Bose-Einstein Condensation in Random Potentials

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1. Introduction: Ultracold Quantum Gases
2. Experimental Realizations of Dirty Bosons
3. Theoretical Description of Dirty Bosons
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6. Hartree-Fock Mean-Field Theory
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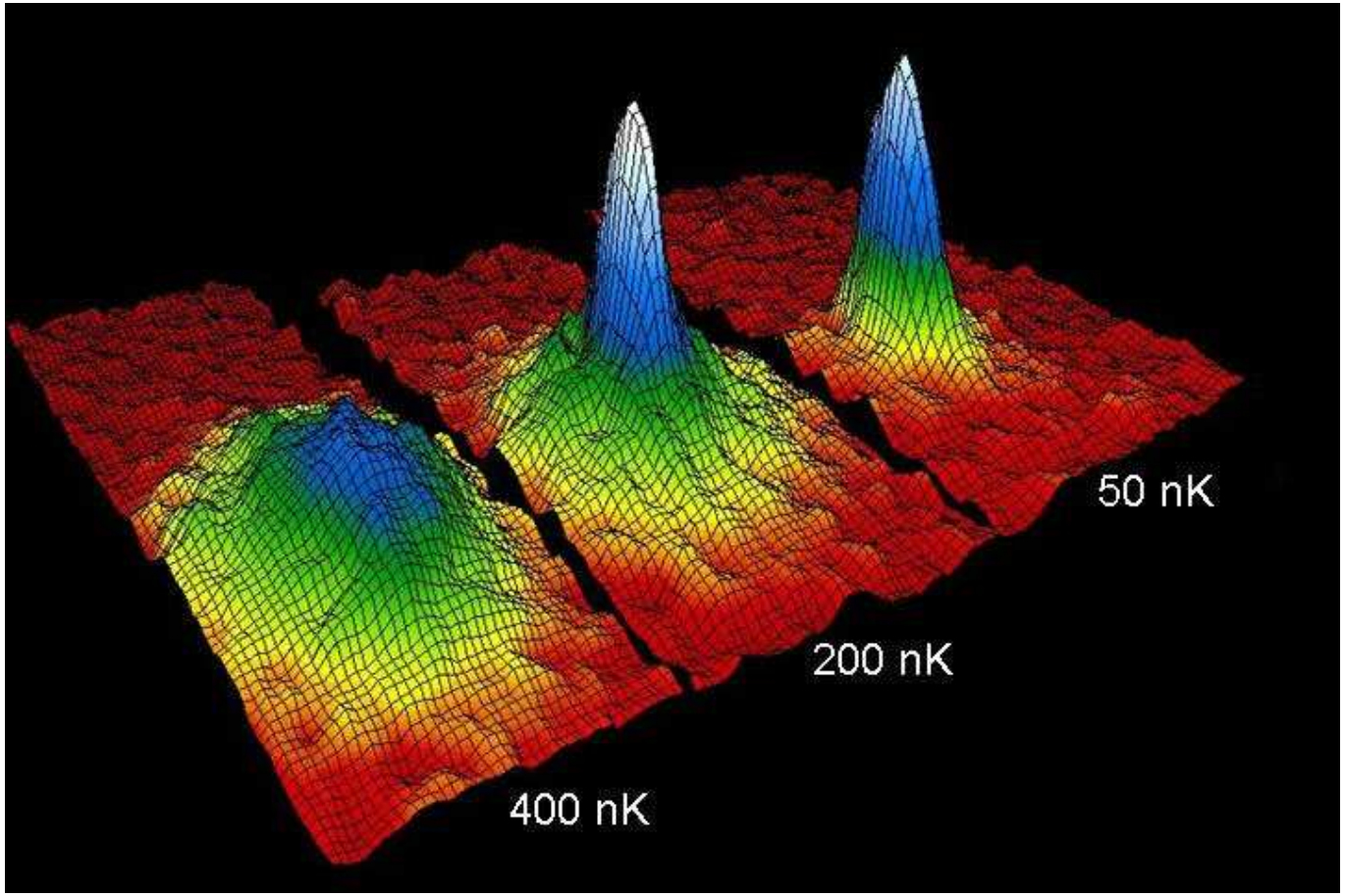
SFB/TR 12: Symmetries and Universality in Mesoscopic Systems

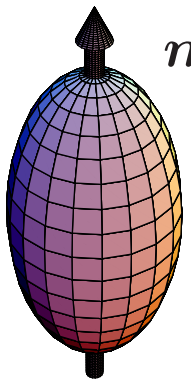
1.1 Experimental Realization



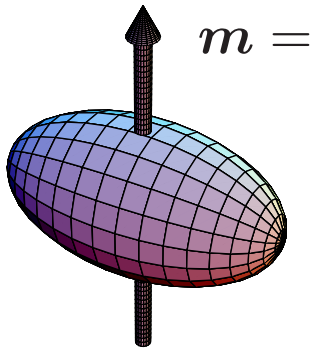
JILA (1995): $^{87}_{37}$Rb , N=20 000 , $\omega_1 = \omega_2 = \omega_3/\sqrt{8} = 2\pi\times$ 120 Hz

1.2 Stuttgart Chrom Condensate 2005

$$V^{(\mathrm{int})}(\mathbf{x}-\mathbf{x}') = \frac{4\pi\hbar^2 a}{M}\delta(\mathbf{x}-\mathbf{x}') - \frac{\mu_0}{4\pi}\left\{\frac{3\left[\mathbf{m}\left(\mathbf{x}-\mathbf{x}'\right)\right]^2}{|\mathbf{x}-\mathbf{x}'|^5} - \frac{\mathbf{m}^2}{|\mathbf{x}-\mathbf{x}'|^3}\right\}$$



