Evolution of BEC in Gravito-optical surface trap

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Presentation for PhD Enrollment

Outline

De Broglie (1929) proposed that all matter is composed of waves. Their wavelengths are given by

$$\lambda = \frac{h}{mv}$$

BEC against our Intuition

- In most everyday matter, the de Broglie wavelength is much shorter than the distance separating the atoms. In this case, the wave nature of atoms cannot be noticed, and they behave as particles.
- The wave nature of atoms becomes noticeable when the de Broglie wavelength is roughly the same as the atomic distance. This happens when the temperature is low enough, so that they have low velocities.
- In this case, the wave nature of atoms will be described by quantum physics.

Bose and Einstein

- In 1924 an Indian physicist named Bose studied the quantum behavior of a collection of photons. Bose sent his work to Einstein, who realized that it was important.
- Einstein generalized the idea to atoms, considering them as quantum particles with mass. Einstein found that when the temperature is high, they behave like ordinary gases. However, when the temperature is very low, they will gather together at the lowest quantum state. This is called Bose-Einstein condensation.

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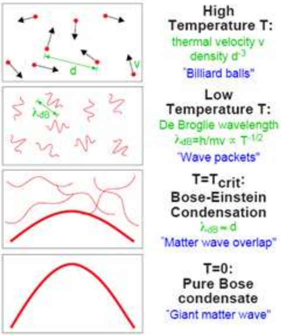

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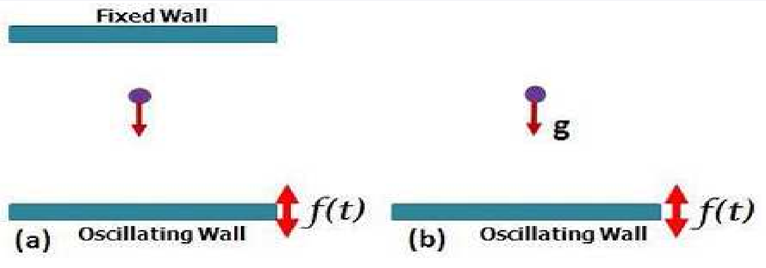
- Some atoms contain an even number of fermions. They have a total spin of integer number. They are called bosons.
- Bosons show strong social behavior, and can have BEC. Example: A $^{23}\mathrm{Na}$ atom has 11 protons, 12 neutrons and 11 electrons.

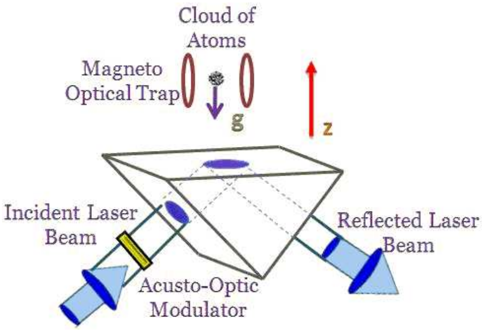
Enrico Fermi

Enrico Fermi provides a mechanism through which charged particles can be accelerated by collisions with moving magnetic field structures. [1]

(a) Fermi-Ulam Accelerator (1961)
(b) Fermi-Pustylnikov Accelerator (1995)

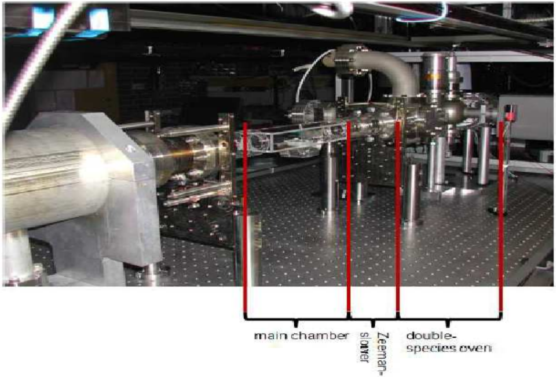


[1] E. Fermi Phys. Rev. **75**, 1169 (1949).



