Hybrid systems	Dimensional reduction from 3D to 1D	Static results 00000000000	Time of flight	Outlook

Analytical and Numerical Study of Localized Impurity in Bose-Einstein Condensate

Javed Akram

Department of Physics Free University of Berlin, Germany

New year seminar, 2015

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- Dimensional reduction from 3D to 1D
 - Derivation of Lagrangian from 3D to 1D
 - Gross-Pitaevskii equation
 - Dimensionless Gross-Pitaevskii equation

Static results

- Thomas-Fermi variational ansatz
- Numerical density profile
- Impurity height/depth and width

4 Time of flight

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Dimensional reduction from 3D to 1D 000000000

Outlook

• First Experiment: Coupling a single-electron/Rydberg atom to a Bose-Einstein condensate,



J.B. Balewski, et al., Nature **502**, 664 (2013).



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• Second Experiment: Coupling a single-ion (Ytterbium-Yb⁺) to a Bose-Einstein condensate,





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 Third experiment: Coupling a single-Cs atom to a Bose-Einstein condensate,



N. Spethmann, et al., Phys. Rev. Lett. 109, 235301 (2012).



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• This talk-outlook





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Derivation of Lagrangi	an from 3D to 1D			

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Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook
	00000000			
Derivation of Lagrangian from 3D to 1D				

3D Lagrangian

$$\begin{split} \mathcal{L}_{3D} = & \frac{i\hbar}{2} \left(\psi^{\star}\left(\mathbf{r},t\right) \frac{\partial\psi\left(\mathbf{r},t\right)}{\partial t} - \psi\left(\mathbf{r},t\right) \frac{\partial\psi^{\star}\left(\mathbf{r},t\right)}{\partial t} \right) \\ &+ \frac{\hbar^{2}}{2m_{\mathrm{B}}} \psi^{\star}\left(\mathbf{r},t\right) \bigtriangleup \psi\left(\mathbf{r},t\right) - V\left(\mathbf{r}\right) \psi^{\star}\left(\mathbf{r},t\right) \psi\left(\mathbf{r},t\right) \\ &- \frac{G_{\mathrm{B}}^{\mathrm{3D}}}{2} \parallel \psi\left(\mathbf{r},t\right) \parallel^{4} - G_{\mathrm{IB}}^{\mathrm{3D}} \parallel \psi_{\mathrm{I}}\left(\mathbf{r},t\right) \parallel^{2} \parallel \psi\left(\mathbf{r},t\right) \parallel^{2} \end{split}$$

•
$$G_{\rm B}^{\rm 3D} = \frac{N4\pi\hbar^2 a_{\rm B}}{m_{\rm B}}$$

• $G_{\rm IB}^{\rm 3D} = \frac{2\pi\hbar^2 a_{\rm IB}}{m_{\rm IB}}$ and $m_{\rm IB} = \frac{m_{\rm I}m_{\rm B}}{m_{\rm I}+m_{\rm B}}$



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3D Lagrangian

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Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook
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Derivation of Lagrangian from 3D to 1D				

•
$$\omega_z \ll \omega_r$$
 and $V(\mathbf{r}) = \frac{m_B \omega_z^2 z^2}{2} + \frac{m_B \omega_r^2 (x^2 + y^2)}{2}$
• $\psi(\mathbf{r}, t) = \psi(z, t)\phi(r_{\perp}, t)$

Wave-functions

$$\begin{split} \phi(r_{\perp},t) &= \frac{e^{-\frac{x^2+y^2}{2l_r^2}}}{\sqrt{\pi}l_r} e^{-i\omega_r t} \\ \phi_{\rm I}(r_{\perp},t) &= \frac{e^{-\frac{x^2+y^2}{2l_r^2}}}{\sqrt{\pi}l_{\rm rI}} e^{-i\omega_{\rm rI} t} \end{split}$$

•
$$l_{\rm r} = \sqrt{\hbar/m_{\rm B}\omega_{\rm r}}$$
 and $l_{\rm rl} = \sqrt{\hbar/m_{\rm IB}\omega_{\rm rl}}$



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Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook
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$$\phi_{\rm I}(r_{\perp}, t) = \frac{e^{-\frac{x^2+y^2}{2l_r^2}}}{\sqrt{\pi}l_{\rm rI}}e^{-i\omega_{\rm rI} t}$$