$\boldsymbol{m} = m\,\boldsymbol{e}_z$



$\boldsymbol{m} = m\,\boldsymbol{e}_z$

$\omega_x^{(\mathrm{I})} = \omega_y^{(\mathrm{I})} \equiv \omega_\perp\,, \qquad \omega_z^{(\mathrm{I})} \equiv \omega_\parallel$

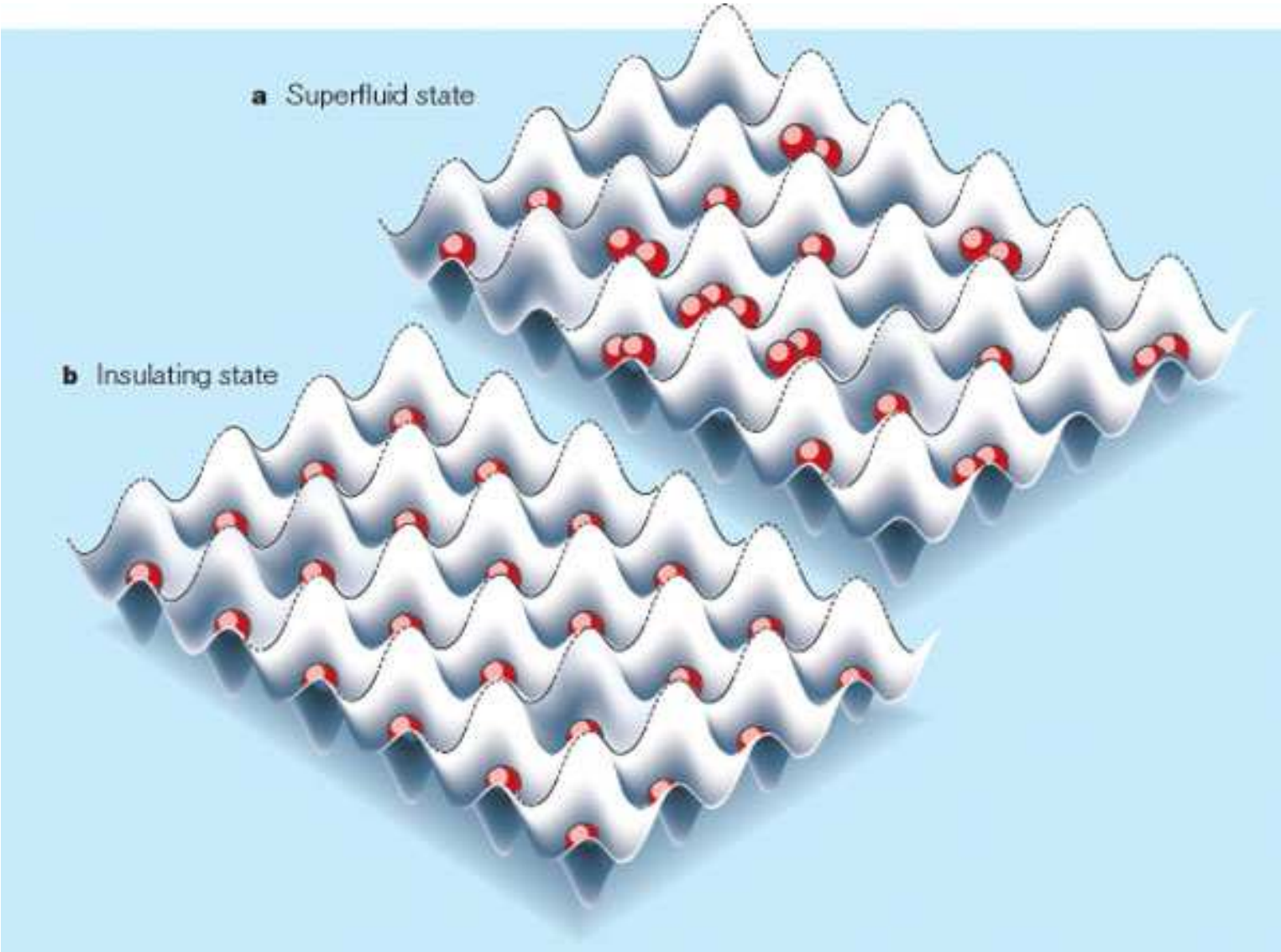
$\omega_y^{(\mathrm{II})} = \omega_z^{(\mathrm{II})} \equiv \omega_\perp\,, \qquad \omega_x^{(\mathrm{II})} \equiv \omega_\parallel$

Goal: difference of critical temperatures $T_c^{(\mathrm{I})} - T_c^{(\mathrm{II})} \sim \dfrac{\mu_0 m^2}{4\pi}$

Glaum, Pelster, Kleinert, and Pfau, PRL **98**, 080407 (2007)

Glaum and Pelster, PRA **76**, 063604 (2007)