Experimental Model

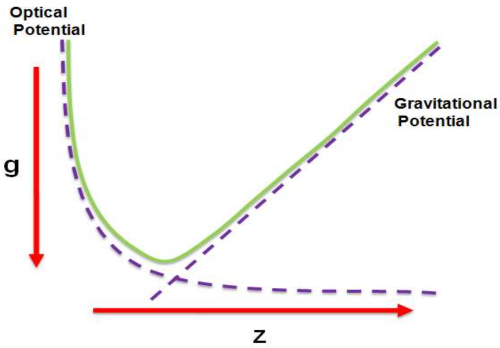
A cloud of atoms is trapped and cooled in a magneto-optical trap (MOT) to a few micro-Kelvin. An evanescent wave created by the total internal reflection of the incident laser beam from the surface of the dielectric serves as a mirror for the atoms.



Experimental Setup at the University of Innsbruck Austria

They used fused-silca prism (n=1.5), maximum potential height of $50\mu K$, and the decay length is $1.4\mu m$. The angle of incidence $\theta = 4.2mrad$ above the critical angle. [2]

[2] D. Rychtarik, et al., Phys. Rev. Lett. **92**, 17 (2004).



Gravitational Cavity

BEC starts its motion from an initial height. It moves under the influence of the linear gravitational potential towards the evanescent wave atomic mirror. Close to the surface of the mirror the effect of the evanescent light field is dominant and BEC experiences an exponential repulsive optical potential.

Gross-Pitaevskii (GP) Equation

The dynamics of a Bose-Einstein condensate at zero temperature is determined by the time dependent Gross-Pitaevskii (GP) equation [3]

$$\imath\hbar\frac{\partial}{\partial t}\Psi(z,t) = \left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + V(z,t) + G\|\Psi(z,t)\|^2\right\}\Psi(z,t)$$

Potential Energy and Strength of two particles

$$V(z,t) = V_0 e^{-\kappa z} + mgz$$

The last term represents the two-particle interaction of BEC atoms, where its strength $\boxed{G = \frac{4\pi\hbar^2 a}{m}}$ is related to the s-wave scattering length *a*.

[3] A. Griffin, et al., *Bose-Einstein Condensation* **1995**.

$$\Psi(z,t) = \sqrt{\frac{2n_0}{\sqrt{\pi}A(t)\left(1+\mathrm{Erf}\left(\frac{z_0(t)}{A(t)}\right)\right)}}\exp\left\{-\frac{(z-z_0(t))^2}{2A(t)^2} -\iota R(t)\,(z-z_0(t))^2 - \iota\alpha(t)\,(z-z_0(t))\right\}$$

Gaussian trial Function

Here $z_0(t)$ is the mean height of BEC from oscillating surface, $A(t)$ is the width of the BEC, n_0 represents the number of atoms. $R(t)$ and $\alpha(t)$ are the variational parameters. Here

$$\mathrm{Erf}\left(\frac{z_0(t)}{A(t)}\right) = \frac{2}{\sqrt{\pi}}\int_0^{\frac{z_0(t)}{A(t)}} e^{-x^2}dx$$

Lagrange density

$$\mathcal{L} = \imath\hbar\Psi^*(z,t)\frac{\partial\Psi(z,t)}{\partial t} - \frac{\hbar^2}{2m}\frac{\partial\Psi^*(z,t)}{\partial z}\frac{\partial\Psi(z,t)}{\partial z} - V(z,t)\Psi^*(z,t)\Psi(z,t) - \frac{G}{2}\Psi^*(z,t)^2\Psi(z,t)^2$$

$$L = \int_0^\infty dz\,\mathcal{L}$$

Dimensionless Lagrangian

$$\tilde{L} = -\frac{n_0}{4\left(1+\mathrm{Erf}\left(\frac{\tilde{z}_0}{\tilde{A}}\right)\right)^2}\left(\tilde{L}_{int} + \tilde{L}_{pot} + \tilde{L}_{kin} + \tilde{L}_{time}\right)$$

Dimensionless Parameters

- I measure energies in units of the gravitational energy mg/κ. Dimensionless Lagrange $\tilde{L} = \frac{L\kappa}{gm}$.

