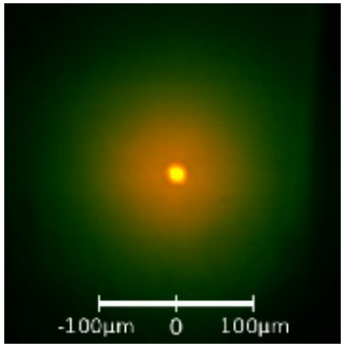
PHYSIK

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Thermooptische Wechselwirkung und dimensionaler Übergang in Photon-Bose-Einstein-Kondensaten



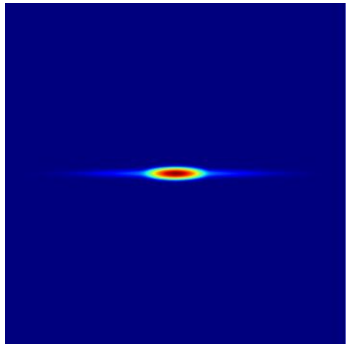
2D

Klaers et al.,
Nature **468**, 545 (2010)

Wissenschaftliche Aussprache

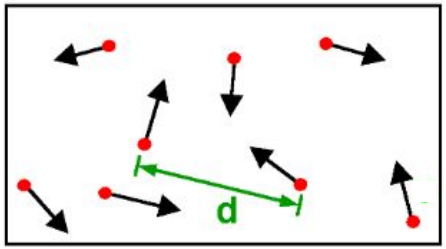
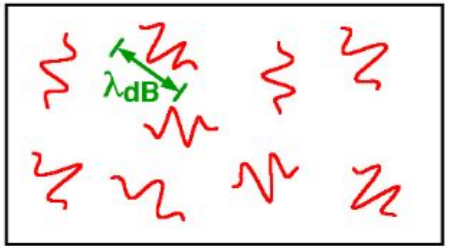
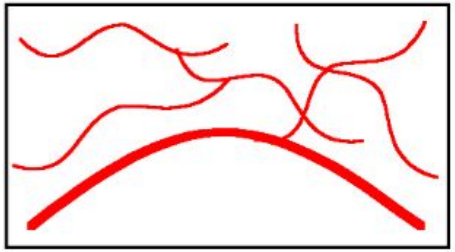
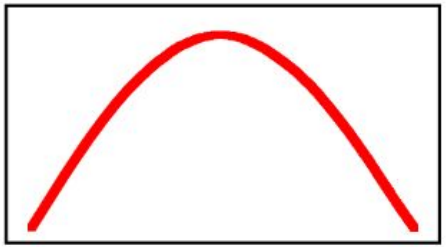
Enrico Stein

09. November 2022



1D

Bose-Einstein-Kondensation

$T \sim T_{\text{Raum}}$	$T \ll T_{\text{Raum}}$	$T = T_C$	$T = 0$
			
Billardkugeln	**Wellenpakete**	**Überlapp von Materiewellen**	**Große Materiewelle**

Thermische de-Broglie Wellenlänge:

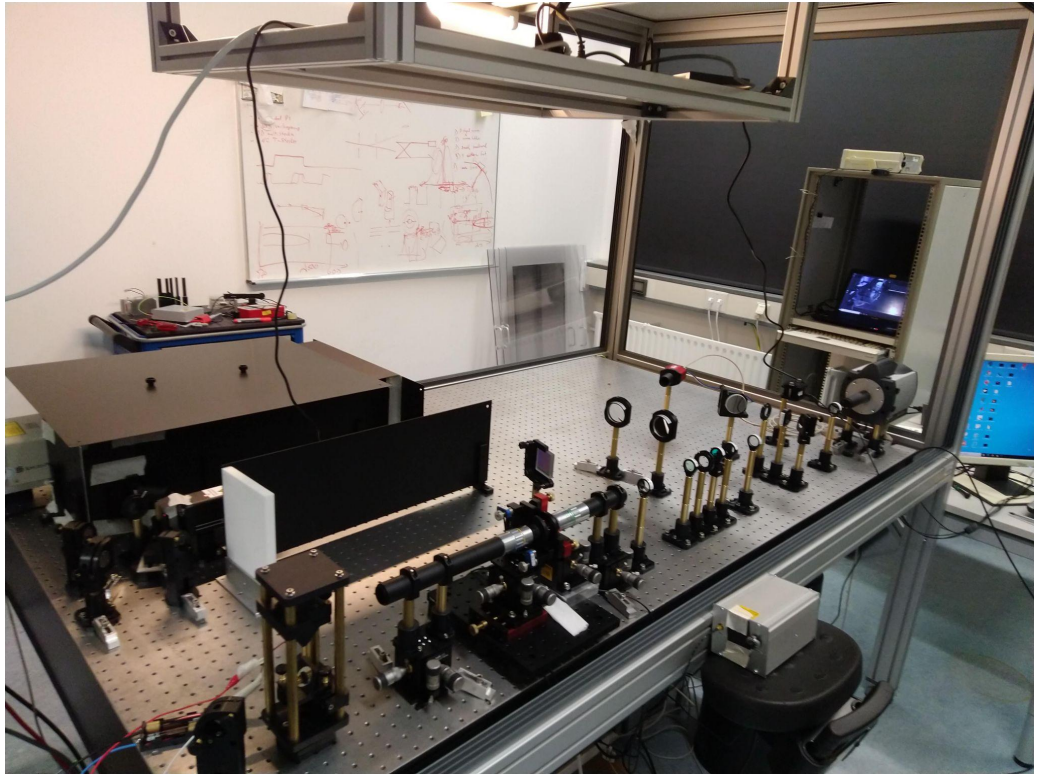
$$\lambda_{\text{dB}} = \hbar\sqrt{\frac{2\pi}{mk_{\text{B}}T}}$$

- vorhergesagt: Bose (1924), Einstein (1925)
- realisiert: Cornell (1995), Ketterle (1995)
- Nobelpreis 2001 für Cornell, Wieman und Ketterle

Atomare BECs

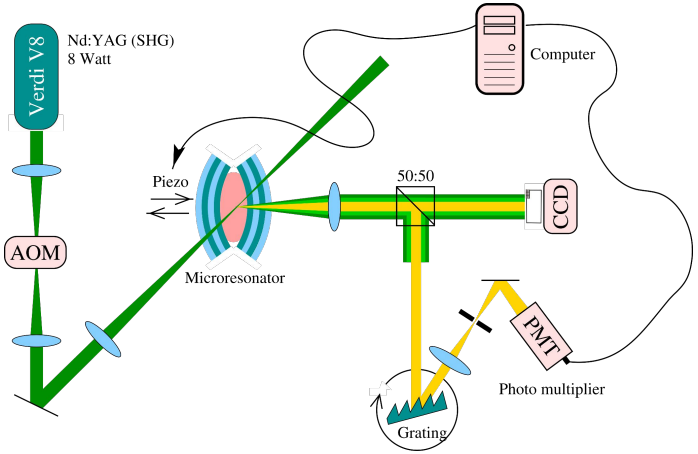
Experiment von W. Ketterle

Photon BECs



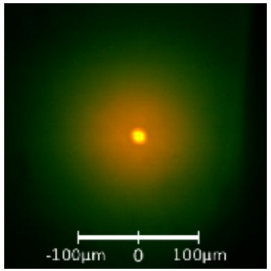
Experiment in Gruppe von Freymann

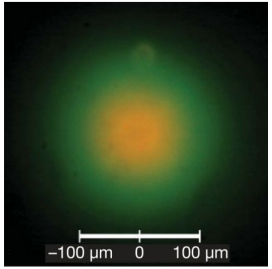
- Raumtemperatur
- kein Vakuum
  ⇒ tabletop-Experiment



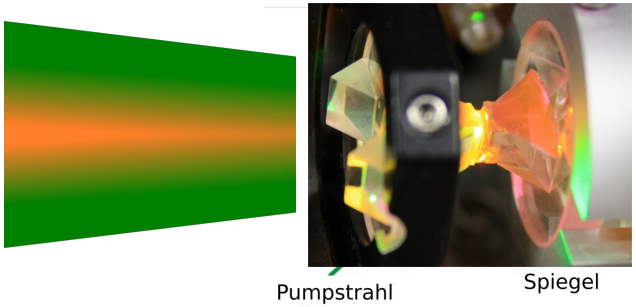
Experiment

Räumlich

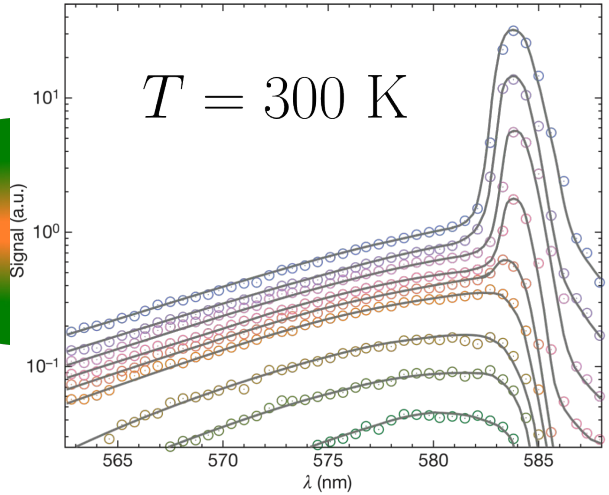




Kavität



Spektrum



Bose-Einstein-Kondensation

Klaers, et alii, Nature **468**, 545 (2010)