1.3 Bosons in Optical Lattices

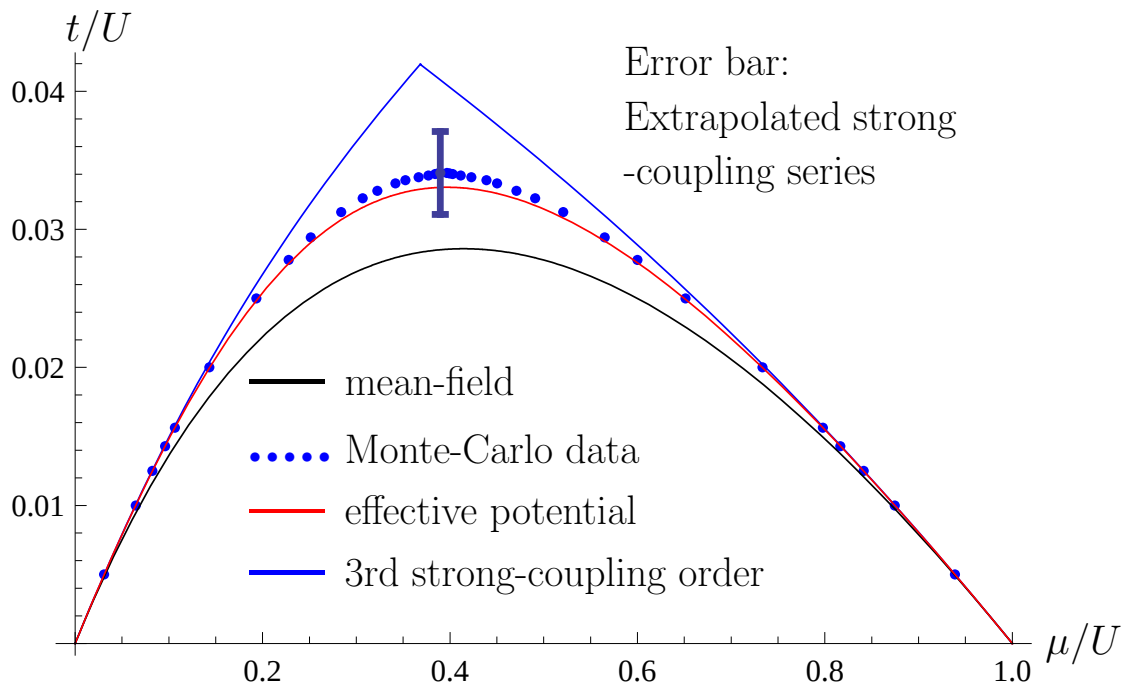


1.4 Quantum Phase Diagram

Bose-Hubbard Hamiltonian:

$$\hat{H}_{\mathrm{BH}} = -t \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \left[\frac{U}{2} \hat{n}_i \left(\hat{n}_i - 1 \right) - \mu \hat{n}_i \right] \; ; \qquad \hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$$

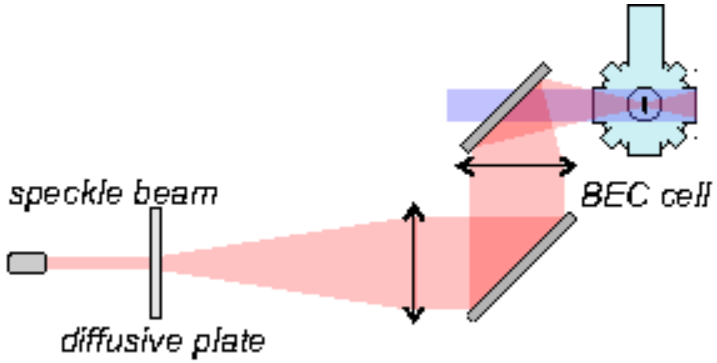
Analytical Improvement of Mean-Field Result:

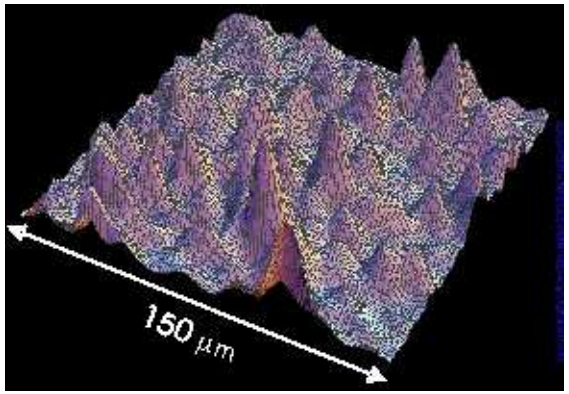


Santos and Pelster, `arXiv:0806.2812`

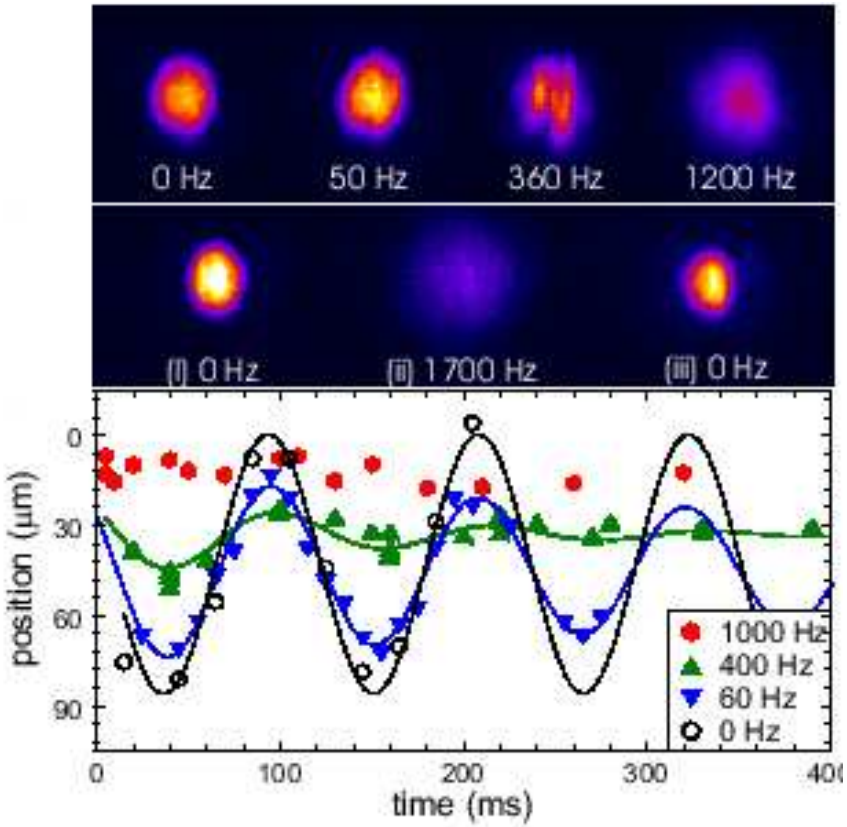
2.1 Magneto-Optical Trap

Laser Speckles:



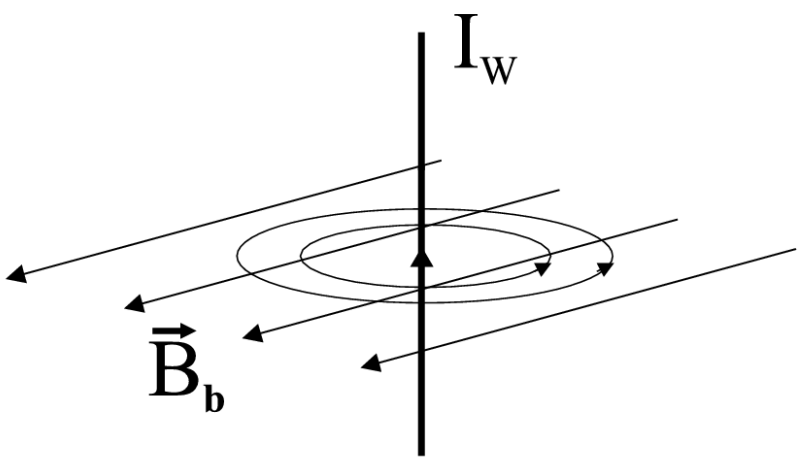


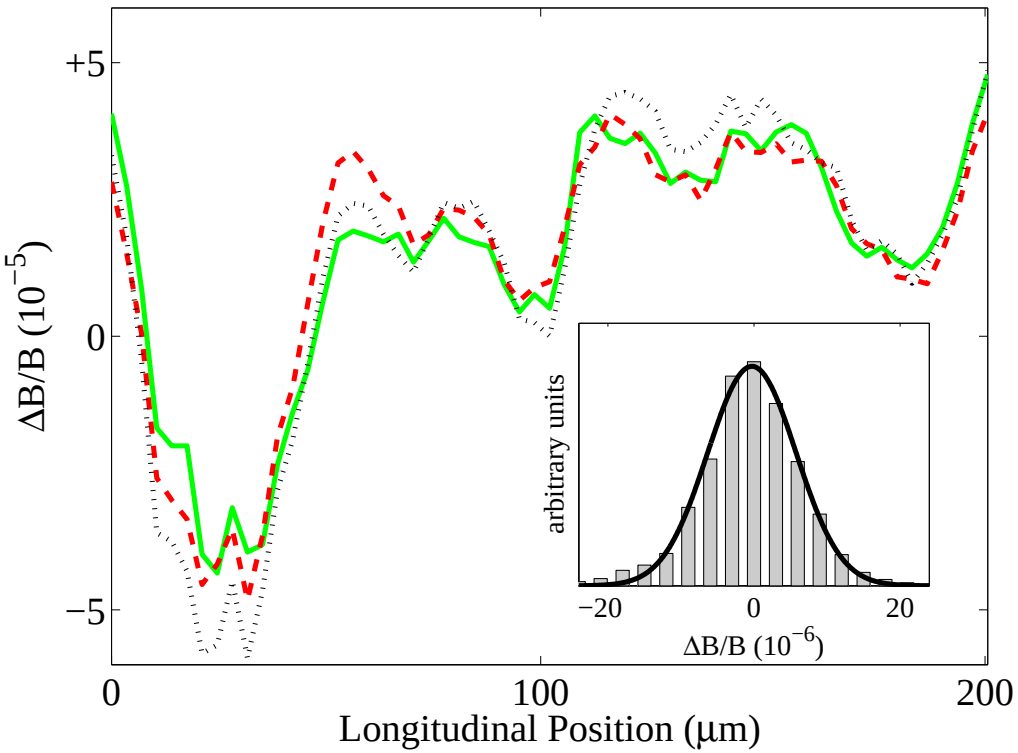
Lye *et al.*, PRL **95**, 070401 (2005)



global condensate vanishes

2.2 Wire Trap





Distance: $d = 10\ \mu$m

Wire Width: $100\ \mu$m

Magnetic Field: 10 G, 20 G, 30 G

Deviation: $\Delta B/B \approx 10^{-4}$

Krüger *et al.*, PRA **76**, 063621 (2007)

Fortàgh and Zimmermann, RMP **79**, 235 (2007)

3.1 Model System

Action of a Bose Gas:

$$\mathcal{A} = \int_0^{\hbar\beta} d\tau \int d^3x \left\{ \psi^* \left[\hbar \frac{\partial}{\partial \tau} - \frac{\hbar^2}{2M} \boldsymbol{\Delta} + \boldsymbol{U}(\mathbf{x}) + \boldsymbol{V}(\mathbf{x}) - \boldsymbol{\mu} \right] \psi + \frac{\boldsymbol{g}}{2} \psi^{*2} \psi^2 \right\}$$