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- $\tilde{R} = \frac{R(t)}{\kappa^2}$ and $\tilde{\alpha} = \frac{\alpha(t)}{\kappa}$ as dimensionless variational parameters.

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- $\tilde{\omega} = \frac{\hbar\kappa}{gm}\omega$ as a dimensionless frequency.
- $\tilde{V}_0 = \frac{\kappa V_0}{gm}$ as a dimensionless strength of the evanescent field, and $\tilde{k} = \frac{\hbar^2\kappa^3}{gm^2}$ as a dimensionless kinetic energy.

Interaction Lagrange Function

$$\tilde{L}_{int} = 8\sqrt{2\pi}\frac{\left(1+\mathrm{Erf}\left(\frac{\sqrt{2}\tilde{z}_0}{\tilde{A}}\right)\right)n_0}{\tilde{A}}\,\tilde{k}\tilde{a}$$

Potential Energy Lagrange Function

$$\begin{aligned}\tilde{L}_{pot} &= 4\left(1+\mathrm{Erf}\left(\frac{\tilde{z}_0}{\tilde{A}}\right)\right)\left\{\tilde{V}_0 e^{\frac{\tilde{A}^2}{4}-\tilde{z}_0}\mathrm{Erfc}\left(\frac{\tilde{A}}{2}-\frac{\tilde{z}_0}{\tilde{A}}\right)\right.\\ &\quad\left.+\left(1+\mathrm{Erf}\left(\frac{\tilde{z}_0}{\tilde{A}}\right)\right)\tilde{z}_0+\frac{\tilde{A}e^{-\frac{\tilde{z}_0^2}{\tilde{A}^2}}}{\sqrt{\pi}}\right\}\end{aligned}$$

here

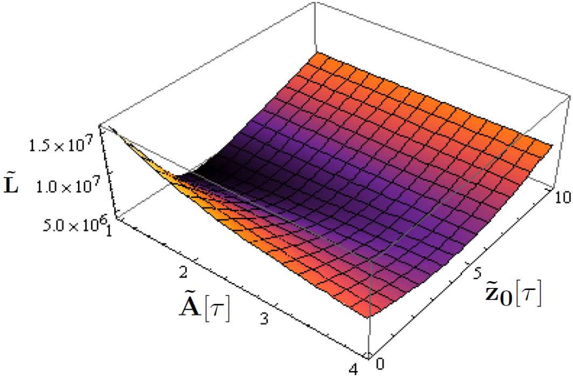
$$\mathrm{Erfc}\left(\frac{\tilde{z}_0}{\tilde{A}}\right) = 1-\mathrm{Erf}\left(\frac{\tilde{z}_0}{\tilde{A}}\right)$$

Kinetic Energy Lagrange Function

$$
\begin{aligned}
\tilde{L}_{kin} \quad = \quad & \frac{\tilde{k}\left(1+\operatorname{Erf}\left(\frac{\tilde{z}_0}{\tilde{A}}\right)\right)}{\sqrt{\pi}\tilde{A}^3}\Big\{\sqrt{\pi}\tilde{A}+4\sqrt{\pi}\tilde{A}^5\tilde{R}^2+2\sqrt{\pi}\tilde{A}^3\tilde{\alpha}^2 \\
& -2e^{-\frac{\tilde{z}_0^2}{\tilde{A}^2}}\left(\tilde{z}_0+4\tilde{A}^4\tilde{R}^2\tilde{z}_0-4\tilde{A}^4\tilde{R}\tilde{\alpha}\right) \\
& +\sqrt{\pi}\tilde{A}\operatorname{Erf}\left(\frac{\tilde{z}_0}{\tilde{A}}\right)\left(1+4\tilde{A}^4\tilde{R}^2+2\tilde{A}^2\tilde{\alpha}^2\right)\Big\}
\end{aligned}
$$