• $l_{\rm r} = \sqrt{\hbar/m_{\rm B}\omega_{\rm r}}$ and $l_{\rm rl} = \sqrt{\hbar/m_{\rm IB}\omega_{\rm rl}}$



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Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook
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Derivation of Lagrangian from 3D to 1D				

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$$\omega_z \ll \omega_r$$
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• $\psi(\mathbf{r}, t) = \psi(z, t)\phi(r_{\perp}, t)$

Wave-functions

$$\phi(r_{\perp}, t) = \frac{e^{-\frac{x^2 + y^2}{2l_r^2}}}{\sqrt{\pi}l_r} e^{-i\omega_r t}$$
$$\phi_{\rm I}(r_{\perp}, t) = \frac{e^{-\frac{x^2 + y^2}{2l_{\rm rl}^2}}}{\sqrt{\pi}l_{\rm rl}} e^{-i\omega_{\rm rl} t}$$

•
$$I_{\rm r} = \sqrt{\hbar/m_{\rm B}\omega_{\rm r}}$$
 and $I_{\rm rl} = \sqrt{\hbar/m_{\rm IB}\omega_{\rm rl}}$

		00000000000	Time of hight	Outiool
Derivation of Lagrar	ngian from 3D to 1D			
1D Lag	rangian			
	$\mathcal{L}_{1D} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty}$	$\int_{\infty}^{\infty} \mathcal{L}_{3D} dx dy.$		
\mathcal{L}_{1L}	$D = \frac{i\hbar}{2} \left(\psi^{\star}(z,t) \frac{\partial \psi(z,t)}{\partial t} + \frac{\hbar^2}{2m_0} \psi^{\star}(z,t) \frac{\partial^2 \psi(z,t)}{\partial z^2} \right)$	$-\psi(z,t)\frac{\partial\psi^{\star}}{\partial t}$	$\frac{(z,t)}{\partial t}$ $(z,t) \psi(z,t)$	
	$-\frac{G_{\rm B}}{2} \parallel \psi(z,t) \parallel^4 -G_{\rm B}$	$_{B}\parallel\psi_{I}\left(z,t ight)\parallel^{2}$	$\left\ \psi\left(z,t ight) ight\ ^{2}$	
• G _B	$s = 2N_{\rm B}a_{\rm B}\hbar\omega_{\rm r}$			
• G _{IE}	$a_{\rm B} = \frac{2\pi^2 a_{\rm B}}{m_{\rm B}(l_r^2 + l_{\rm rl}^2)}$, how we can	increase or dec	crease G _{IB} (nex	xt

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Deriv	ation of Lagrangia	an from 3D to 1D			
	1D Lagra	angian			
		$\mathcal{L}_{1D} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty}$	$\int_{\infty}^{\infty} \mathcal{L}_{3D} dx dy.$		
	\mathcal{L}_{1D}	$=\frac{i\hbar}{2}\left(\psi^{\star}\left(z,t\right)\frac{\partial\psi\left(z,t\right)}{\partial t}-\right.$	$-\psi(z,t)\frac{\partial\psi^{*}(z)}{\partial t}$	$\left(\frac{z,t}{t}\right)$	L
		$+\frac{\hbar^2}{2m_{\rm B}}\psi^{\star}(z,t)\frac{\partial^2\psi(z,t)}{\partial z^2}$	$\left(t ight) -V\left(z ight) \psi ^{\star }$	$(z,t)\psi(z,t)$	L
		$-\frac{G_{B}}{2} \parallel \psi(z,t) \parallel^{4} - G_{I}$	$\mathbf{B} \parallel \psi_{I}(z,t) \parallel^2$	$\parallel \psi(z,t) \parallel^2$	
	• G _B =	$= 2N_{\rm B}a_{\rm B}\hbar\omega_{\rm r}$			1
	• G _{IB}	$= \frac{2\hbar^2 a_{\text{IB}}}{m_{\text{IB}}(l_r^2 + l_{\text{rl}}^2)}, \text{ how we can}$	increase or dec	rease G _{IB} (next	
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3D to 1D

Dimensional reduction from





• In our calculation we are using $\omega_{
m lr}/\omega_{
m r}=1$







• In our calculation we are using $\omega_{\rm lr}/\omega_{\rm r}=1$



Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook
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Gross-Pitaevskii equatic	n			

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Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook
	00000 0 0000			
Gross-Pitaevskii equation				