Properties:

- harmonic trap potential: $\boldsymbol{U}(\mathbf{x}) = \frac{M}{2}\omega^2 \mathbf{x}^2$
- disorder potential: $\boldsymbol{V}(\mathbf{x})$; bounded from below, i.e. $V(\mathbf{x}) \geq V_0$

$$\overline{V(\mathbf{x}_1)} = 0\,, \qquad \overline{V(\mathbf{x}_1)V(\mathbf{x}_2)} = R(\mathbf{x}_1 - \mathbf{x}_2)\,, \qquad \ldots$$

- chemical potential: $\boldsymbol{\mu}$
- repulsive interaction: $\boldsymbol{g} = \frac{4\pi\hbar^2 a}{M}$
- periodic Bose fields: $\psi(\mathbf{x}, \tau + \hbar\beta) = \psi(\mathbf{x}, \tau)$

3.2 Grand-Canonical Potential

Aim:

$$\begin{aligned}\Omega &= -\frac{1}{\beta}\overline{\ln \mathcal{Z}} \\ \mathcal{Z} &= \oint D\psi D\psi^* \, e^{-\mathcal{A}[\psi^*,\psi]/\hbar}\end{aligned}$$

Problem:

$$\overline{\ln \mathcal{Z}} \neq \ln \overline{\mathcal{Z}}$$

Solution: Replica Trick

$$\Omega = -\frac{1}{\beta}\lim_{N\to 0}\frac{\overline{\mathcal{Z}^N}-1}{N}$$

3.3 Replica Trick

Disorder Averaged Partition Function:

$$\overline{\mathcal{Z}^N} = \overline{\oint \left\{ \prod_{\alpha=1}^{N} D\psi_\alpha D\psi_\alpha^* \right\} e^{-\sum_{\alpha=1}^{N} \mathcal{A}([\psi_\alpha^*,\psi_\alpha])/\hbar}} = \oint \left\{ \prod_{\alpha=1}^{N} D\psi_\alpha D\psi_\alpha^* \right\} e^{-\mathcal{A}^{(N)}/\hbar}$$

Replicated Action:

$$\begin{aligned}
\mathcal{A}^{(N)} \quad = \quad & \int_0^{\hbar\beta} d\tau \int d^3x \sum_{\alpha=1}^{N} \left\{ \psi_\alpha^*(\mathbf{x},\tau) \left[\hbar \frac{\partial}{\partial \tau} - \frac{\hbar^2}{2M} \mathbf{\Delta} + U(\mathbf{x}) - \mu \right] \psi_\alpha(\mathbf{x},\tau) \right. \\
& \left. + \frac{g}{2} \psi_\alpha^*(\mathbf{x},\tau)^2 \psi_\alpha(\mathbf{x},\tau)^2 \right\} \\
& - \frac{1}{2\hbar} \int_0^{\hbar\beta} d\tau \int_0^{\hbar\beta} d\tau' \int d^3x \int d^3x' \sum_{\alpha=1}^{N} \sum_{\alpha'=1}^{N} \\
& \qquad \times \psi_\alpha^*(\mathbf{x},\tau) \psi_\alpha(\mathbf{x},\tau) R(\mathbf{x}-\mathbf{x}') \psi_{\alpha'}^*(\mathbf{x}',\tau') \psi_{\alpha'}(\mathbf{x}',\tau') + \ldots
\end{aligned}$$

Similar: disorder averaged correlation functions

4.1 Condensate Density

Assumptions:

homogeneous Bose gas: $U(\mathbf{x}) = 0$

δ-correlated disorder: $R(\mathbf{x}) = R\,\delta(\mathbf{x})$

Bogoliubov Theory:

background method: $\psi_\alpha(\mathbf{x},\tau) = \Psi_\alpha + \delta\psi_\alpha(\mathbf{x},\tau)$

replica symmetry: $\Psi_\alpha = \sqrt{n_0}$

Result:

$$n_0 = n - \frac{8}{3\sqrt{\pi}}\sqrt{a\,n_0}^{\,3} - \frac{M^2 R}{8\pi^{3/2}\hbar^4}\sqrt{\frac{n_0}{a}}$$

Huang and Meng, PRL **69**, 644 (1992)

Falco, Pelster, and Graham, PRA **75**, 063619 (2007)

4.2 Superfluid Density

Galilei Boost:

$$\Delta\mathcal{A} = \int_0^{\hbar\beta} d\tau \int d^3x\, \psi^*(\mathbf{x},\tau)\, \mathbf{u}\, \frac{\hbar}{i}\, \boldsymbol{\nabla}\, \psi(\mathbf{x},\tau)$$

$$d\Omega = -S\, dT - p\, dV - N\, d\mu - \mathbf{p}\, d\mathbf{u}$$

$$\mathbf{p} = -\left.\frac{\partial\Omega(T,V,\mu,\mathbf{u})}{\partial\mathbf{u}}\right|_{T,V,\mu} = MV\, n_n\, \mathbf{u} + \ldots$$

Result:

$$n_s = n - n_n = n - \frac{4}{3}\frac{M^2 R}{8\pi^{3/2}\hbar^4}\sqrt{\frac{n_0}{a}}$$

Huang and Meng, PRL **69**, 644 (1992)

Falco, Pelster, and Graham, PRA **75**, 063619 (2007)