Time Lagrange Function

$$\tilde{L}_{time} = 2\tilde{\omega}\left(1+\mathrm{Erf}\left(\frac{\tilde{z}_0}{\tilde{A}}\right)\right)\left\{\left(-2+\mathrm{Erfc}\left(\frac{\tilde{z}_0}{\tilde{A}}\right)\right)\right.$$
$$\left.\times\left(\tilde{A}^2\tilde{R}'-2\tilde{\alpha}\tilde{z}_0'\right)+\frac{2e^{-\frac{\tilde{z}_0^2}{\tilde{A}^2}}\tilde{A}\left(\tilde{z}_0\tilde{R}'+2\tilde{R}\tilde{z}_0'-\tilde{\alpha}'\right)}{\sqrt{\pi}}\right\}$$



Time Independent Lagrange

Dependence of the dimensionless Lagrange function on the dimensionless width of the BEC wave packet $\tilde{A}$ and the dimensionless mean height of the BEC $\tilde{z}_0$ for the static case $\tilde{R}(\tau) = \tilde{\alpha}(\tau) = 0$.

Euler-Lagrange Equations

$$\frac{\partial \tilde{L}}{\partial \tilde{A}} - \frac{d}{d\tau}\frac{\partial \tilde{L}}{\partial \tilde{A}'} = 0$$

$$\frac{\partial \tilde{L}}{\partial \tilde{R}} - \frac{d}{d\tau}\frac{\partial \tilde{L}}{\partial \tilde{R}'} = 0$$

$$\frac{\partial \tilde{L}}{\partial \tilde{\alpha}} - \frac{d}{d\tau}\frac{\partial \tilde{L}}{\partial \tilde{\alpha}'} = 0$$

$$\frac{\partial \tilde{L}}{\partial \tilde{z}_0} - \frac{d}{d\tau}\frac{\partial \tilde{L}}{\partial \tilde{z}_0'} = 0$$

Euler Lagrange Equations Results

Click Here

Physical Approximation and approximated Lagrange Function

We conclude from our experimental setup that $\dfrac{\tilde{z}_0}{\tilde{A}} \gg 1$,

then we can furthermore use $\mathrm{Erf}\left(\dfrac{\tilde{z}_0}{\tilde{A}}\right) \simeq 1$.

Approximated Lagrange Function

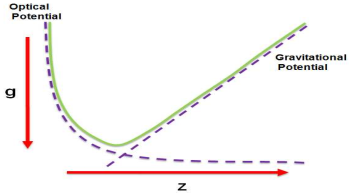
$$\begin{aligned}\tilde{L} \;\simeq\;& -\frac{n_0}{16}\Bigg\{16\sqrt{2\pi}\frac{n_0}{\tilde{A}}\,\tilde{k}\tilde{a} + 8\left(2e^{\frac{1}{4}\tilde{A}^2-\tilde{z}_0}\tilde{V}_0 + 2\tilde{z}_0\right)\\ &+8\tilde{\omega}\left(\tilde{A}^2\tilde{R}' - 2\tilde{\alpha}\tilde{z}_0'\right)\\ &+\frac{4\tilde{k}}{\sqrt{\pi}\tilde{A}^3}\left(\sqrt{\pi}\tilde{A} + 4\sqrt{\pi}\tilde{A}^5\tilde{R}^2 + 2\sqrt{\pi}\tilde{A}^3\tilde{\alpha}^2\right)\Bigg\}\end{aligned}$$

Euler-Lagrange equation for width and height of BEC

$$\tilde{A}'' + \tilde{A}\tilde{V}_0 e^{\frac{1}{4}\tilde{A}^2 - \tilde{z}_0} = \frac{\tilde{k}}{\tilde{A}^3} + 2\sqrt{2\pi}\frac{\tilde{k}\tilde{a}}{\tilde{A}^2} n_0$$

$$\tilde{z}_0'' + 1 = \tilde{V}_0 e^{\frac{1}{4}\tilde{A}^2 - \tilde{z}_0}$$

Note that above equations have been simplified by choosing $\tilde{\omega} = \sqrt{\tilde{k}}$, i.e. we have $\omega = \sqrt{g\kappa}$ for the frequency for the time scale. The frequency ω coincides with the classical frequency when the gravitational potential is harmonically approximated around its minimum.