Photonen in Kavität

- **effektive Photonenmasse:**

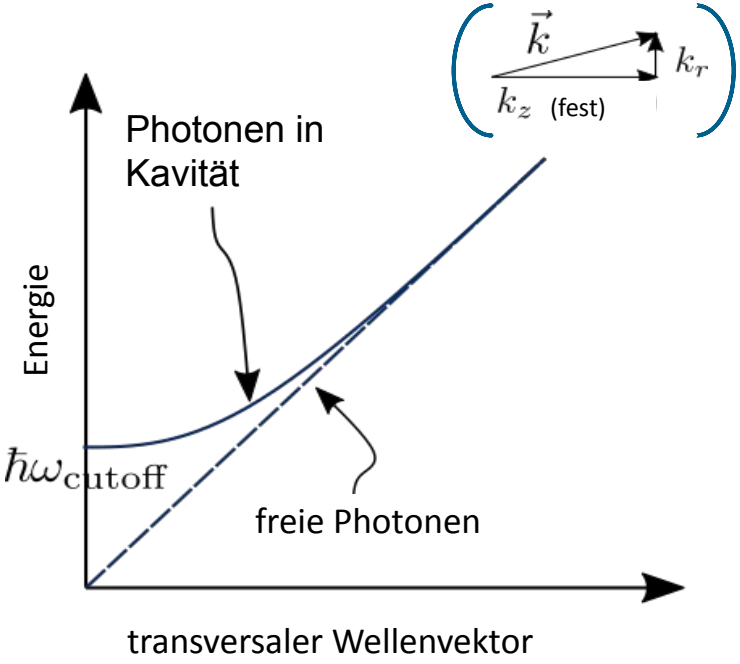
  $$m \sim 10^{-9} m_{\mathrm{Proton}}$$

- **Fallenfrequenz:**

  $$\Omega \sim 10^{11}\,\mathrm{Hz}$$

- **relative Skala ähnlich zu Atomen**

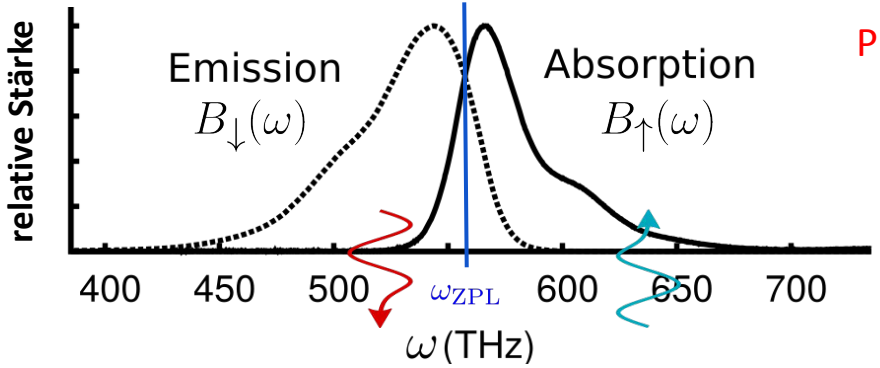
  $$\frac{\hbar\Omega}{k_{\mathrm{B}} T_{\mathrm{Raum}}} \approx 6 \times 10^{-3}$$

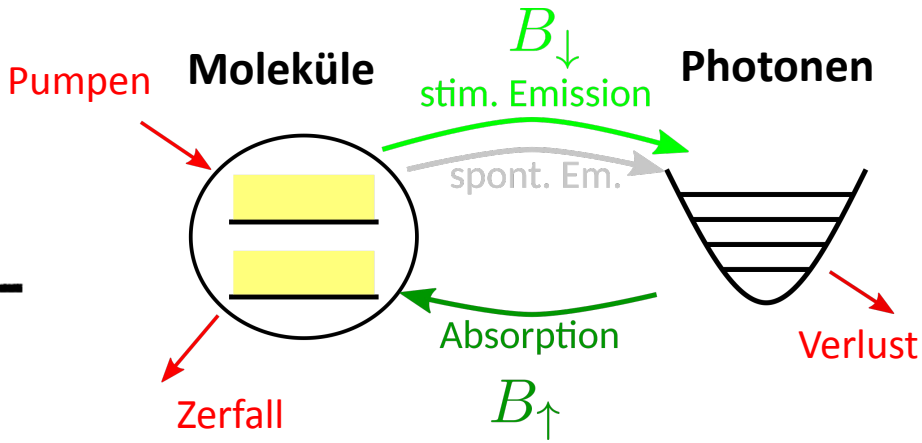


⇒ **massive Bosonen in 2D harmonischem Potential**

Thermalisierung

Farbstoffspektrum:





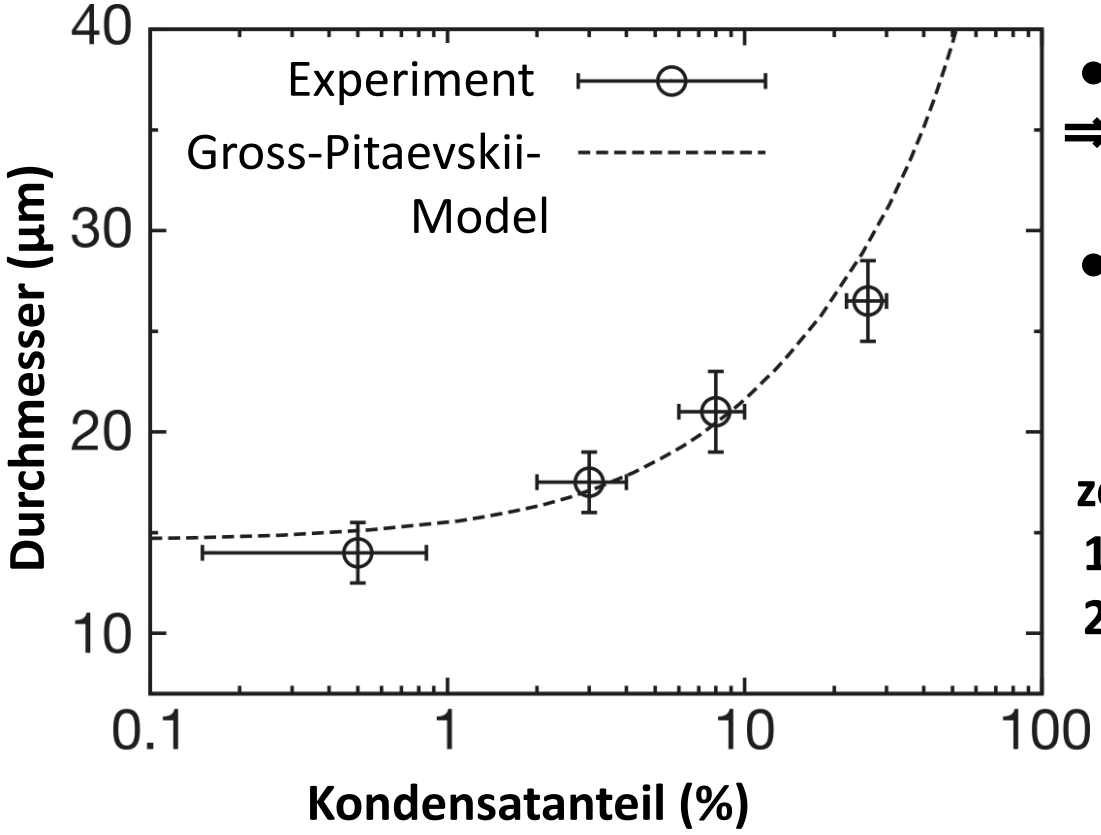
Kennard-Stepanov-Relation: $B_\downarrow(\omega) = B_\uparrow(\omega) e^{-\frac{\hbar(\omega-\omega_{\mathrm{ZPL}})}{k_{\mathrm{B}}T}}$

Kennard, Phys. Rev. 11, 29 (1918)
Stepanov, Dokl. Akad. Nauk 112, 839 (1957)

⇒ Gleichgewichtszustand verhält sich wie ideales Bosegas:

$$n_i = \left[e^{(E_i-\mu)/(k_{\mathrm{B}}T)} - 1\right]^{-1} \qquad \mu = -k_{\mathrm{B}}T \ln\left(N_\downarrow/N_\uparrow\right)$$

Ideales Bosegas?



