CRITICAL TEMPERATURE OF A BOSE-EINSTEIN CONDENSATE WITH $1/r$ INTERACTIONS

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We investigate a trapped Bose-Einstein condensate underlying besides a short-range contact interaction especially a long-range $1/r$ interaction. The latter one can artificially be created by the radiation field of a certain laser configuration via induced electric dipoles. For this system we calculate the leading shift of the critical temperature with respect to the ideal gas.

After its first experimental realization in 1995¹ the field of Bose-Einstein condensation (BEC) has experienced an enormous increase of interest both experimentally and theoretically. Although BEC is actually an atomic physics phase transition, it represents such a versatile and controllable many-particle system, that the research in BEC could be extended to several other fields of physics. In all early BEC experiments the atoms were alkali metals. So it was sufficient to model the van-der-Waals interaction in the gas by using the short-range pseudopotential

$$V_{\delta}^{(\text{int})}(\mathbf{r} - \mathbf{r}') = \frac{4\pi\hbar^2 a}{M} \delta(\mathbf{r} - \mathbf{r}'). \quad (1)$$

Its strength is characterized by the s-wave scattering length a . In 2005, the first dipolar BEC was realized in a gas of ^{52}Cr , where besides (1) also a magnetic dipole-dipole interaction² is present. An even more dramatic long-range interaction would be supplied by a $1/r$ potential

$$V_{1/r}^{(\text{int})}(\mathbf{r} - \mathbf{r}') = -\frac{4\pi^2\hbar^2}{Ma_G} \frac{1}{|\mathbf{r} - \mathbf{r}'|}. \quad (2)$$

Here, we basically follow an idea of Ref.³ to artificially create an attractive $1/r$ interaction due to a certain laser configuration which is similar to gravitation but up to 17 orders of magnitude stronger. The characteristic length scale a_G contains all information about the laser setup. As the interaction is attractive, a self-binding situation can occur where the attractive $1/r$ interaction balances both the kinetic energy and the contact interaction. Such tremendous attractive gravitational forces are only possible in nature on stellar scales. Thus, such ultracold quantum gases can be used to simulate cosmology in the laboratory and, in particular, to investigate the possibility of a Bose star which, so far, has only been discussed theoretically.⁴

An effective $1/r$ interaction results from the interaction of neutral atoms with a radiation field via induced electric dipoles. A quantum-electrodynamic fourth-order perturbative calculation leads to the interatomic contribution⁵

$$U(\mathbf{r}) = \frac{I}{\varepsilon_0 c} \hat{e}_i^{(\lambda)*}(\mathbf{k}) \hat{e}_j^{(\lambda)}(\mathbf{k}) \alpha^2(k) V_{ij}(\mathbf{r}, k) \cos(\mathbf{k} \cdot \mathbf{r}). \quad (3)$$

Here, $\alpha(k)$ denotes the polarizability, I the intensity of the radiation field, $\hat{e}_i^{(\lambda)*}(\mathbf{k})$, $\hat{e}_j^{(\lambda)}(\mathbf{k})$ are the polarization vectors, and $\lambda = -, +$ stands for right or left circular polarization. Furthermore, the retarded dipole-dipole interaction tensor $V_{ij}(\mathbf{r}, k)$ turns out to be

$$\frac{1}{4\pi\varepsilon_0 r^3} [(\delta_{ij} - 3\hat{r}_i\hat{r}_j)(\cos kr + kr \sin kr) - (\delta_{ij} - \hat{r}_i\hat{r}_j)k^2 r^2 \cos kr]. \quad (4)$$

In the near zone $kr \ll 1$ and a rotational average, Eq. (3) reduces to an attractive $1/r$ interaction.⁵

The first experimental proposal, how such an orientation average could be realized for ultracold gases, has been made in Ref.³ The simplest model that suppresses the $1/r^3$ part consists of three orthogonal, circularly polarized laser beams: $\mathbf{k}_1 = k\hat{\mathbf{e}}_x$, $\mathbf{k}_2 = k\hat{\mathbf{e}}_y$, $\mathbf{k}_3 = k\hat{\mathbf{e}}_z$. In the near zone $kr \ll 1$ we simply superpose the three terms to get the resulting potential

$$U_3(\mathbf{r}) = -\frac{3Ik^2\alpha^2}{16\pi c\varepsilon_0^2} \frac{1}{r} \left[\frac{7}{3} + (\sin\vartheta \cos\varphi)^4 + (\sin\vartheta \sin\varphi)^4 + (\cos\vartheta)^4 \right]. \quad (5)$$