Euler-Lagrange equation for width and height of BEC

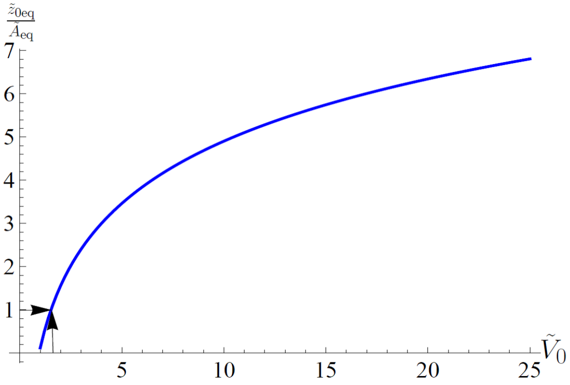
$$\begin{aligned} 2\frac{\tilde{k}}{\tilde{A}_{\mathrm{eq}}} + 4\sqrt{2\pi}\tilde{k}\tilde{a}n_0 - 2\tilde{A}_{\mathrm{eq}}^3 \tilde{V}_0 e^{\frac{1}{4}\tilde{A}_{\mathrm{eq}}^2 - \tilde{z}_{0\mathrm{eq}}} &= 0 \\ -1 + \tilde{V}_0 e^{\frac{1}{4}\tilde{A}_{\mathrm{eq}}^2 - \tilde{z}_{0\mathrm{eq}}} &= 0 \end{aligned}$$

Dimensionless width and height of BEC

$$\begin{aligned} \tilde{A}_{-\mathrm{eq}} &= \sqrt{\tilde{a}\tilde{k}\sqrt{2\pi}n_0 - \sqrt{\tilde{k} + 2\pi\tilde{a}^2\tilde{k}^2 n_0^2}} \quad \text{Unphysical} \\ \tilde{A}_{+\mathrm{eq}} &= \sqrt{\tilde{a}\tilde{k}\sqrt{2\pi}n_0 + \sqrt{\tilde{k} + 2\pi\tilde{a}^2\tilde{k}^2 n_0^2}} \end{aligned}$$

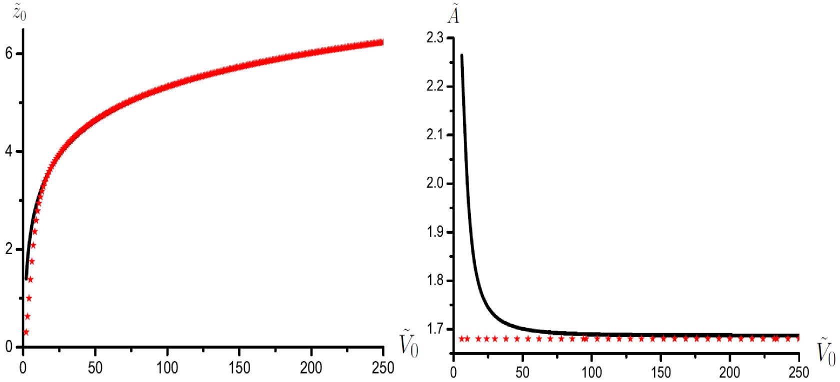
and

$$\tilde{z}_{0\mathrm{eq}} = \frac{1}{4}\tilde{A}_{+\mathrm{eq}} + \ln \tilde{V}_0$$



Aspect ratio of the equilibrium

In order to make our proposed model experimentally realizable i.e. for $\tilde{z}_{0\text{eq}} \gg \tilde{A}_{\text{eq}}$, we need $\tilde{V}_0 \gg 3$.