Klaers, et alii, Nature **468**, 545 (2010)

- Verbreiterung des Kondensats

⇒ **Photon-Photon-Wechselwirkung**

- Größenordnung:

$$mg/\hbar^2 \sim 10^{-4}$$

zentrale Fragen:

1. **Ursprung der Wechselwirkung**
2. **Verstärkung der Wechselwirkung**

Inhalt

$T \neq 0$
$g = 0$

- Effektive Dimension am Übergang 2D ⇔ 1D
  New J. Phys. ***24****, 023013 (2022)*

$T \neq 0$
$g \neq 0$

- Hartree-Fock-Analogon
  arXiv: 2203.16955 (2022)
  arXiv: 2204.08818 (2022)

$T = 0$
$g = 0$

QM1: Harmonischer Oszillator

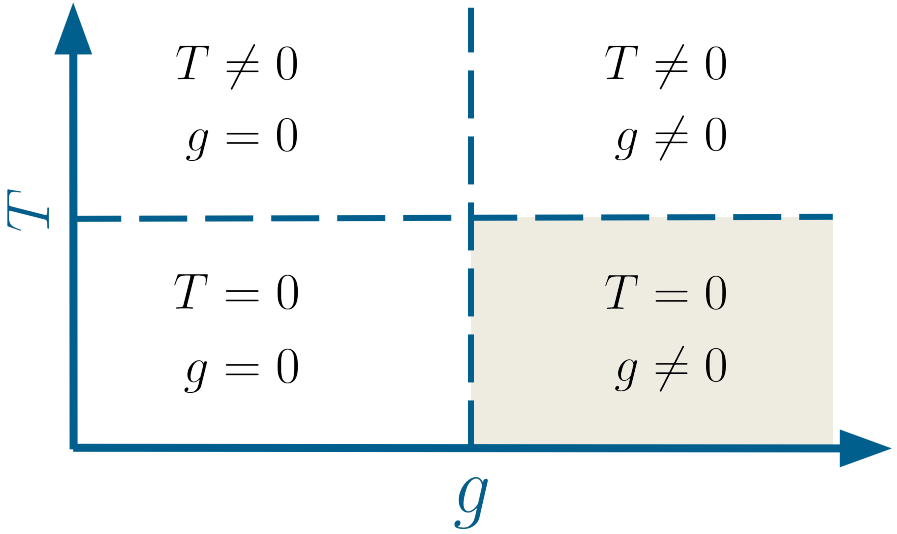
$T = 0$
$g \neq 0$

- Kollektive Anregungen
  New J. Phys. ***21****, 103044 (2019)*
- Dimensionaler Übergang
  New J. Phys. ***24****, 023032 (2022)*

Temperatur T

Wechselwirkung g

Kollektive Anregungen



Ziele:

- Verständnis der effektiven Photon-Photon-Wechselwirkung
- Untersuchung der kollektiven Anregungen

⇒ Unterschied zu atomaren BECs

Stein und Pelster, New J. Phys. **21**, 103044 (2019)

Wechselwirkungseffekte

Photonenergie: $E(t) = E_{\mathrm{HO}} + \hbar\omega_{\mathrm{c}} \int d^2x \; [\Delta n_{\mathrm{K}} + \Delta n_{\mathrm{T}}(t)] \, |\psi|^2$

Kerr-Effekt

- Intensitätsabhängige Änderung des Brechungsindex :

$$\Delta n_{\mathrm{K}} \sim |\psi|^2$$

⇒ Kontaktwechselwirkung

übliche Gross-Pitaevskii-Gleichung

Thermooptischer Effekt

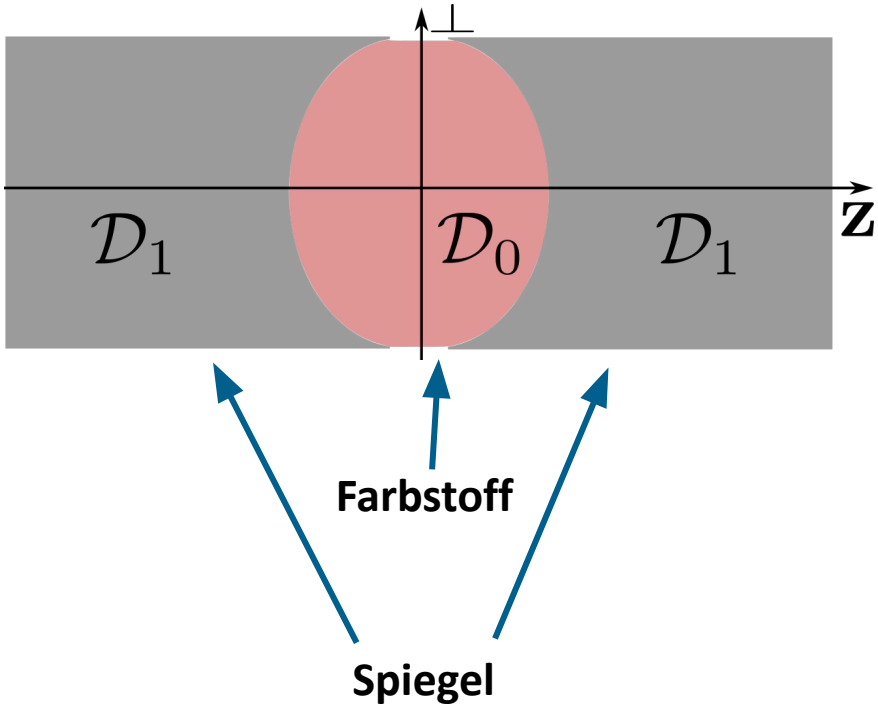
- Aufheizung des Farbstoffmediums
- Wärmediffusion:

$$\Delta n_{\mathrm{T}}(t) \sim \int_0^t dt' \int d^2x' \, \mathcal{G}(\mathbf{x}-\mathbf{x}', t-t') |\psi(\mathbf{x}', t')|^2$$

⇒ Nichtlokale, retardierte Wechselwirkung

Maßgeblicher Effekt

Temperaturdiffusion



3D Diffusion:

$$\partial_t \Delta T = \left[\mathcal{D}(z)\nabla_\perp^2 + \partial_z\left(\mathcal{D}(z)\,\partial_z\right)\right]\Delta T$$

Gleichgewicht entlang optischer Achse und Spiegel

$$\tau \approx 0.1\ \mathrm{s} \gg 1/\Omega$$

2D Diffusion :

$$\partial_t \Delta T_\perp = \left[\mathcal{D}_0\nabla_\perp^2 - \frac{1}{\tau}\right]\Delta T_\perp$$

Skala der Retardierung

Kollektive Anregungen

Grundgleichungen:

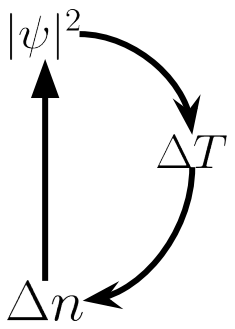
Harmonischer Oszillator

$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2\nabla_\perp^2}{2m} + \frac{m\Omega^2}{2}(x^2+y^2) + \gamma\Delta T\right]\psi$$

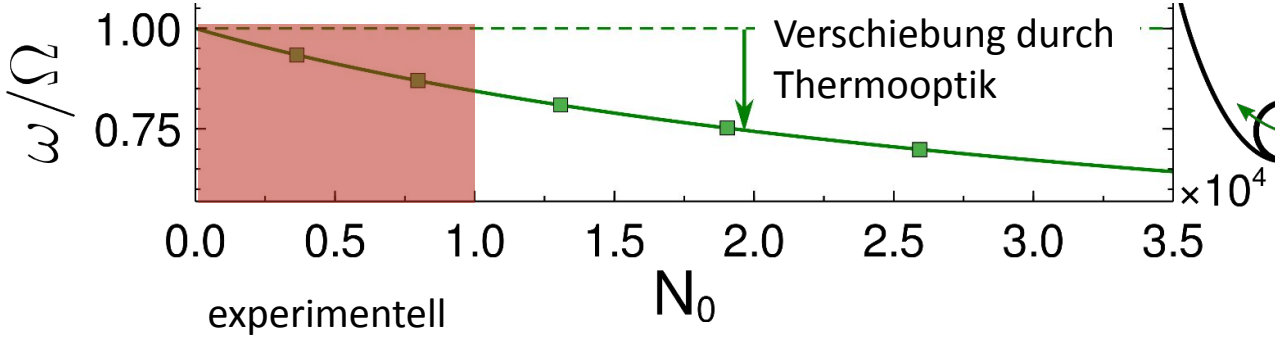
thermooptische Kopplung

$$\partial_t\Delta T = \left(\mathcal{D}\nabla_\perp^2 - \frac{1}{\tau}\right)\Delta T + B|\psi|^2$$

