Trapped Bose-Einstein Condensates with Strong Disorder*

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Disorder: Realizations and characterization

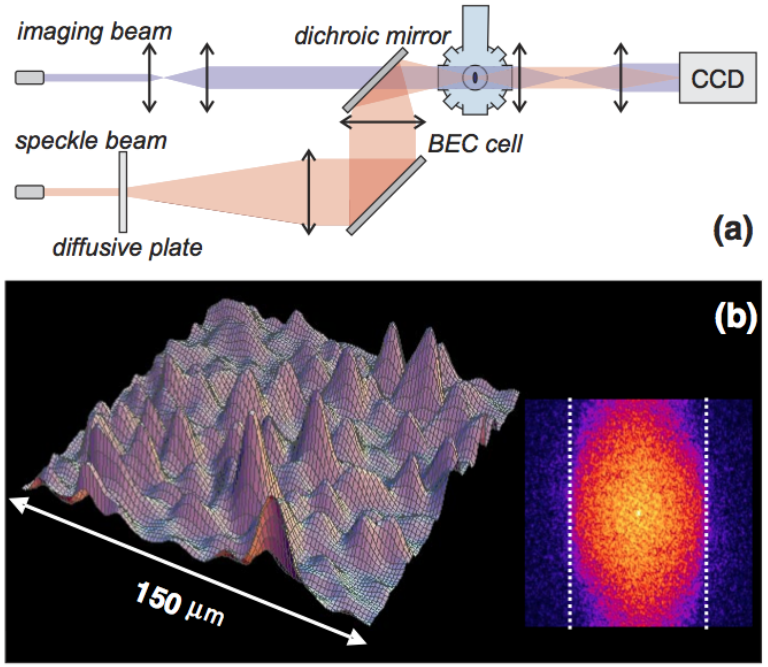
- Natural disorder due to impurities
- Superfluid helium in porous media: persistence of superfluidity

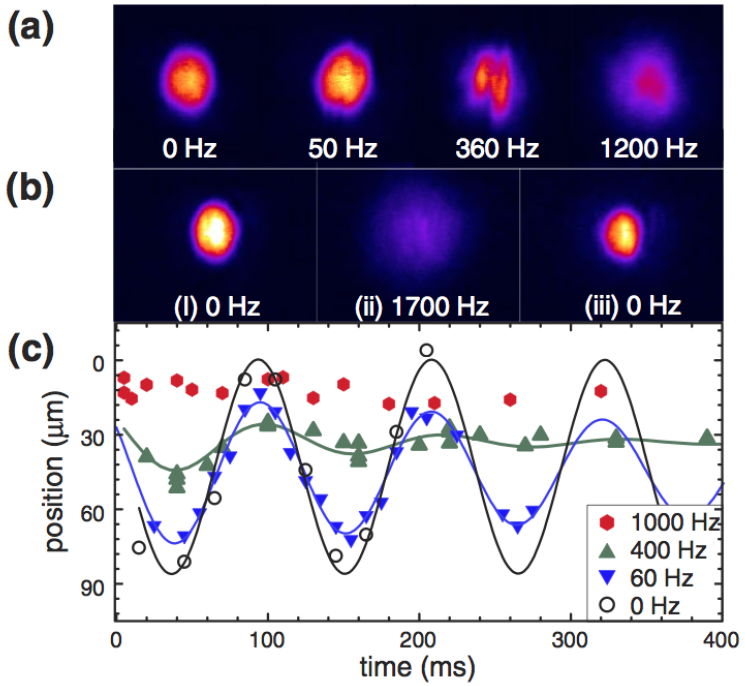
  Crooker et al., PRL **51**, 666 (1983)
- Laser speckles: controlled randomness
  Lye et al., PRL **95**, 070401 (2005); Clément et al., PRL **95**, 170409 (2005); Billy et al., Nature **453**, 891 (2008)
- Wire traps: undesired randomness
  Krüger et al., PRA **76**, 063621 (2007)
  Fortágh and Zimmermann, RMP **79**, 235 (2007)
- Incommensurate lattices: quasi-randomness
  Damski et al., PRL **91**, 080403 (2003); Schulte et al., PRL **95**, 170411 (2005)
  Roati et al., Nature **453**, 895 (2008)
- Disorder potential $U(\mathbf{x})$ characterized by the correlators

$$\langle U(\mathbf{x})\rangle = 0, \quad \langle U(\mathbf{x})U(\mathbf{x}')\rangle = R(\mathbf{x}-\mathbf{x}'), \ldots$$

Laser speckles

- Lye et al., PRL **95**, 070401 (2005)





- Condensate depletion due to disorder
- Global condensate vanishes for sufficiently strong disorder: Bose-glass phase transition