Equilibrium Values

Equilibrium values for (a) mean height $\tilde{z}_0$ and (b) width $\tilde{A}$ of the BEC. Exact values (solid lines) compared with the approximations (dashed lines).

Dynamics for small deflections around equilibrium positions

Let us assume for the width of the BEC $\tilde{A} = \tilde{A}_{\text{eq}} + \delta\tilde{A}$

and for the mean position of the BEC $\tilde{z}_0 = \tilde{z}_{0\text{eq}} + \delta\tilde{z}_0$.

Dynamic equations for width and height of BEC

$$\begin{aligned}\delta\tilde{A}'' &= -3\frac{\tilde{k}\,\delta\tilde{A}}{\tilde{A}_{\text{eq}}^4} - 4\sqrt{2\pi}\frac{\tilde{k}\tilde{a}n_0\delta\tilde{A}}{\tilde{A}_{\text{eq}}^3} - e^{\frac{\tilde{A}_{\text{eq}}^2}{4} - \tilde{z}_{0\text{eq}}}\tilde{V}_0 \\ &\quad \times \left(\frac{1}{2}\tilde{A}_{\text{eq}}^2\delta\tilde{A} - \delta\tilde{z}_0\tilde{A}_{\text{eq}}\right) - e^{\frac{1}{4}\tilde{A}_{\text{eq}}^2 - \tilde{z}_{0\text{eq}}}\tilde{V}_0\delta\tilde{A}\end{aligned}$$

and

$$\delta\tilde{z}_0'' - \frac{1}{2}\tilde{A}_{\text{eq}}\delta\tilde{A} + \delta\tilde{z}_0 \simeq 0$$

Deflections around equilibrium points describe dynamics

$$\begin{pmatrix} \delta\tilde{z}_0'' \\ \delta\tilde{A}'' \end{pmatrix} + M \begin{pmatrix} \delta\tilde{z}_0 \\ \delta\tilde{A} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

where M is

$$M = \begin{pmatrix} 1 & -\frac{1}{2}\tilde{A}_{\mathrm{eq}} \\ -\tilde{A}_{\mathrm{eq}} & \left(1 + 3\frac{\tilde{k}}{\tilde{A}_{\mathrm{eq}}^4} + 4\sqrt{2\pi}\frac{\tilde{k}\tilde{a}n_0}{\tilde{A}_{\mathrm{eq}}^3} + \frac{\tilde{A}_{\mathrm{eq}}^2}{2}\right) \end{pmatrix}$$

Characteristic equation

To determine the eigenvalues of the matrix, we find the solution of the characteristic equation $\boxed{\det(M - \zeta I) = 0}$ which is

$$\zeta^2 - S\zeta + T = 0$$

$$\zeta_\pm = \frac{S \pm \sqrt{S^2 - 4T}}{2}$$

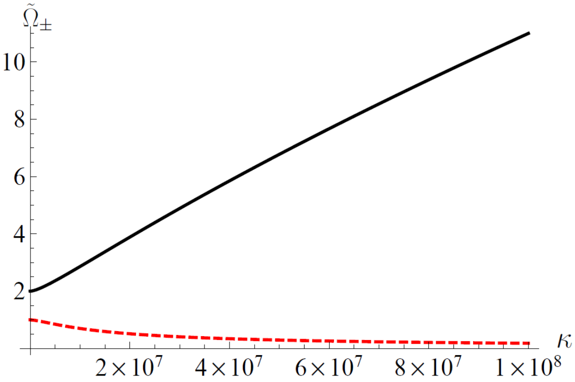
where we introduce the abbreviations

$$S = 2 + 3\frac{\tilde{k}}{\tilde{A}_{\text{eq}}^4} + 4\sqrt{2\pi}\frac{\tilde{k}\tilde{a}n_0}{\tilde{A}_{\text{eq}}^3} + \frac{\tilde{A}_{\text{eq}}^2}{2}$$

$$T = 1 + 3\frac{\tilde{k}}{\tilde{A}_{\text{eq}}^4} + 4\sqrt{2\pi}\frac{\tilde{k}\tilde{a}n_0}{\tilde{A}_{\text{eq}}^3}$$