Diffusion

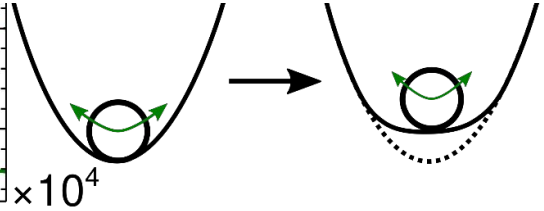


Variationsansatz:

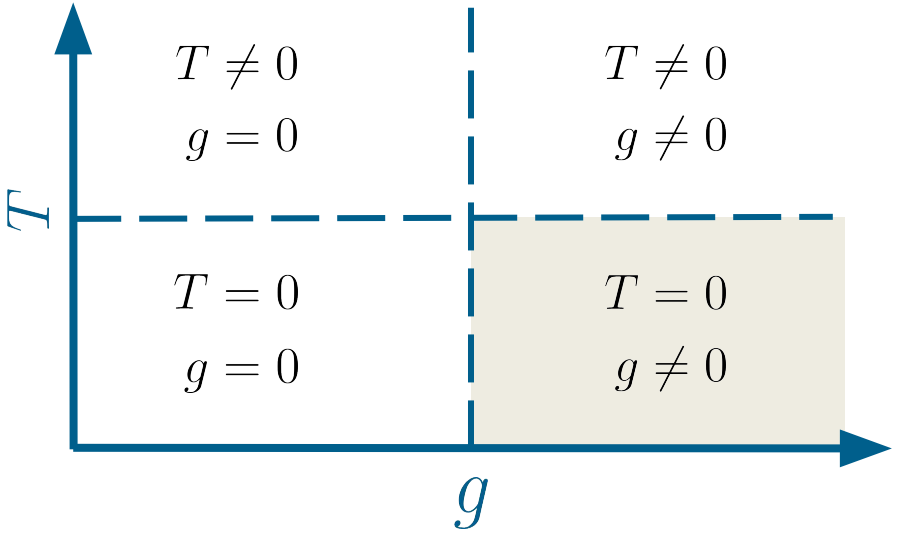


experimentell relevant

Dipolmode



Dimensionaler Übergang



Ziele:

- Verhalten der Photon-Photon-Wechselwirkung am dimensionalen Übergang 2D ⇔ 1D

⇒ Verstärkung der effektiven Wechselwirkung

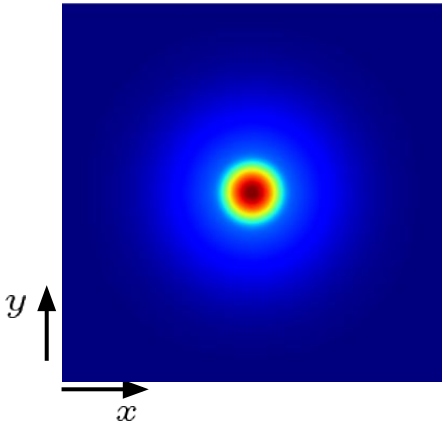
Stein und Pelster, New J. Phys. **24**, 023032 (2022)

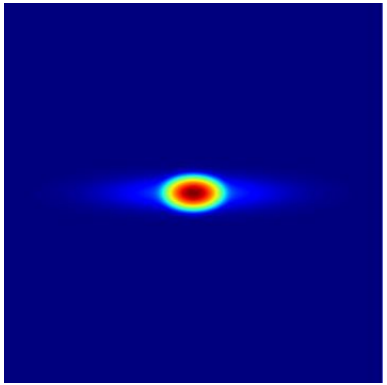
Übergang 2D ⇔ 1D

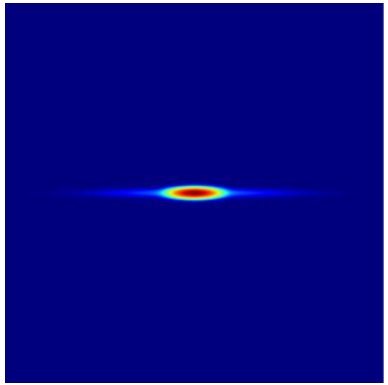
- **Fallenverhältnis:** $\lambda = \sqrt{\Omega_y/\Omega_x}$