4.3 Collective Excitations

Hydrodynamic Equation in Trap With Disorder:

$$m\frac{\partial^2}{\partial t^2}\delta n(\mathbf{x},t) - \boldsymbol{\nabla}\Big[g n_{\mathrm{TF}}(\mathbf{x})\boldsymbol{\nabla}\delta n(\mathbf{x},t)\Big]$$
$$= -\boldsymbol{\nabla}^2\Big[3g n_R(\mathbf{x})\delta n(\mathbf{x},t)\Big] - \boldsymbol{\nabla}\left[\frac{4g}{3}n_R(\mathbf{x})\boldsymbol{\nabla}\delta n(\mathbf{x},t)\right]$$

$n_R(\mathbf{x})$: Huang-Meng depletion in trap

$n_{\mathrm{TF}}(\mathbf{x}) = [\mu - V(\mathbf{x})]/g$: Thomas-Fermi density

Violation of Kohn Theorem:

Surface dipole mode ($n = 0\,, l = 1$):

$$\frac{\delta\omega_{\mathrm{dip}}(\xi = 0)}{\omega_{\mathrm{dip}}} = -\frac{5\pi}{16}\frac{M^2 R}{8\pi^{3/2}\hbar^4\sqrt{n_{\mathrm{TF}}(\mathbf{0})a}}$$

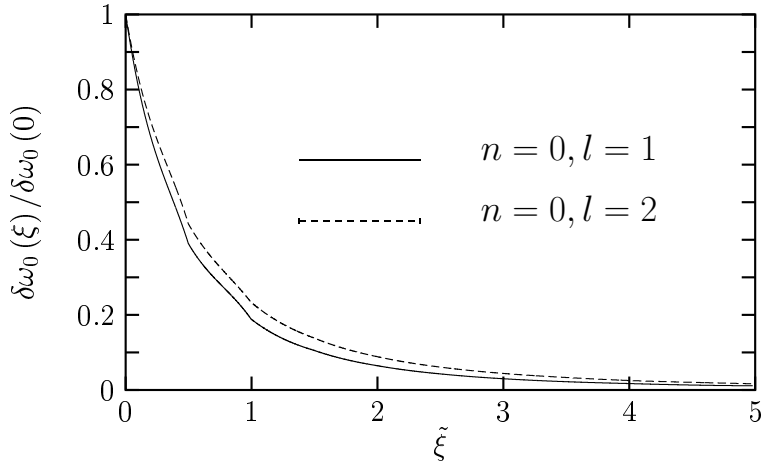
Falco, Pelster, and Graham, PRA **76**, 013624 (2007)

4.4 Comparison With Experiment

Typical Values:

Inguscio *et al.*, PRL **95**, 070401 (2005)

$$\left.\begin{array}{rcl} \xi & = & 10\ \mu\text{m} \\ R_{\text{TF}} & = & 100\ \mu\text{m} \\ l_{\text{HO}} & = & 10\ \mu\text{m} \end{array}\right\} \quad \tilde{\xi} = \frac{\xi R_{\text{TF}}}{l_{\text{HO}}^2\sqrt{2}} \approx 7$$



$\Longrightarrow$ **Disorder effect vanishes in laser speckle experiment**

Improvement:

laser speckle setup with correlation length $\xi = 1\ \mu$m

Aspect *et al.*, NJP **8**, 165 (2006)

$\Longrightarrow$ **Disorder effect should be measurable**

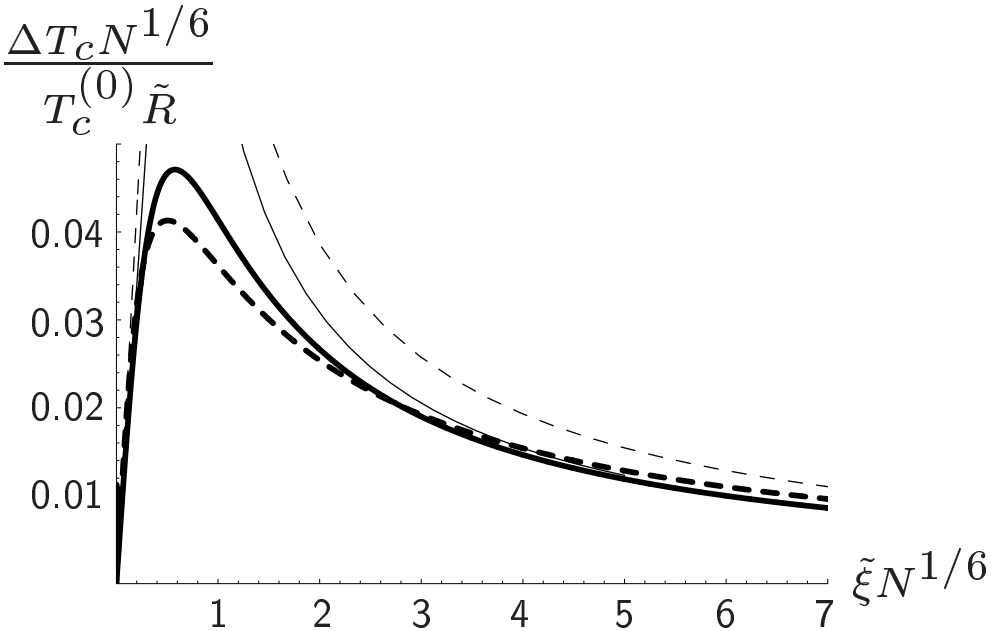
Falco, Pelster, and Graham, PRA **76**, 013624 (2007)