The angles φ and ϑ describe the orientation of the atoms with respect to the incident beam. Although the perturbative derivation of the interaction is only valid for static perturbations, we suggest here as a second experimental proposal a setup of rotating lasers within a quasi-static frame, under the constraint $\omega_{\text{excitation}} \ll \omega_{\text{rot}} \ll \omega_{\text{laser}}$. Instead of a static wave vector $\mathbf{k}$ we now use the time-dependent one $\mathbf{k}(t) = k(\sin\gamma \cos\omega t, \sin\gamma \sin\omega t, \cos\gamma)$

with a free parameter γ for which the best choice turns out to be the magic angle $\gamma = \arccos \sqrt{1/3}$.^{6,8} Thus, the averaged potential reduces to

$$U_{1 \times \text{rot}}(\mathbf{r}) = -\frac{I\alpha^2 k^2}{96\pi c \varepsilon_0^2} \frac{1}{r} (17 + 6 \cos^2 \vartheta - 7 \cos^4 \vartheta). \quad (6)$$

The rotating setup does not depend on φ and is more spherical than (5). A totally spherical-symmetric $1/r$ potential is obtained either from 18 static lasers as suggested in Ref.³ or from three rotating lasers as proposed here.⁷

Using the functional-integral formalism of many-body physics, we apply Feynman's diagrammatic perturbation expansion to calculate the leading shift of the critical temperature for a Bose gas in an isotropic harmonic trap. As there occurs an infrared divergence in the exchange contribution, it is not possible to apply the usual semiclassical equal-time correlation function. Instead we have to use the full quantum-mechanical one $G^{(0)}(\mathbf{x}, 0; \mathbf{x}', 0)$:

$$\sum_{n=1}^{\infty} e^{n\beta(\mu - 3\hbar\omega/2)} \left[\frac{M\omega}{\pi\hbar(1 - e^{-2\hbar\beta\omega n})} \right]^{3/2} e^{-\frac{M\omega}{2\hbar \sinh \hbar\beta\omega n} [(\mathbf{x}^2 + \mathbf{x}'^2) \cosh \hbar\beta\omega n - 2\mathbf{x} \cdot \mathbf{x}']}. \quad (7)$$

Herewith we solve the Feynman diagrams of the free energy

$$\mathcal{F} = \mathcal{F}^{(0)} - \frac{1}{\beta} \left\{ \frac{1}{2} \text{ (diagram 1) } + \frac{1}{2} \text{ (diagram 2) } + \dots \right\} \quad (8)$$

in order to determine the particle number $N = -\partial\mathcal{F}/\partial\mu$ and the self-energy

$$\Sigma(\mathbf{x}, \tau; \mathbf{x}', \tau') = \text{ (diagram 1) } + \text{ (diagram 2) } + \dots, \quad (9)$$

which defines the critical chemical potential.⁸ Collecting the terms properly, we obtain the following result for the leading shift of the critical temperature caused by both interactions⁷

$$\frac{\Delta T_c}{T_c^{(0)}} = -c_\delta(N) \frac{a}{\lambda_{T_c^{(0)}}} + \frac{\lambda_{T_c^{(0)}}}{a_G} \left[c_E(N) + c_D(N) \frac{1}{(\hbar\beta_c^{(0)}\omega)^2} \right] \quad (10)$$

with the thermodynamic de Broglie wavelength $\lambda_{T_c^{(0)}} = \sqrt{2\pi\hbar^2\beta_c^{(0)}/M}$. The coefficients c_δ , c_D , c_E are shown as functions of the particle number in Fig. 1. In the semiclassical approximation we execute the limit $\hbar\beta_c^{(0)}\omega \ll 1$. For the contact interaction we obtain $c_\delta = 3.42603$ which is known from literature and has also been confirmed experimentally.⁹ The direct term becomes $c_D = 2.0951$ but for the exchange contribution c_E we find the

Fig. 1. 1(a)–1(c): Numerical evaluation of the coefficients in (10). 1(a), 1(b): the straight lines show the corresponding semiclassical results. Due to the divergence there is no one in the exchange case; here we added a fitting line $2.929 \cdot N^{2/9}$ that reproduces the values quite well and shows the behavior of the divergence. 1(d): the straight line comes from the result of a contact interaction only, $a_G \rightarrow \infty$, while the dashed lines give the complete shift (10) for different values of a_G ; short to long dashes: $a_G = 0.05, 0.075, 0.1$ m. Chosen values for all plots: mass of ^{87}Rb , $a = 3$ nm, and $\omega = 2\pi \cdot 100$ Hz.

divergence $\sum_{m=1}^{\infty} 1/\sqrt{m}$ which was avoided by our full quantum-mechanical calculation and renders $c_E(N)$ finite. For a realistic experimental situation the complete shift is of the size of about 5 – 10%, see Fig. 1(d), and should thus be measurable.

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