,\downarrow} \hat{\psi}^\dagger_\sigma(x)\left[-\frac{\hbar^2}{2m}\nabla^2 - \mu\right]\hat{\psi}_\sigma(x) + g\hat{\psi}^\dagger_\uparrow(x)\hat{\psi}^\dagger_\downarrow(x)\hat{\psi}_\downarrow(x)\hat{\psi}_\uparrow(x) \right\}$$

quartic fields interaction; contact interaction parameter, $g \propto a_s$ (s-wave scattering length)

- Mean-Field (MF) approach: MF Hamiltonian $\hat{H}^{(\text{MF})}$ approximates Hamiltonian $\hat{H}$ well enough:

Helmholtz-free energy (thermodynamic potential); MF Helmholtz-free energy

$$\hat{H} \approx \hat{H}^{(\text{MF})} \;\Rightarrow\; F \approx F^{(\text{MF})} = -\beta^{-1}\log \mathrm{Tr}\left(e^{-\beta \hat{H}^{(\text{MF})}}\right)$$

- Quadratic fields approximation: Hartree-Fock-Bogoliubov (HFB) theory

$$\hat{\psi}^\dagger_\uparrow\hat{\psi}^\dagger_\downarrow\hat{\psi}_\downarrow\hat{\psi}_\uparrow \approx +\langle\hat{\psi}^\dagger_\uparrow\hat{\psi}_\uparrow\rangle\hat{\psi}^\dagger_\downarrow\hat{\psi}_\downarrow + \langle\hat{\psi}^\dagger_\downarrow\hat{\psi}_\downarrow\rangle\hat{\psi}^\dagger_\uparrow\hat{\psi}_\uparrow - \langle\hat{\psi}^\dagger_\uparrow\hat{\psi}_\uparrow\rangle\langle\hat{\psi}^\dagger_\downarrow\hat{\psi}_\downarrow\rangle \quad \text{Hartree channel}$$

$$-\langle\hat{\psi}^\dagger_\uparrow\hat{\psi}_\downarrow\rangle\hat{\psi}^\dagger_\downarrow\hat{\psi}_\uparrow - \langle\hat{\psi}^\dagger_\downarrow\hat{\psi}_\uparrow\rangle\hat{\psi}^\dagger_\uparrow\hat{\psi}_\downarrow + \langle\hat{\psi}^\dagger_\uparrow\hat{\psi}_\downarrow\rangle\langle\hat{\psi}^\dagger_\downarrow\hat{\psi}_\uparrow\rangle \quad \text{Fock channel}$$

$$+\langle\hat{\psi}^\dagger_\uparrow\hat{\psi}^\dagger_\downarrow\rangle\hat{\psi}_\downarrow\hat{\psi}_\uparrow + \langle\hat{\psi}_\downarrow\hat{\psi}_\uparrow\rangle\hat{\psi}^\dagger_\uparrow\hat{\psi}^\dagger_\downarrow - \langle\hat{\psi}^\dagger_\uparrow\hat{\psi}^\dagger_\downarrow\rangle\langle\hat{\psi}_\downarrow\hat{\psi}_\uparrow\rangle \quad \text{Bogoliubov channel}$$

- Different MF-Theories by selection of channels

Channels	Bogoliubov	Hartree, Fock	HFB
Theory	BCS MF-Theory	Interacting Fermi gas	Interacting superfluid
Order Parameter	$\Delta = g\langle\hat{\psi}_\downarrow\hat{\psi}_\uparrow\rangle$	$\Gamma^{\text{H}}_\sigma = g\langle\hat{\psi}^\dagger_\sigma\hat{\psi}_\sigma\rangle$, $\Gamma^{\text{F}} = g\langle\hat{\psi}^\dagger_\uparrow\hat{\psi}_\downarrow\rangle = g\langle\hat{\psi}^\dagger_\downarrow\hat{\psi}_\uparrow\rangle$	$\Gamma^{\text{H}}_\sigma, \Gamma^{\text{F}}, \Delta$
Diagonalization	Bogoliubov trafo.	Hartree-Fock trafo.	HFB trafo.