$$V = \frac{m\Omega_x^2}{2}\left(x^2 + \lambda^4 y^2\right)$$

$\lambda = 1$ $\qquad$ $\lambda \to \infty$

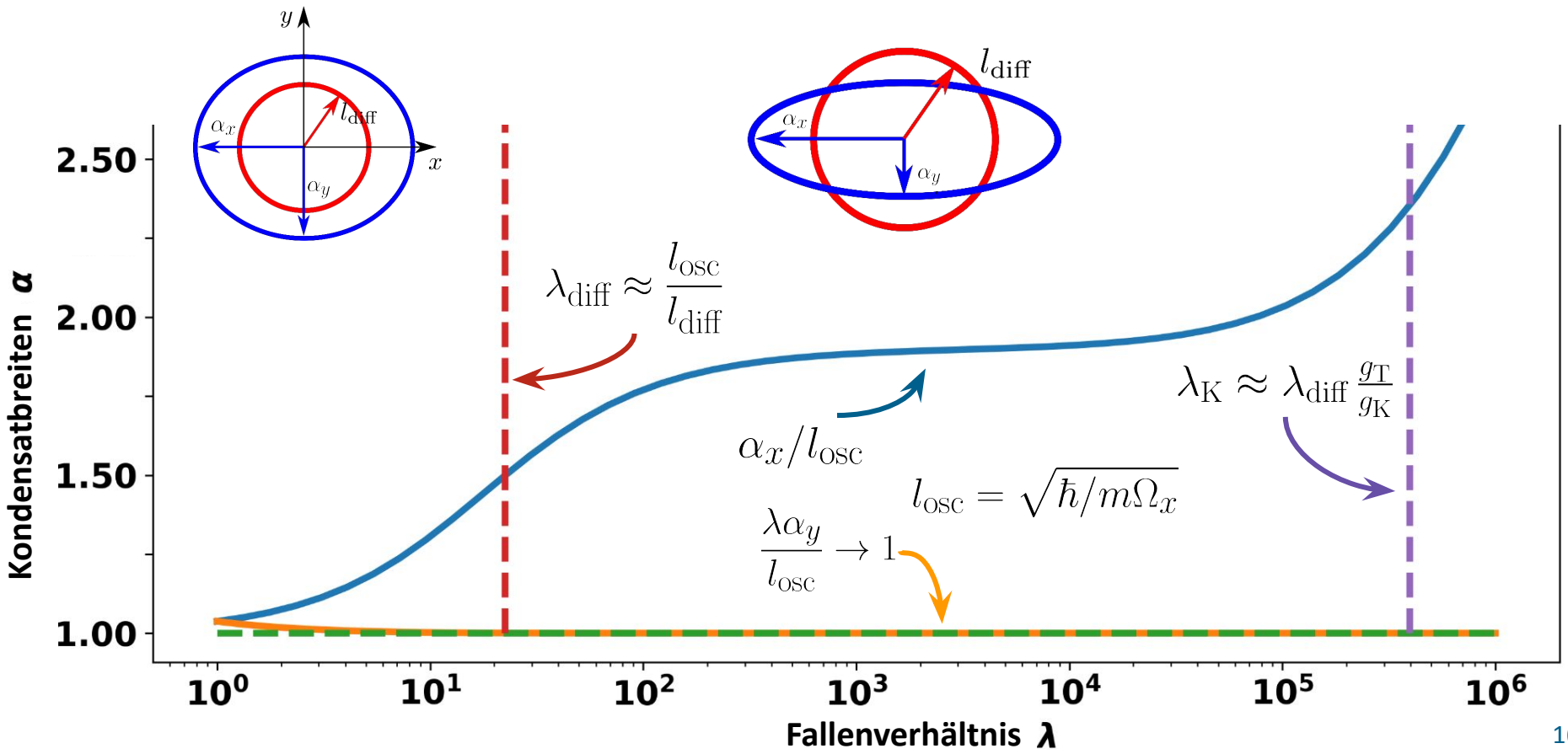




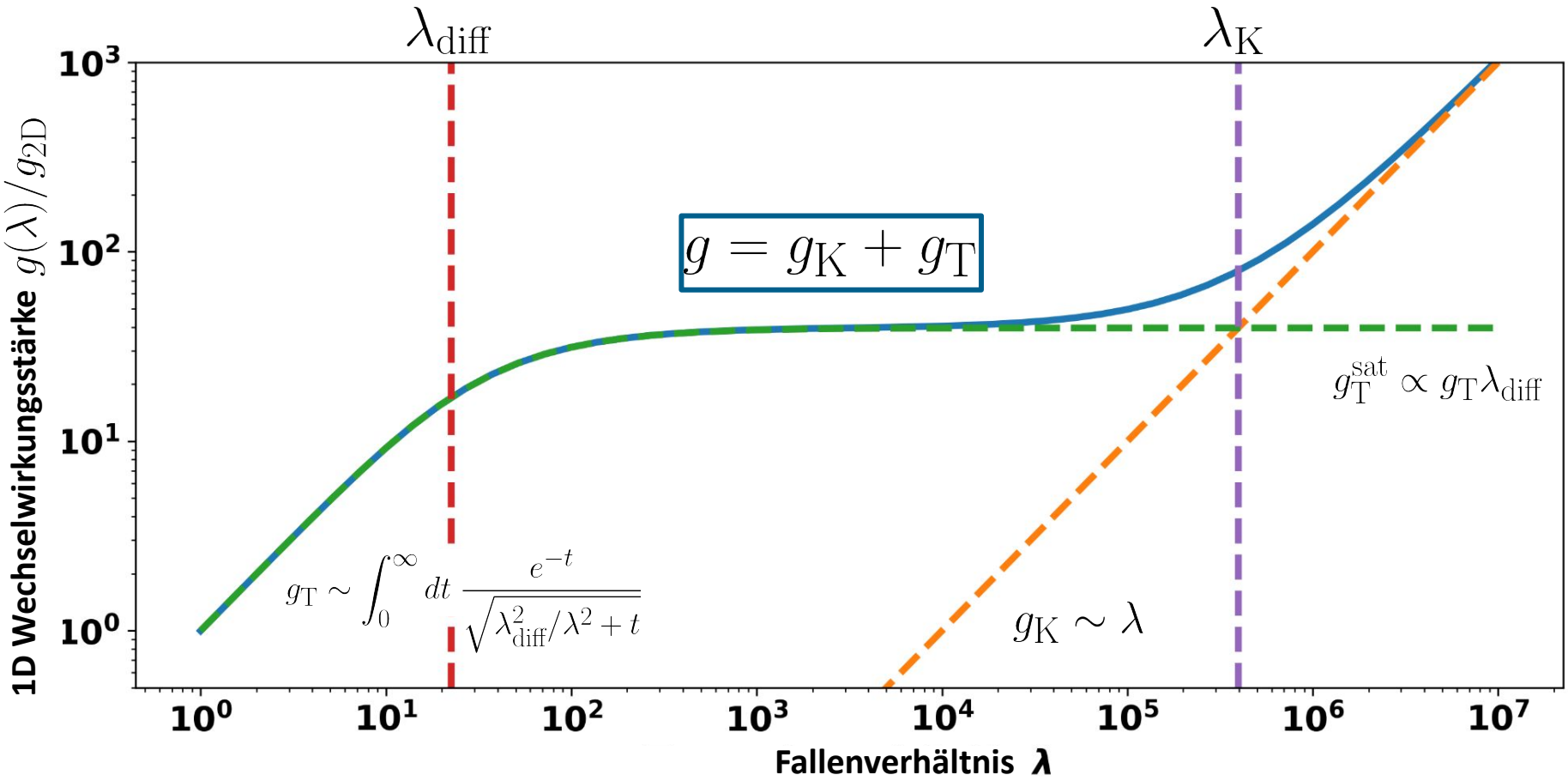


2D → **Dimensionaler Übergang** → 1D

Kondensatbreiten

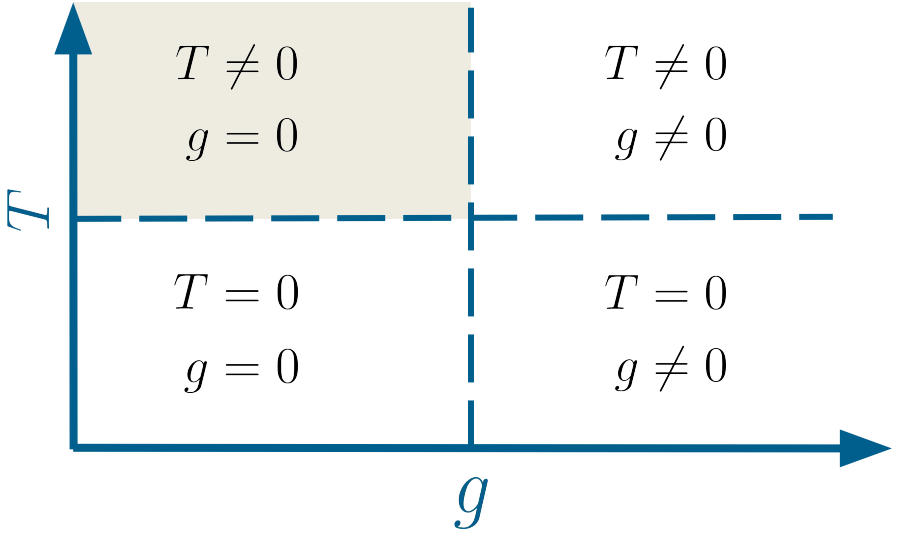


1D Wechselwirkungsstärke



Dimensionaler Übergang

Effektive Dimension



Ziele:

- Verständnis des dimensionalen Übergangs 2D ⇔ 1D
- Untersuchung der Thermodynamik eines idealen Bosegases

⇒ Definition der effektiven Dimension

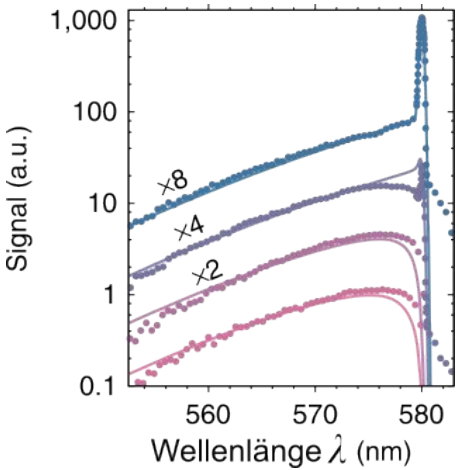
Stein und Pelster, New J. Phys. **24**, 023013 (2022)

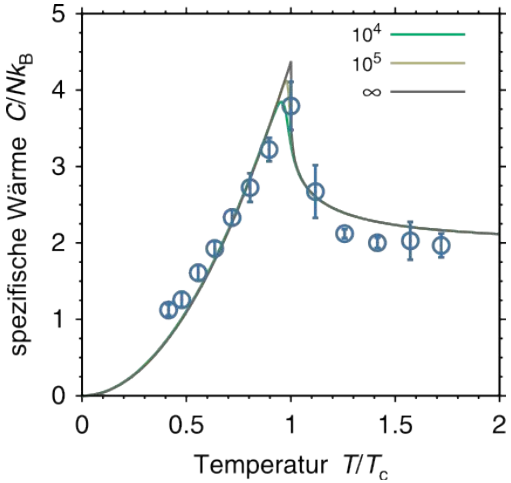
Experimente in 2D

- **Thermodynamik des Photon BECs:**

  Damm et al., Nat. Commun. **7,** 11340 (2016)

- **Spektrum enthält thermodynamische Eigenschaften**





⇒ **Photon BEC als ideales Bosegas in 2D!**

Großkanonisches Potential

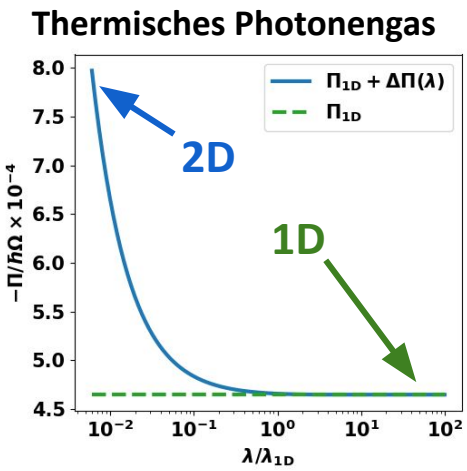
Potential: $$\Pi = -\frac{1}{\beta}\sum_{n,j=0}^{\infty} \ln\left\{1 - e^{-\beta[E_{nj}(\lambda)-\mu]}\right\}$$