5.1 Earlier Results

trapped Bose gas	homogeneous Bose gas
$T_c^{(0)} = \frac{\hbar\omega_g}{k_B}\left[\frac{N}{\zeta(3)}\right]^{1/3}$	$T_c^{(0)} = \frac{2\pi\hbar^2}{k_B M}\left[\frac{n}{\zeta(3/2)}\right]^{2/3}$
$\frac{\Delta T_c}{T_c^{(0)}} = -3.426\,\frac{a}{\lambda_c^{(0)}}$ Giorgini et al., PRA **54**, R4633 (1996) Gerbier et al., PRL **92**, 030405 (2004)	$\frac{\Delta T_c}{T_c^{(0)}} = 1.3\,an^{1/3}$ Kleinert, MPLB **17**, 1011 (2003) Kastening, PRA **69**, 043613 (2004)
$R(\mathbf{x}) = ?$ $\frac{\Delta T_c}{T_c^{(0)}} = ?$	$R(\mathbf{x}) = R\,\delta(\mathbf{x})$ $\frac{\Delta T_c}{T_c^{(0)}} = -\frac{M^2 R}{3\pi[\zeta(3/2)]^{2/3}\hbar^2 n^{1/3}}$ Lopatin and Vinokur, PRL **88**, 235503 (2002)

Procedure: $n = n(\mu), \quad \mu \nearrow \mu_c \quad \Rightarrow \quad T_c$

5.2 Our Results



solid: Gaussian

dashed: Lorentzian

Length Scale: $$l_{\mathrm{HO}} = \sqrt{\frac{\hbar}{M\omega_{\mathrm{g}}}} \quad , \qquad \omega_{\mathrm{g}} = (\omega_1\omega_2\omega_3)^{1/3}$$

Dimensionless Units: $$\tilde{\xi} = \frac{\xi}{l_{\mathrm{HO}}} \quad , \qquad \tilde{R} = \frac{R}{\left(\frac{\hbar^2}{M l_{\mathrm{HO}}^2}\right)^2 l_{\mathrm{HO}}^3} = \frac{M^{3/2} R}{\hbar^{7/2} \omega_{\mathrm{g}}^{1/2}}$$

Timmer, Pelster, and Graham, EPL **76**, 760 (2006)

Klünder, Pelster, and Graham, to be published

6.1 Order Parameters

Definition:

$$\lim_{|\mathbf{x}-\mathbf{x}'|\to\infty} \overline{\langle\psi(\mathbf{x},\tau)\psi^*(\mathbf{x}',\tau)\rangle} = n_0$$

$$\lim_{|\mathbf{x}-\mathbf{x}'|\to\infty} \overline{|\langle\psi(\mathbf{x},\tau)\psi^*(\mathbf{x}',\tau)\rangle|^2} = (n_0+q)^2$$

Note:

q is similar to Edwards-Anderson order parameter of spin-glass theory

Hartree-Fock Mean-Field Theory:

Self-consistent determination of n_0 and q for $R(\mathbf{x}-\mathbf{x}') = R\,\delta(\mathbf{x}-\mathbf{x}')$

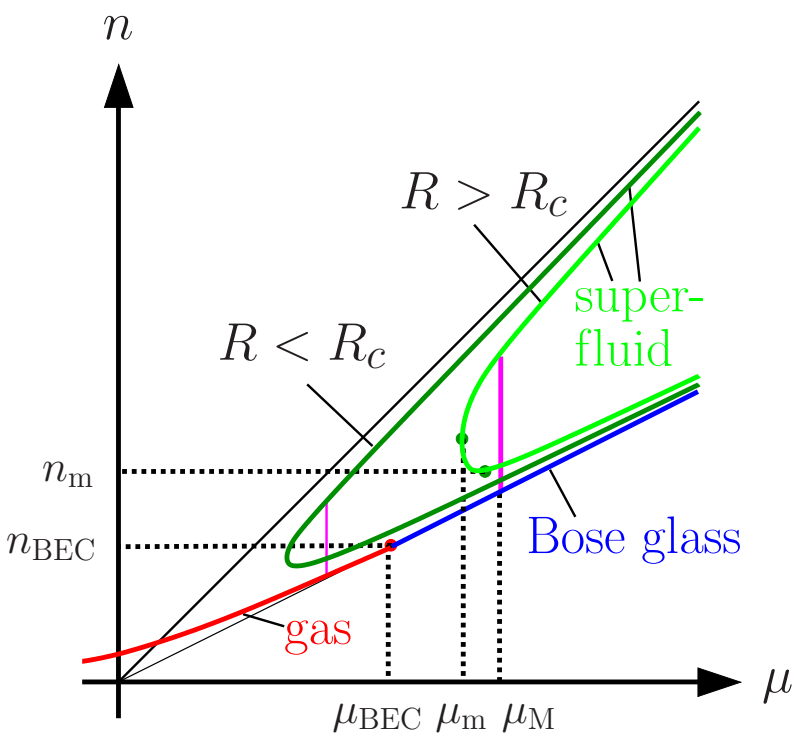
Phase Classification:

gas	Bose glass	superfluid
$q = n_0 = 0$	$q > 0, n_0 = 0$	$q > 0, n_0 > 0$

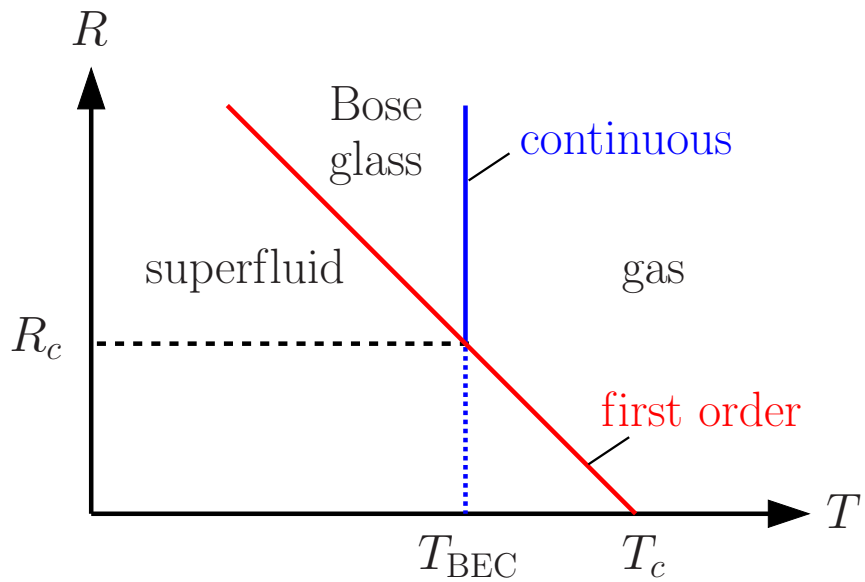
6.2 Hartree-Fock Results

Isotherm: $T = \text{const.}$

disorder strength R = const.



Phase Diagram: $\mu = \text{const.}$



Graham and Pelster, submitted to IJBC

7 Summary and Outlook

- **Frozen Disorder Potential**:

  arises both artificially (laser speckles) or naturally (wire trap)

- **Bosons:**

  local condensates in minima + global condensate + thermally excited

- **Localization Versus Transport:**

  disorder reduces superfluidity

- **Phase Diagram:**

  yet unknown for strong disorder

  Navez, Pelster, and Graham, APB **86**, 395 (2007)

- **Disordered Bosons in Lattice:**

  Bose Glass versus Mott phase

  Krutitsky, Pelster, and Graham, NJP **8**, 187 (2006)

8 Acknowledgements

University Duisburg-Essen AG Robert Graham	Free University of Berlin AG Hagen Kleinert
Giovanni Falco (Köln) Ben Klünder (LMU) Konstantin Krutitsky Patrick Navez (Leuven) Wieland Ronalter Matthias Timmer	Konstantin Glaum (Ulm) Alexander Hoffmann (LMU) Sebastian Kling (Bonn) Aristeu Lima (DAAD) Pascal Mattern Matthias Ohliger Steffen Röthel (Münster) Ednilson Santos (DAAD) Sebastian Schmidt (Yale) Moritz Schütte (Golm) Parvis Soltan-Panahi (Hamburg)
SFB/TR 12: Symmetries and Universality in Mesoscopic Systems	**SPP 1116: Interactions in Ultra-Cold Atomic and Molecular Gases**

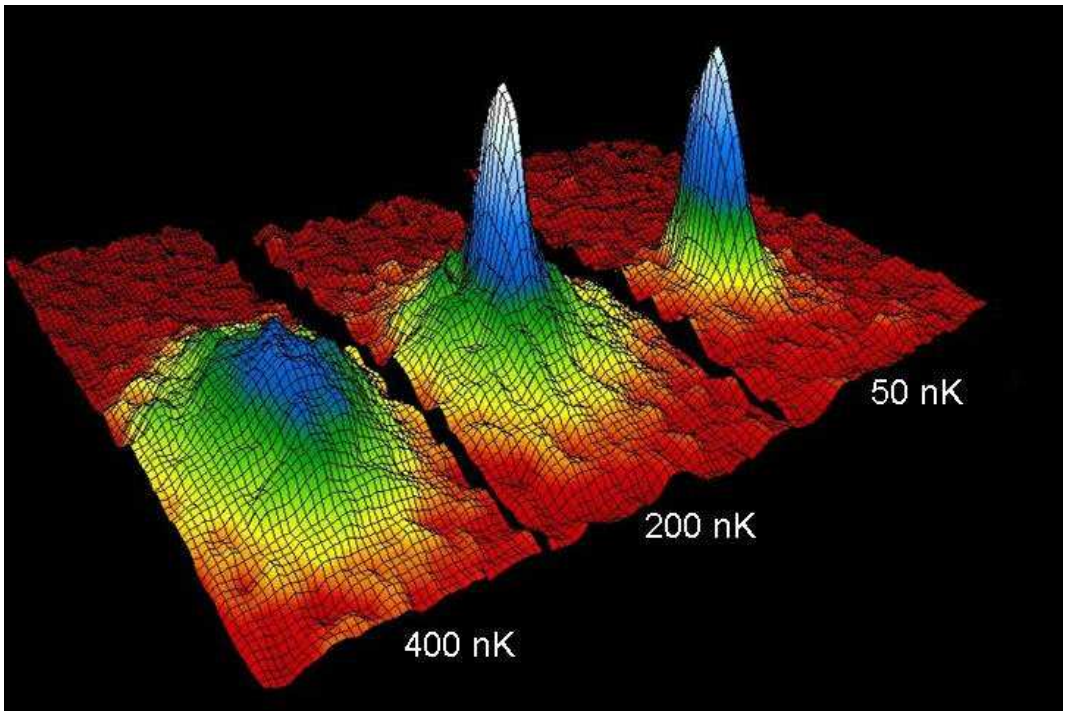
422nd Wilhelm and Else Heraeus Seminar

Quo Vadis BEC?

Bad Honnef, October 29 - 31, 2008

Scientific Coordinators: Martin Holthaus (Oldenburg) and Axel Pelster (Duisburg-Essen)

Research Topics: BEC/BCS Crossover, Dipolar Gases, **Disorder**, Dynamics, Quantum Information, Spinor Bose and Fermi Gases, Strong Correlations, Tunneling



http://www.theo-phys.uni-essen.de/tp/ags/pelster_dir/Heraeus/index.html