Bogoliubov channel gives regular BCS theory [4] ⇒ Superfluidity
Hartree-Fock describes interaction effects ⇒ Contact interaction in spin-balanced gas: No Fock contribution [5]
HFB allows interaction in superfluid phase ⇒ Separation of thermal and superfluid density

- Extremal thermodynamic potential ⇒ Self-consistent order parameter equations

$$\frac{\partial F^{(\text{MF})}}{\partial \Gamma^{\text{H}}} = 0 \;\Rightarrow\; \frac{\Gamma^{\text{H}}}{g} = \frac{n}{2}$$

Hartree-order parameter equation (density equation)

$$\frac{\partial F^{(\text{MF})}}{\partial \Delta^*} = 0 \;\Rightarrow\; \frac{\Delta}{g_r} = \frac{\Delta}{2V}\sum_k \left[\frac{1}{\epsilon_k} - \frac{1}{E(k)}\tanh\left(\frac{\beta}{2}E(k)\right)\right]$$

Renormalized BCS-order parameter equation

$$E(k) = \sqrt{\left(\epsilon_k - \mu + \frac{g}{2}n\right)^2 + \Delta^2}, \quad k_F = \sqrt{\frac{2m\epsilon_F}{\hbar^2}}, \quad \lambda_F = \frac{2\pi}{k_F}, \quad \lambda = \sqrt{\frac{2\pi\hbar^2\beta}{m}}$$

- Hartree-shift: $\mu \to \mu - \Gamma^{\text{H}}$
- Critical temperature depends on density n.

$$\frac{T_{\text{pair}}}{T_{\text{F}}} = \left[\frac{\lambda_{\text{F}}^3 n}{-2\pi^{3/2}\mathrm{Li}_{3/2}\left(-e^{(\beta\mu)_{\text{pair}}}\right)}\right]^{2/3}, \quad \text{compare:} \quad \frac{T_{\text{BEC}}}{T} = \left[\frac{\lambda^3 n}{\mathrm{Li}_{3/2}\left(e^0\right)}\right]^{2/3}$$

$\beta\mu$ at $T = T_{\text{pair}}$, determined by solving equations above at $\Delta = 0$; $\mathrm{Li}_{3/2}(e^0) = \zeta(3/2)$

- T_{pair} changes due to Hartree-interaction: $n \to n\left(\Gamma^{\text{H}}\right) \;\Rightarrow\; T_{\text{pair}} \to T_{\text{pair}}\left(\Gamma^{\text{H}}\right)$

Experiment

- Confining potential: Harmonic potential in three dimensions

$$V_{\text{HO}}(x) = \frac{m}{2}\sum_{i=1}^{3}\omega_i^2 x_i^2 \;\Rightarrow\; \mu(x) = \mu - V_{\text{HO}}(x) \quad \text{(Local Density Approximation)}$$

- Calculation of spatial dependent self-consistent equations ⇒ density profile

$$n(x) = \frac{1}{V}\sum_k \left[1 - \frac{\epsilon_k - \mu(x) + \frac{g}{2}n(x)}{E(k,x)}\tanh\left(\frac{\beta}{2}E(k,x)\right)\right]$$

$$E(k,x) = \sqrt{\left(\epsilon_k - \mu(x) + \frac{g}{2}n(x)\right)^2 + \Delta(x)^2}$$