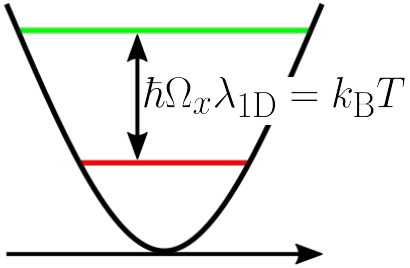
Energie: $$E_{nj}(\lambda) = \hbar\Omega_x[n + \lambda j + (1+\lambda)/2]$$

“Dimensionale Entwicklung”:

$$\Pi = \Pi_{1\mathrm{D}} + \Delta\Pi(\lambda)$$

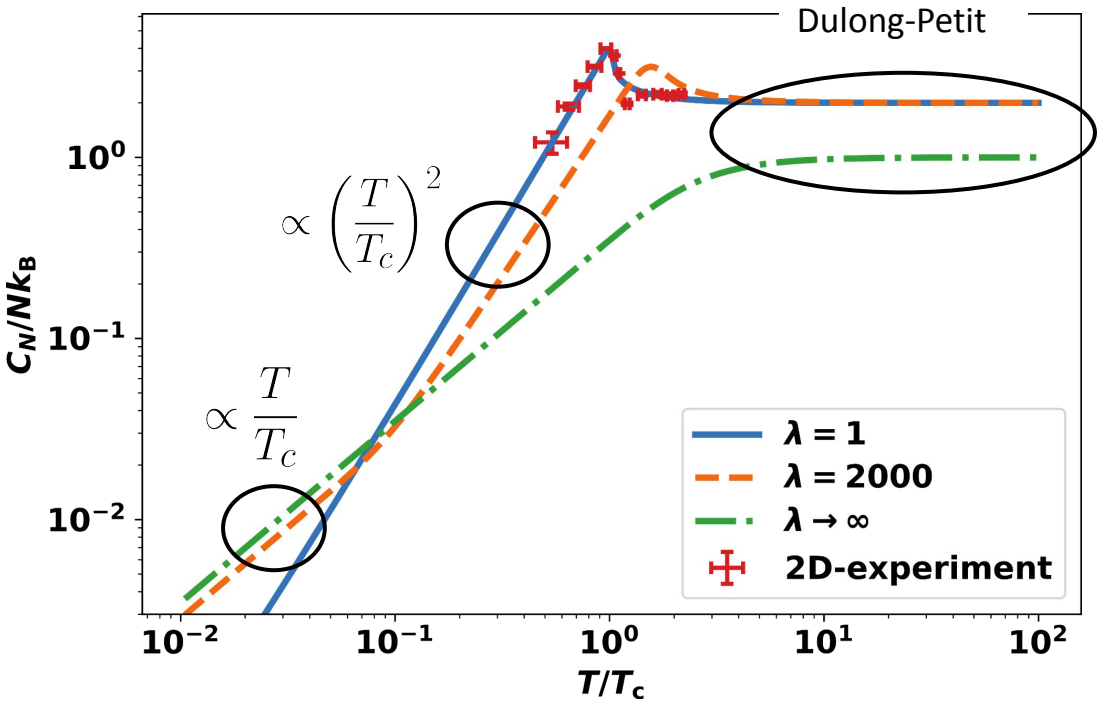
Analytisch approximierbar!

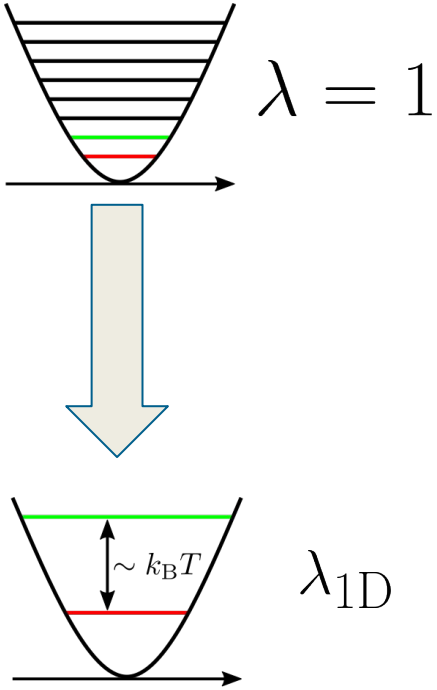




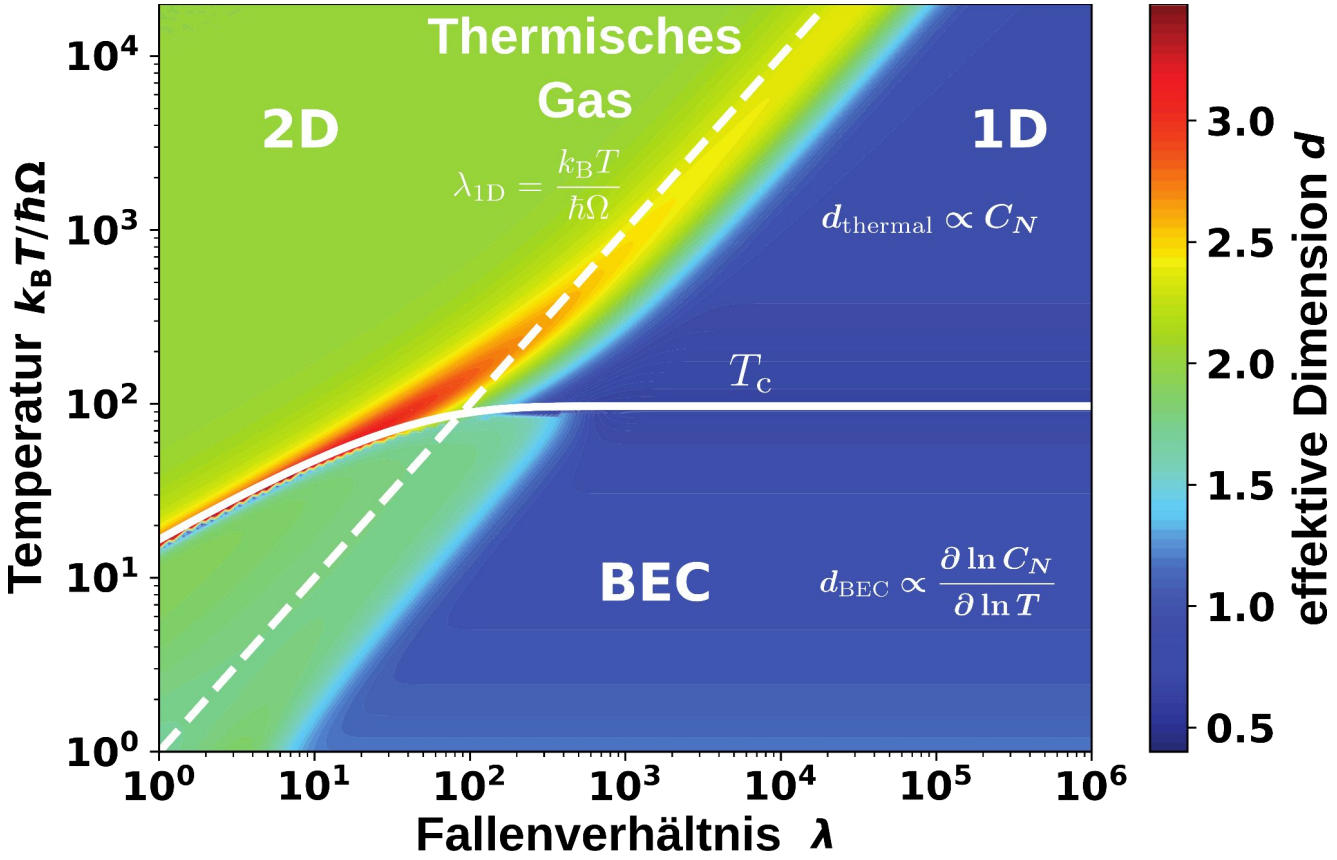
Spezifische Wärme

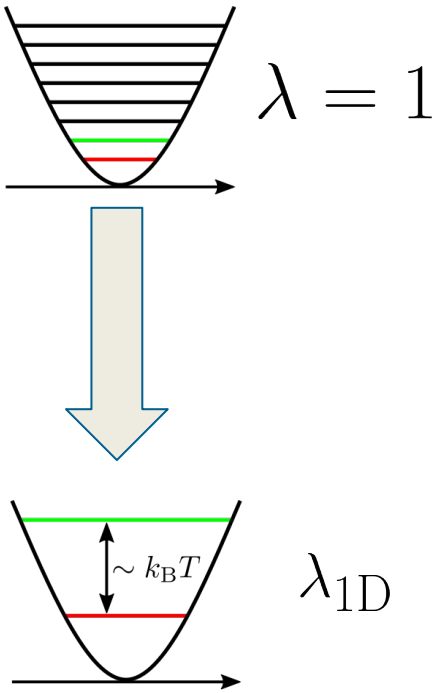
$$C_N = \left(\frac{\partial U}{\partial T}\right)_N$$