Euler-Lagrangian equation

$$\frac{\delta \mathcal{A}\left[\psi^{\star},\psi\right]}{\delta \psi^{\star}\left(z,t\right)} = \frac{\partial \mathcal{L}_{1D}}{\partial \psi^{\star}\left(z,t\right)} - \frac{\partial}{\partial z} \frac{\partial \mathcal{L}_{1D}}{\partial \frac{\partial \psi^{\star}\left(z,t\right)}{\partial z}} - \frac{\partial}{\partial t} \frac{\partial \mathcal{L}_{1D}}{\partial \frac{\partial \psi^{\star}\left(z,t\right)}{\partial t}} = 0$$

1D Gross-Pitaevskii equation (1DGPE)

$$i\hbar\frac{\partial}{\partial t}\psi(z,t) = \left\{-\frac{\hbar^2}{2m_B}\frac{\partial^2}{\partial z^2} + \frac{m_B\omega_z^2 z^2}{2} + G_{\rm IB} \parallel \psi_{\rm I}(z,t) \parallel^2 + G_{\rm B}|\psi(z,t)|^2\right\}\psi(z,t)$$



Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook
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Gross-Pitaevskii equation				

Euler-Lagrangian equation

$$\frac{\delta \mathcal{A}\left[\psi^{\star},\psi\right]}{\delta \psi^{\star}\left(z,t\right)} = \frac{\partial \mathcal{L}_{1D}}{\partial \psi^{\star}\left(z,t\right)} - \frac{\partial}{\partial z} \frac{\partial \mathcal{L}_{1D}}{\partial \frac{\partial \psi^{\star}\left(z,t\right)}{\partial z}} - \frac{\partial}{\partial t} \frac{\partial \mathcal{L}_{1D}}{\partial \frac{\partial \psi^{\star}\left(z,t\right)}{\partial t}} = 0$$

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$$\begin{split} i\hbar\frac{\partial}{\partial t}\psi(z,t) = \\ \left\{-\frac{\hbar^2}{2m_B}\frac{\partial^2}{\partial z^2} + \frac{m_B\omega_z^2 z^2}{2} + G_{\text{IB}} \parallel \psi_{\text{I}}(z,t) \parallel^2 + G_{\text{B}}|\psi(z,t)|^2\right\}\psi(z,t) \end{split}$$



Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook
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Dimensionless Gross-Pi	taevskii equation			

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Dimensionless Gross-Pi	taevskii equation			

1D Dimensionless Gross-Pitaevskii equation

$$i\frac{\partial}{\partial\tau}\tilde{\psi}\left(\tilde{z}\right) = \left\{-\frac{1}{2}\frac{\partial^{2}}{\partial\tilde{z}^{2}} + \frac{\tilde{z}^{2}}{2} + \tilde{G}_{\mathsf{IB}}|\tilde{\psi}_{\mathsf{I}}\left(\tilde{z}\right)|^{2} + \tilde{G}_{\mathsf{B}}|\tilde{\psi}\left(\tilde{z}\right)|^{2}\right\}\tilde{\psi}\left(\tilde{z}\right)$$

Dimensionless parameters

•
$$\tau = \omega_z t$$
, $\tilde{z} = z/I_z$ and $\tilde{\psi} = \psi/\sqrt{I_z}$

•
$$\tilde{G}_{\rm B} = 2N\omega_{\rm r}a_{\rm B}/\omega_{\rm z}l_{\rm z}$$

•
$$\tilde{G}_{\rm IB} = 2a_{\rm IB}\omega_{\rm r}f\left(\omega_{\rm Ir}/\omega_{\rm r}\right)/\omega_{\rm z}l_{\rm z}$$

- where $I_z = \sqrt{\hbar/m_B\omega_z}$ is the oscillator length
- From here on, we will drop the tildes for simplicity.



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Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook
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Dimensionless Gross-Pi	taevskii equation			

1D Dimensionless Gross-Pitaevskii equation

$$i\frac{\partial}{\partial\tau}\tilde{\psi}\left(\tilde{z}\right) = \left\{-\frac{1}{2}\frac{\partial^{2}}{\partial\tilde{z}^{2}} + \frac{\tilde{z}^{2}}{2} + \tilde{G}_{\mathsf{IB}}|\tilde{\psi}_{\mathsf{I}}\left(\tilde{z}\right)|^{2} + \tilde{G}_{\mathsf{B}}|\tilde{\psi}\left(\tilde{z}\right)|^{2}\right\}\tilde{\psi}\left(\tilde{z}\right)$$