Collective excitation frequencies

The eigenvalues $\zeta_\pm$ correspond to oscillation frequencies $\tilde{\Omega}_\pm$ of the condensate according to $\tilde{\Omega}_\pm = \sqrt{\zeta_\pm}$.
Thus, the collective excitation frequencies of the Bose-Einstein condensate amount to $\Omega_\pm = \omega\tilde{\Omega}_\pm$ and, due to $\omega = \sqrt{g\kappa} = 3.1305 \times 10^3 \mathrm{s}^{-1}$, they are of the order of kHz.



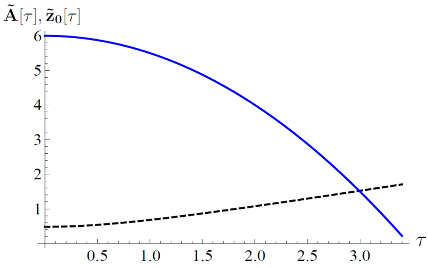
Oscillation frequencies $\tilde{\Omega}_\pm$ of the condensate

Black (Red) line represents the eigenvalue $\tilde{\Omega}_+$ ($\tilde{\Omega}_-$) versus κ, because frequencies are not depending upon $\tilde{V}_0$.

$$\mathbf{X}_\pm = \left\{ \frac{1}{4\tilde{A}_{\mathrm{eq}}^5}\left(\pm\sqrt{\frac{\tilde{A}_{\mathrm{eq}}^6 + 6\tilde{k} + 8\tilde{a}\tilde{A}_{\mathrm{eq}}\tilde{k}n_0}{\begin{array}{c} 8\tilde{A}_{\mathrm{eq}}^{10} + \tilde{A}_{\mathrm{eq}}^{12} + 12\tilde{A}_{\mathrm{eq}}^6\tilde{k} + 36\tilde{k}^2 \\ +16\tilde{a}\tilde{A}_{\mathrm{eq}}\tilde{k}n_0\left(\tilde{A}_{\mathrm{eq}}^6 + 6\tilde{k} + 4\tilde{a}\tilde{A}_{\mathrm{eq}}\tilde{k}n_0\right)\end{array}}} \right), 1 \right\}$$

Eigenvectors

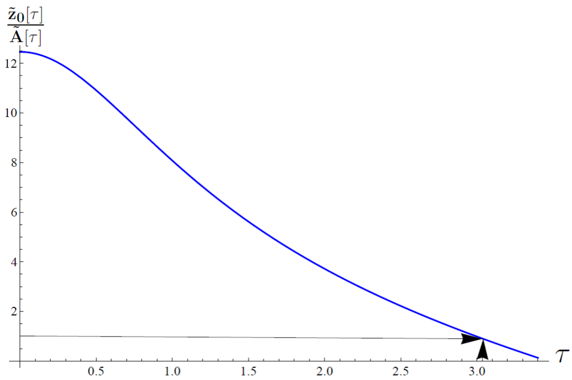
The two eigenvectors represent the two modes. The first mode $\mathbf{X}_+(\mathbf{X}_-)$ has a positive (negative) sign which means that height and width are in(out) phase.



Time of Flight Expansion

Here dashed black line represents the width of the condensate and blue solid line represents the mean height of the BEC for a decay length $\frac{1}{\kappa} = 10^{-6}\mathrm{m}$.

$$\begin{aligned} \tilde{A}'' &= \frac{\tilde{k}}{\tilde{A}^3} + 2\sqrt{2\pi}\frac{\tilde{k}\tilde{a}}{\tilde{A}^2}n_0 \\ \tilde{z}_0'' &= -1 \end{aligned}$$



Aspect ratio of mean height and mean width of the BEC

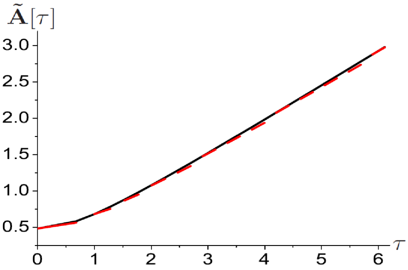
We read off that our approximation $\frac{\tilde{z}_0}{\tilde{A}} \gg 1$ for large value of $\tilde{V}_0$ is valid up to the expansion time $t = \frac{\tau}{\omega} = 0.99\mathrm{ms}$.