Density equation with LDA

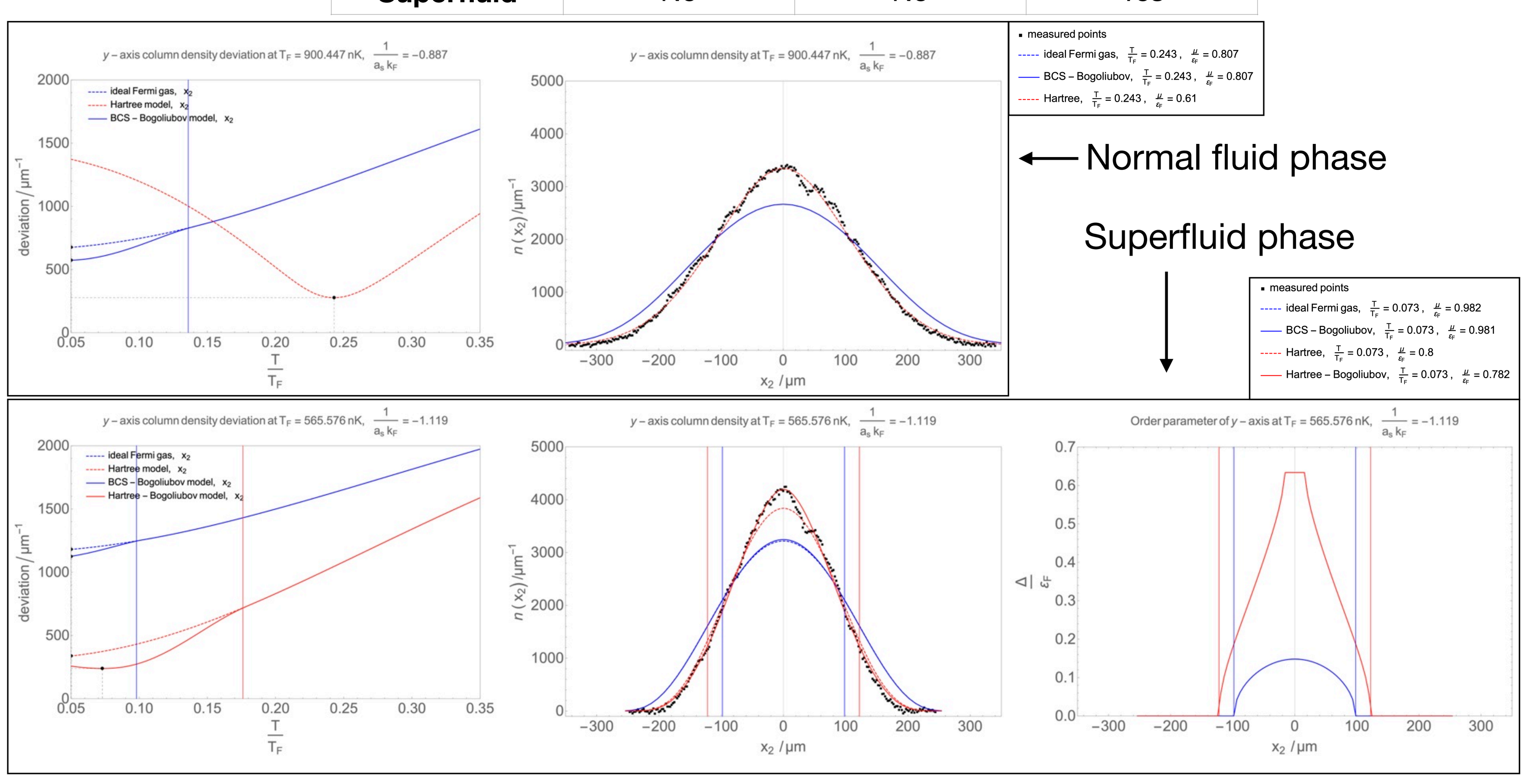
$$\frac{\Delta(x)}{g_r} = \frac{\Delta(x)}{2V}\sum_k \left[\frac{1}{\epsilon_k} - \frac{1}{E(k,x)}\tanh\left(\frac{\beta}{2}E(k,x)\right)\right]$$