Dimensionless parameters

•
$$au=\omega_{\sf z} t$$
, $ilde{z}=z/\mathit{I}_{\sf z}$ and $ilde{\psi}=\psi/\sqrt{\mathit{I}_{\sf z}}$

•
$$\tilde{G}_{\rm B} = 2N\omega_{\rm r}a_{\rm B}/\omega_{\rm z}l_{\rm z}$$

•
$$\tilde{G}_{IB} = 2a_{IB}\omega_{r}f(\omega_{Ir}/\omega_{r})/\omega_{z}l_{z}$$

- where $l_z = \sqrt{\hbar/m_B\omega_z}$ is the oscillator length
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Hybrid systems

Dimensional reduction from 3D to 1D $\circ\circ\circ\circ\circ\circ\circ\circ\circ$

Static results

Time of flight

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Dimensionless Gross-Pitaevskii equation

Impurity wave function in experimental realization



N. Spethmann, et al., Phys. Rev. Lett. 109, 235301 (2012).

Impurity wave function in dimensionless form

$$\psi_{\mathsf{I}}(z) = \frac{1}{\sqrt{\sqrt{\pi}\alpha}} e^{-\frac{z^2}{2\alpha^2}}$$

Here
$$\alpha = I_{zl} / I_z$$



Hybrid systems

Dimensional reduction from 3D to 1D $\circ\circ\circ\circ\circ\circ\circ\circ\circ$

Static results

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Dimensionless Gross-Pitaevskii equation

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Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook
Thomas-Fermi variation	nal ansatz			

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Dimensional reduction from 3D to 1D

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• Thomas-Fermi variational ansatz

- Numerical density profile
- Impurity height/depth and width

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Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook
		0000000000		
Thomas-Fermi variation	nal ansatz			

•
$$\psi(z) = \sqrt{\frac{\mu}{G_{\rm B}} \left(1 - \frac{z^2}{2\mu} - \frac{Ge^{-\frac{z^2}{\alpha^2}}}{\mu\sqrt{\pi\alpha}}\right)} \Theta\left(1 - \frac{z^2}{2\mu} - \frac{Ge^{-\frac{z^2}{\alpha^2}}}{\mu\sqrt{\pi\alpha}}\right)$$

• Here μ and G are variational parameters

Extremizing energy with respect to μ and G yields: • $\mu = \frac{1}{2} \left(\frac{3}{2}\right)^{2/3} (G_{\rm B} + G)^{2/3}$ $G = G_{\rm IB}$ \Rightarrow Thomas-Fermi solution

Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook
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Thomas-Fermi variation	nal ansatz			

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Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook
Thomas-Fermi variation	al ansatz			

Numerical Method

• Our numerical calculation based on the split operator method

•
$$H = T + V$$
, here $T = K.E$ and $V = P.E + I.E$

•
$$\psi(z, \tau + \Delta \tau) = e^{-\frac{T\Delta \tau}{2}} e^{-V\Delta \tau} e^{-\frac{T\Delta \tau}{2}} \psi(z, \tau) + \mathcal{O}(\Delta \tau^3)$$

•
$$\psi(z,\tau) = e^{-\tau H} \psi(z,0) = \sum_j e^{-\tau E_j} c_j \phi_j$$

J. Javanainen, et al., J. Phys. A: Math. Gen. 39, L179 (2006).
R. Barnett, et al., New Journal of Physics 12, 043004 (2010).
D. Vudragovic, et al., Comput. Phys. Commun. 183, 2021 (2012).

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Thomas-Fermi variation	nal ansatz			

Imaginary time evolution

Movie



Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook
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Numerical density profi	le			

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- Thomas-Fermi variational ansatz
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- Impurity height/depth and width

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Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook	
Numerical density profile					

Density profile of BEC



Here we plotted numerically the density profile of the system for different values of the G_{IB} .