Exact solution of dynamic equation for width of BEC

$$\int_0^{\tau} dt' = \int_{\tilde{A}_{\mathrm{eq}}}^{\tilde{A}} \frac{\tilde{A}_{\mathrm{eq}}\tilde{A}(t')d\tilde{A}(t')}{\sqrt{b\tilde{A}(t')^2 + c\tilde{A}(t') + d\tilde{k}\tilde{A}_{\mathrm{eq}}^2}}$$

here

$$\begin{aligned} Y(t') &= b\tilde{A}(t')^2 + c\tilde{A}(t') + d \\ b &= \tilde{k}(1 + 4\sqrt{2\pi}n_0\tilde{a}\tilde{A}_{\mathrm{eq}}) \\ c &= -4\sqrt{2\pi}n_0\tilde{k}\tilde{a}\tilde{A}_{\mathrm{eq}}^2 \quad \text{and} \quad d = -\tilde{k}\tilde{A}_{\mathrm{eq}}^2 \end{aligned}$$



Exact value of Time of Flight expansion

Here numerical value (black line) compared with the exact value (dashed red line)

$$\begin{aligned}\tau &= \frac{\tilde{A}_{\mathrm{eq}}}{b}\left(\sqrt{Y}-\sqrt{b\tilde{A}_{\mathrm{eq}}^2+c\tilde{A}_{\mathrm{eq}}+d}\right)-\frac{c\tilde{A}_{\mathrm{eq}}}{2b^2}\Big\{\ln\Big[2\sqrt{bY}\\ &\quad +2b\tilde{A}+c\Big]-\ln\Big[2\sqrt{b\tilde{A}_{\mathrm{eq}}^2+c\tilde{A}_{\mathrm{eq}}+d}+2b\tilde{A}_{\mathrm{eq}}+c\Big]\Big\}\end{aligned}$$

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- Planck's constant reads $\hbar = 1.054 \times 10^{-34}\ \mathrm{J/s}$, and the gravitational acceleration is $g = 9.8\ \mathrm{ms}^{-2}$.

Experimental Values

- We consider the number $n_0 = 10^6$ of ^{87}Rb atoms,
- One ^{87}Rb atom has the mass $m = 87 \times 1.67 \times 10^{-27}\ \mathrm{kg}$,
- The s-wave scattering length of ^{87}Rb atoms amounts to $a = 90 \times 0.0529 \times 10^{-9}\ \mathrm{m}$,
- Planck's constant reads $\hbar = 1.054 \times 10^{-34}\ \mathrm{J/s}$, and the gravitational acceleration is $g = 9.8\ \mathrm{ms}^{-2}$.
- The evanescent field is assumed to be determined by the inverse decay length $\kappa = 10^6\ \mathrm{m}^{-1}$ and the strength $V_0 = 3.132 \times 10^{-29}\ \mathrm{J}$.

Future work

- I will extend my problem to see the acceleration of BEC in the gravitational cavity. To study the acceleration of the BEC, I will model exponential decaying potential with

$$V(z,t) = V_0 e^{\lambda \sin(\mu t) - \kappa z}$$

Future work

- I will do research work on the topic “How does the stability diagram change with $\tilde{P}(t) = \tilde{k}\tilde{a}(t)$ in dynamic equation $\tilde{A}'' = \frac{\tilde{k}}{\tilde{A}^3} + 2\sqrt{2\pi}\frac{\tilde{P}(t)}{\tilde{A}^2}n_0$ ”.

Question and answer session

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- ***Thanks for your attention***