Renormalized order parameter equation with LDA

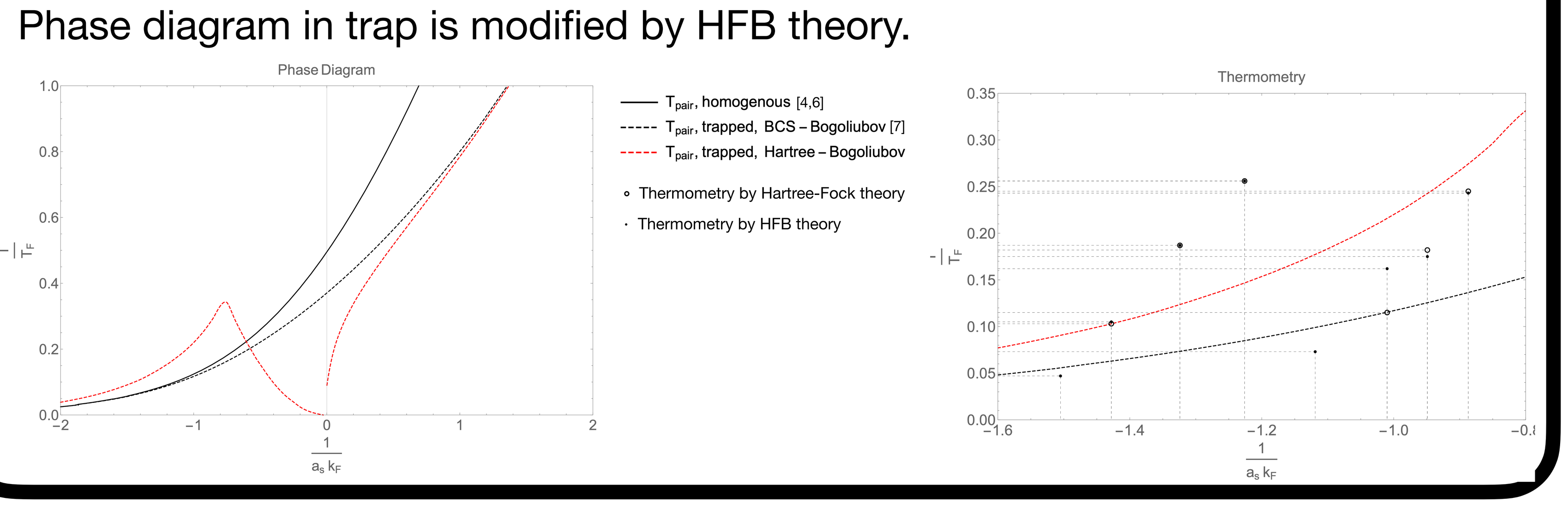
- Hartree-interaction $gn(x)/2$ gives interaction dependency to density distribution and modifies superfluid regime.
- $\mu[n(x)]$ is functional of Hartree-interaction dependent density distribution.
- Temperature is determined by minimizing deviation between measured- and calculated-density profiles at various T.
- No deviation minimum without Hartree-interaction: No thermometry

Thermometry Applicability

Phase\Theory	BCS	Hartree-Fock	HFB
Normal Fluid	No	Yes	No
Superfluid	No	No	Yes



- Phase diagram in trap is modified by HFB theory.



Conclusion

- MF theory is sufficient to directly measure the temperature of the weakly interacting Fermi gas in BCS-regime.
- Temperature and superfluidity can be directly and quantitatively determined by fitting MF-theories to measurement.
- Interaction effect is crucial both in normal fluid- and superfluid-phases.
- Closer to unitarity fluctuations get so important that MF theories fail.

Outlook

- Spin-imbalanced case
- Introduce effective range interaction and momentum cut-off } Poster by N. Kaschewski
- Beyond-MF HFB theory

References:

[1] C.A.R. Sá de Melo, Physics Today **61**(10), 45 (2008).
[2] B. Gänger, J. Phieler, B. Nagler, and A. Widera, Rev. Sci. Instr. **89**, 093105 (2018).
[3] C. A. Regal, M. Greiner, and D. S Jin, Phys. Rev. Lett., **92**(4), 040403 (2004).
[4] A. Becker, Master Thesis, TU Kaiserslautern (2021).
[5] G. Czycholl, *Solid State Theory*, Vol. 1 Springer (2023).
[6] C.A.R. Sá de Melo, M. Randeria, and J.R. Engelbrecht, Phys. Rev. Lett. **71**, 3202 (1993).
[7] A. Perali, P. Pieri, L. Pisani, and G.C. Strinati, Phys. Rev. Lett. **92**, 220404 (2004).

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