Non-perturbative approach

- Cumulants of the wave function and the disorder potential

$$\langle \Psi(\mathbf{x})\Psi(\mathbf{x}')\rangle_c = \langle \Psi(\mathbf{x})\Psi(\mathbf{x}')\rangle - \langle \Psi(\mathbf{x})\rangle\langle \Psi(\mathbf{x}')\rangle \equiv G_{\Psi\Psi}(\mathbf{x},\mathbf{x}')$$

$$\langle U(\mathbf{x})\Psi(\mathbf{x}')\rangle_c = \langle U(\mathbf{x})\Psi(\mathbf{x}')\rangle \equiv G_{U\Psi}(\mathbf{x},\mathbf{x}')$$

- Particle density: $n(\mathbf{x}) = \langle \Psi(\mathbf{x})\Psi(\mathbf{x})\rangle$
- Condensate density: $n_0(\mathbf{x}) = \langle \Psi(\mathbf{x})\rangle^2$
- Condensate depletion: $q(\mathbf{x}) = n(\mathbf{x}) - n_0(\mathbf{x})$
- Homogeneous case: off-diagonal long-range order

$$\lim_{|\mathbf{x}-\mathbf{x}'|\to\infty} \left\langle \Psi(\mathbf{x})\Psi(\mathbf{x}')\right\rangle = n_0$$

$$\lim_{|\mathbf{x}-\mathbf{x}'|\to\infty} \left\langle |\Psi(\mathbf{x})\Psi(\mathbf{x}')|^2\right\rangle = (n_0+q)^2$$

Graham and Pelster, Int. J. Bifurc. Chaos **19**, 2745 (2009)

Self-consistency equations (1)

- Averaging the Gross-Pitaevskii equation and using the Gaussian approximation, we get

$$\left\{-\frac{\hbar^2}{2M}\Delta + V(\mathbf{x}) - \mu + 3gn(\mathbf{x}) - 2gn_0(\mathbf{x})\right\}\psi(\mathbf{x}) = -G_{U\Psi}(\mathbf{x},\mathbf{x})$$

$$\left\{-\frac{\hbar^2}{2M}\Delta + V(\mathbf{x}) - \mu + 3gn(\mathbf{x})\right\}G_{U\Psi}(\mathbf{x}',\mathbf{x}) = -R(\mathbf{x}-\mathbf{x}')\psi(\mathbf{x})$$

$$\left\{-\frac{\hbar^2}{2M}\Delta + V(\mathbf{x}) - \mu + 3gn(\mathbf{x})\right\}G_{\Psi\Psi}(\mathbf{x},\mathbf{x}') = -G_{U\Psi}(\mathbf{x},\mathbf{x}')\psi(\mathbf{x})$$

- Condensate and particle density

$$n_0(\mathbf{x}) = \psi(\mathbf{x})^2$$

$$n(\mathbf{x}) = n_0(\mathbf{x}) + G_{\Psi\Psi}(\mathbf{x},\mathbf{x})$$

Self-consistency equations (2)

- Using Fourier transformation and semiclassical approximation, we get

$$\left\{-\frac{\hbar^2}{2M}\Delta + V(\mathbf{x}) - \mu + 3gn(\mathbf{x}) - 2gn_0(\mathbf{x})\right\}\psi(\mathbf{x}) = \psi(\mathbf{x})\int \frac{d^D k}{(2\pi)^D}\frac{R(\mathbf{k})}{\frac{\hbar^2\mathbf{k}^2}{2M} + V(\mathbf{x}) - \mu + 3gn(\mathbf{x})}$$

$$n(\mathbf{x}) = n_0(\mathbf{x}) + n_0(\mathbf{x})\int \frac{d^D k}{(2\pi)^D}\frac{R(\mathbf{k})}{\left(\frac{\hbar^2\mathbf{k}^2}{2M} + V(\mathbf{x}) - \mu + 3gn(\mathbf{x})\right)^2}$$

$$n_0(\mathbf{x}) = \psi(\mathbf{x})^2$$

- Fixing the chemical potential: $N = \int d^D x\, n(\mathbf{x})$
- Number of atoms in global condensate: $N_0 = \int d^D x\, n_0(\mathbf{x})$
- Weak disorder: reproduces Huang-Meng theory
  Nikolić, Balaž, Pelster, PRA **88**, 013624 (2013)