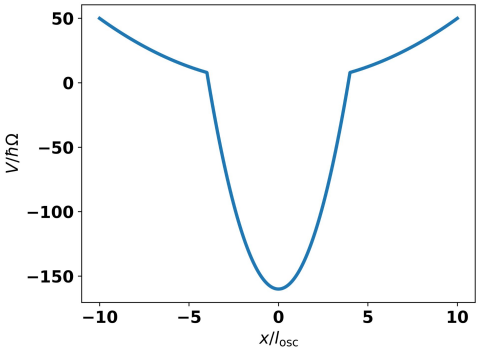
Effektive Dimension



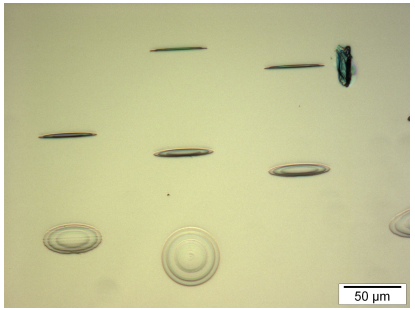


Experimentelle Realisierung

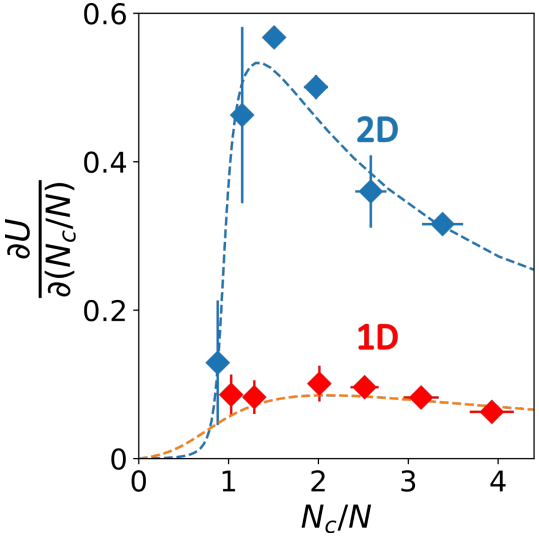
Dimple-Potential



Mikrostrukturierung von Julian Schulz

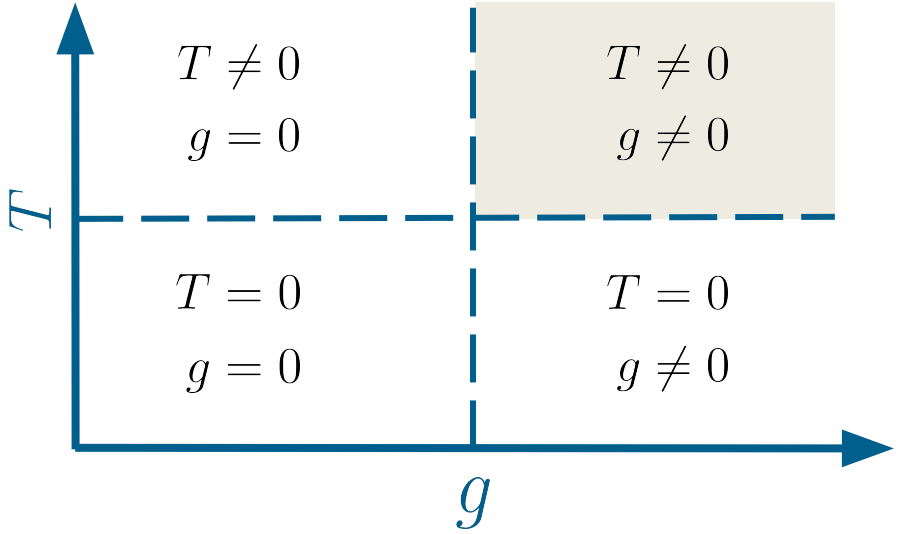


Messung der spezifischen Wärme von Kiran Umesh



$$\frac{N_c}{N} = \left(\frac{T}{T_c}\right)^D$$

Hartree-Fock-Analogon



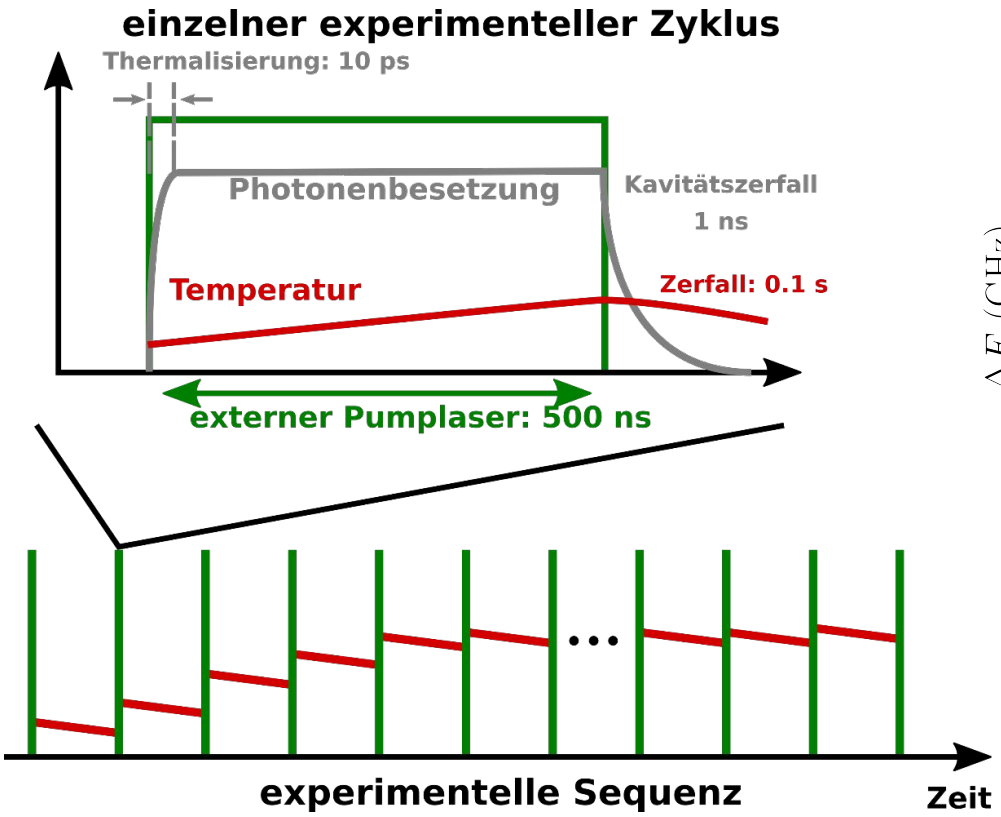
Ziele:

- ideales Gas vs. Wechselwirkung
- experimentelle Zeitskalen

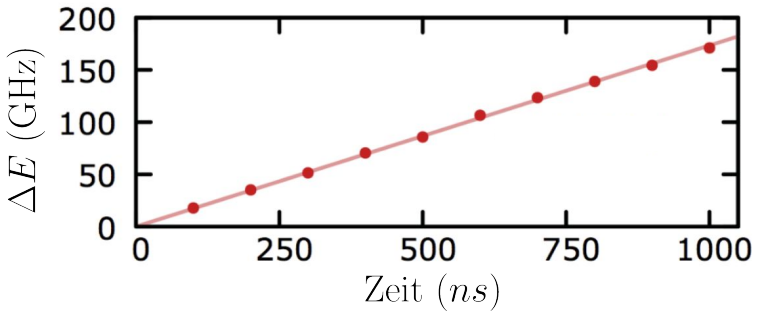
⇒ Hartree-Fock-Analogon

Stein und Pelster, arXiv: 2203.16955 (2022)
Stein und Pelster, arXiv: 2204.08818 (2022)

Zeitskalen



Relative Grundzustandsenergie



Dung et al., Nature Photonics **11**, 565 (2017)

⇒ relative Verschiebung durch Thermooptik

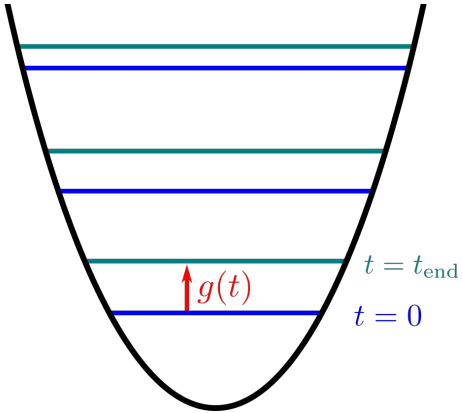
Kurzzeitapproximation

- Hamiltonian eines experimentellen Zyklus:

$$\hat{H}(t) = \sum_m E_m(0)\hat{a}_m^\dagger \hat{a}_m + g(t) \sum_{mn} F_{mn}\hat{a}_m^\dagger \hat{a}_n$$

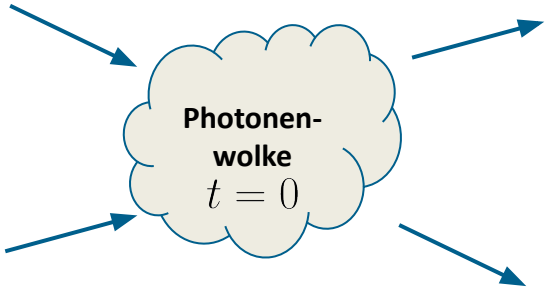
$g(t) \propto t$ adiabatisch!