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Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook
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Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook	
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Impurity height/depth and width					

Impurity height/depth (IHD)

• IHD =
$$\begin{cases} |\Psi(0)|_{G_{\mathsf{IB}}}^2 - |\Psi(0)|_{G_{\mathsf{IB}}=0}^2 & G_{\mathsf{IB}} \le 0\\ \mathsf{Max}\left(|\Psi(z)|_{G_{\mathsf{IB}}}^2\right) - |\Psi(0)|_{G_{\mathsf{IB}}}^2 & G_{\mathsf{IB}} > 0 \end{cases}$$

Impurity full width half maximum width (IW) defined as

•
$$|\Psi(IW)|^2_{G_{IB}} = \begin{cases} \left(|\Psi(0)|^2_{G_{IB}} + |\Psi(0)|^2_{G_{IB}=0}\right)/2 & G_{IB} \le 0 \\ \left(Max\left(|\Psi(z)|^2_{G_{IB}}\right) + |\Psi(0)|^2_{G_{IB}}\right)/2 & G_{IB} > 0 \end{cases}$$

Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook	
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Impurity height/depth and width					

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• IHD =
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Impurity full width half maximum width (IW) defined as

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Impurity height/depth and width

Impurity height/depth (IHD)



Height/depth of impurity bump/dip versus impurity-BEC coupling constant G_{IB} for the BEC coupling constant $G_{B} = 4718.15$.



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Dimensional reduction from 3D to 1D 000000000

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Impurity height/depth and width

Impurity full width half maximum width (IW)



Width of impurity bump/dip versus impurity-BEC coupling constant G_{IB} for the BEC coupling constant $G_{\text{B}} = 4718.15$.



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Dimensional reduction from 3D to 1D $_{\rm OOOOOOOOO}$

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Critical impurity-BEC coupling constant

For a specific value of $G_{\rm IB}$ when no more ⁸⁷Rb atoms left at the center of the trap, the Impurity dip reaches its maximum value, we called this value critical impurity-BEC coupling constant $G_{\rm IBc}$.

$$G_{\rm IBc} = \sqrt{\pi} \alpha / 2 (3/2)^{2/3} (G_{\rm B} + G_{\rm IBc})^{2/3}$$

$$G_{\rm IBc} \stackrel{\rm Variational}{=} 275 \stackrel{\rm Numerical}{\simeq} 286$$

 $\mathsf{IHD}_{\mathsf{c}} = G_{\mathsf{IBc}} / \sqrt{\pi} \alpha G_{\mathsf{B}} \stackrel{\text{Variational}}{=} 0.0401 \stackrel{\text{Numerical}}{\simeq} 0.0393$

$$IW_{c} \stackrel{\text{Variational}}{=} 1.3444 \stackrel{\text{Numerical}}{\simeq} 1.400$$

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Real time evolution

Movie



Hybrid systems

Dimensional reduction from 3D to 1D 000000000

Static results

Time of flight

Outlook

Impurity height/depth (IHD)



We plotted the IHD vs time for different values of impurity-BEC coupling constant G_{IB} .



Dimensional reduction from 3D to 1D 000000000

Static results

Time of flight

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Impurity height/depth (IHD)



We plotted the IHD vs time for different values of impurity-BEC coupling constant G_{IB} .



Hybrid systems	Dimensional reduction from 3D to 1D	Static results 00000000000	Time of flight	Outloo
We will	work out more realistic mo	dels by explorir	ng in more de	tail
the pro	perties of a 2D and 3D syst	em.		

Emergence of Phonons by moving Impurity

Move vortex with using Impurity



Hybrid systems	Dimensional reduction from 3D to 1D	Static results 00000000000	Time of flight	Outlo

We will work out more realistic models by exploring in more detail the properties of a 2D and 3D system.

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Static results

Time of flight

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Outlook

Impurity and the three-body recombination

$$i\hbar\frac{\partial}{\partial t}\psi(z,t) = \left\{-\frac{\hbar^2}{2m_B}\frac{\partial^2}{\partial z^2} + \frac{m_B\omega_z^2 z^2}{2}\right\}$$

 $+G_{\rm IB} \| \psi_{\rm I}(z,t) \|^{2} + G_{\rm B} |\psi(z,t)|^{2} - i\hbar \frac{K_{3}}{2} |\psi(z,t)|^{4} \Big\} \psi(z,t)$

Bloch Group, Phys. Rev. Lett. **102**, 030408 (2009). Widera Group, Phys. Rev. Lett. **109**, 235301 (2012).

mpurity and far field vortex-vortex interaction

R. Navarro, et al., Phys. Rev. Lett. **110**, 225301 (2013).

Solve two coupled GPEs for 100 ¹³⁷Cs atoms as an Impurity and 10^{6 87}Rb atoms as a BEC



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Static results

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Hybrid systems	Dimensional reduction from 3D to 1D	Static results	Time of flight	Outlook

Thanks