- zeitabhängige Energieverschiebung



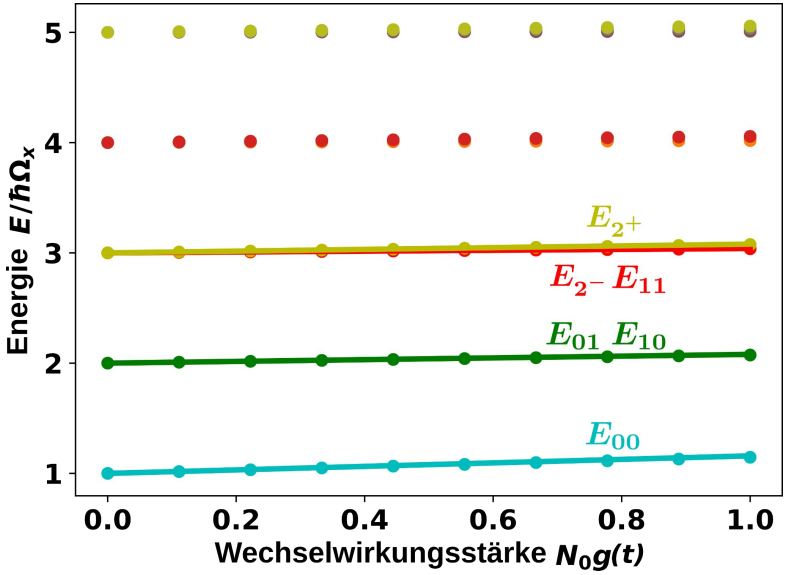
- Wechselwirkung $\tau\Omega \gg 1$ $\Rightarrow$ effektives Potential durch Anfangswolke

$F_{mn} =$

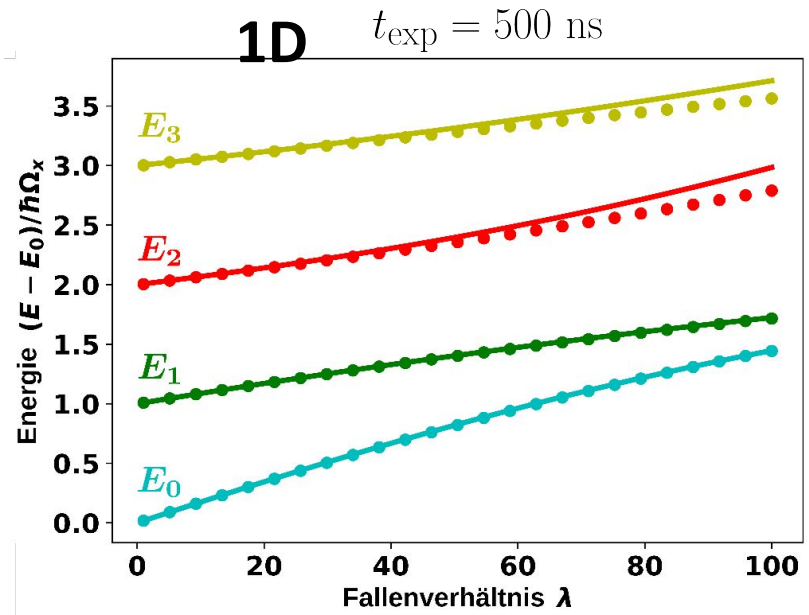


Spektrum

2D



Übergang 2D ⇔ 1D



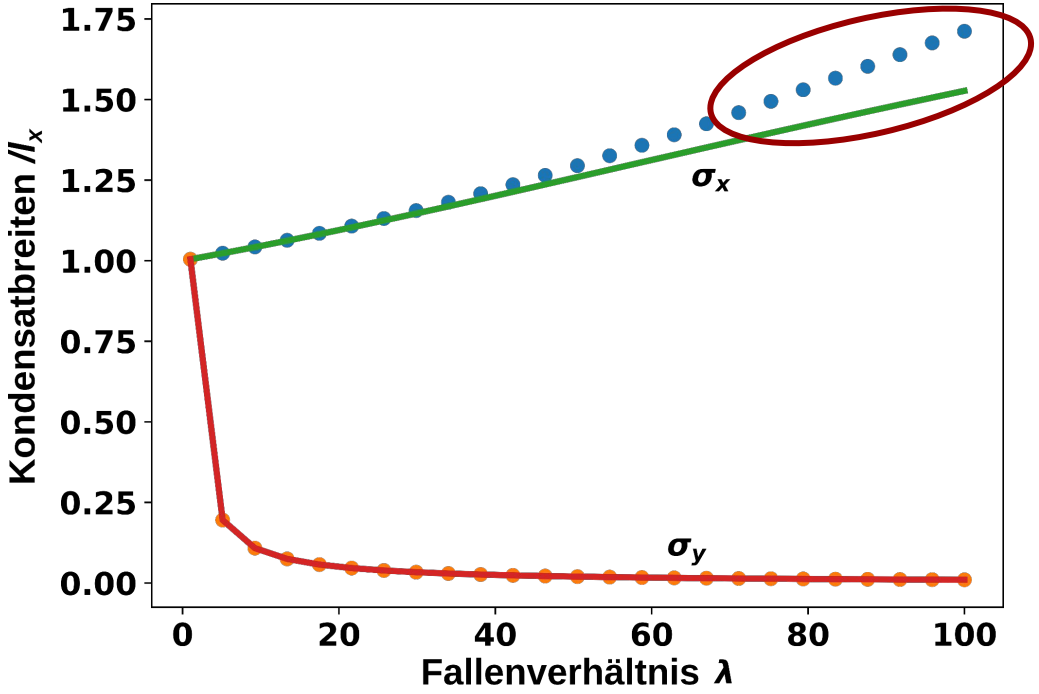
$N = 10.000,$
$N_0/N \approx 1,$

- Punkte: Exakte Diagonalisierung
- Linien: Störungstheorie

- Effektive 1D Wechselwirkungsstärke:

$$g_{1\mathrm{D}} \propto \lambda$$

Kondensatbreiten



Abweichung durch thermische Wolke!

- Effektive 1D Wechselwirkungsstärke:

$$g_{1\mathrm{D}} \propto \lambda$$

- Punkte: Exakte Diagonalisierung
- Linien: Variationsansatz bei *T=0*

$$V = \frac{m\Omega^2}{2}\left(x^2 + \lambda^4 y^2\right)$$

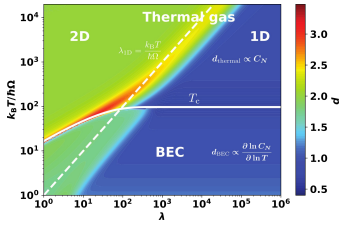
$$N = 10.000, \; N_0/N \approx 1,$$
$$mg/\hbar^2 = 10^{-5}, t_{\mathrm{exp}} = 500 \text{ ns}$$

Zusammenfassung

Temperatur T

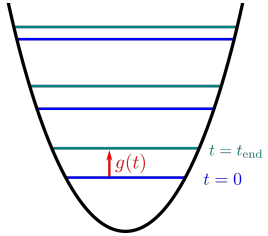
$T \neq 0$
$g = 0$

- Effektive Dimension am Übergang 2D ⇔ 1D



$T \neq 0$
$g \neq 0$

- Hartree-Fock-Analogon



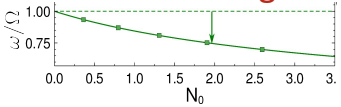
$T = 0$
$g = 0$

QM1: Harmonischer Oszillator

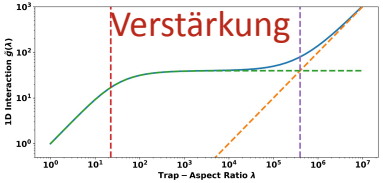
$T = 0$
$g \neq 0$

- Kollektive Anregungen
- Dimensionaler Übergang

Retardierung



Verstärkung



Wechselwirkung g