Lorentzian-correlated disorder in $D = 3$

- Disorder correlation function

$$R(\mathbf{k}) = \frac{R}{1 + \xi^2 \mathbf{k}^2}$$

- Equations in $D = 3$

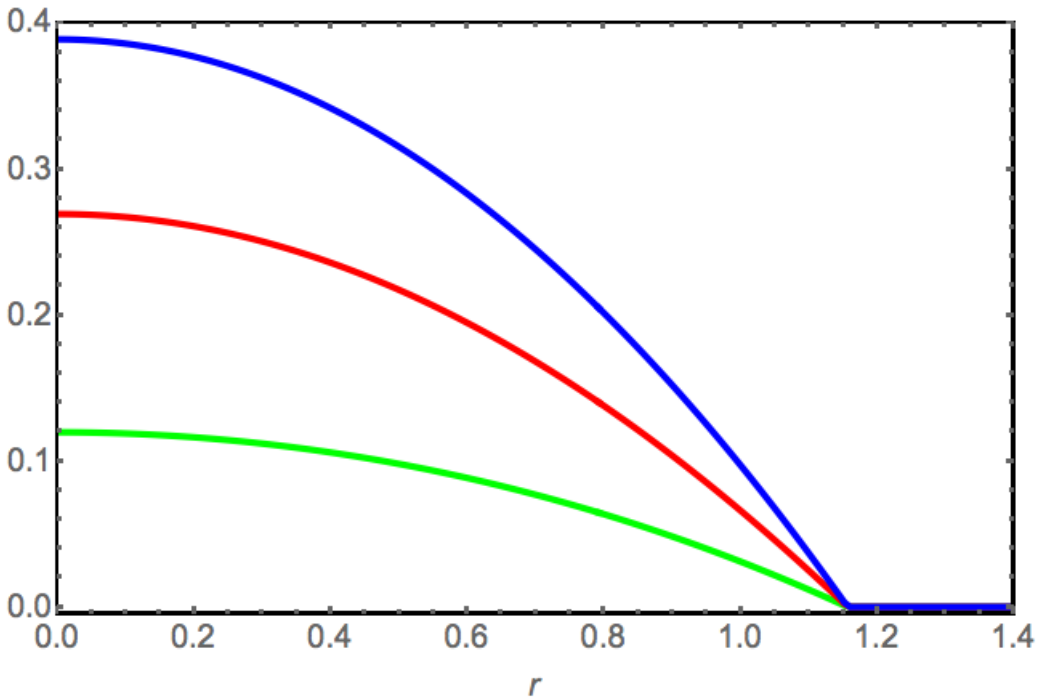
$$\left\{ -\frac{\hbar^2}{2M}\Delta + V(\mathbf{x}) - \mu + 3gn(\mathbf{x}) - 2gn_0(\mathbf{x}) \right\} \psi(\mathbf{x}) = \frac{MR}{2\pi\hbar^2\xi} \frac{\psi(\mathbf{x})}{1 + \sqrt{V(\mathbf{x}) - \mu + 3gn(\mathbf{x})} / \sqrt{\frac{\hbar^2}{2M\xi^2}}}$$

$$n(\mathbf{x}) = n_0(\mathbf{x}) + \frac{M^2 R}{2\pi\hbar^2 \sqrt{V(\mathbf{x}) - \mu + 3gn(\mathbf{x})} / \sqrt{\frac{\hbar^2}{2M}}} \frac{n_0(\mathbf{x})}{\left(1 + \sqrt{V(\mathbf{x}) - \mu + 3gn(\mathbf{x})} / \sqrt{\frac{\hbar^2}{2M\xi^2}}\right)^2}$$

Spatially homogeneous case

- $V(\mathbf{x}) = 0$, $n(\mathbf{x}) = n$, $n_0(\mathbf{x}) = n_0$
- Limit of the strong disorder
  - In $D = 3$, $n_0/n \to 2/3$, $\tilde{\mu} \to -\left(\frac{2\tilde{R}}{\tilde{\xi}^2}\right)^{2/3}$
  - In $D = 2$, $n_0/n \to 1/2$, $\tilde{\mu} \to \tilde{\mu}_\infty$, $(-\tilde{\mu}_\infty)^2 = \frac{\tilde{R}}{\tilde{\xi}^2} \ln[(-\tilde{\mu}_\infty)\tilde{\xi}^2]$
  - In $D = 1$, $n_0/n =\to 1/2$, $\tilde{\mu} \to -\sqrt{\frac{2\tilde{R}}{\tilde{\xi}}}$
- No Bose-glass phase

Trapped case: Thomas-Fermi approximation



Particle density (blue), condensate density (red), and condensate depletion (green) as a function of the radial coordinate in $D = 3$ for $\tilde{R} = 1000$, $\tilde{\xi} = 7$.

- Similar behavior in $D = 1, 2$
- Again, elusive Bose-glass phase is nowhere to be found

Search for a Bose-glass phase

- Using the TF radius as a length scale, in $D = 3$ we get the following dimensionless equations:

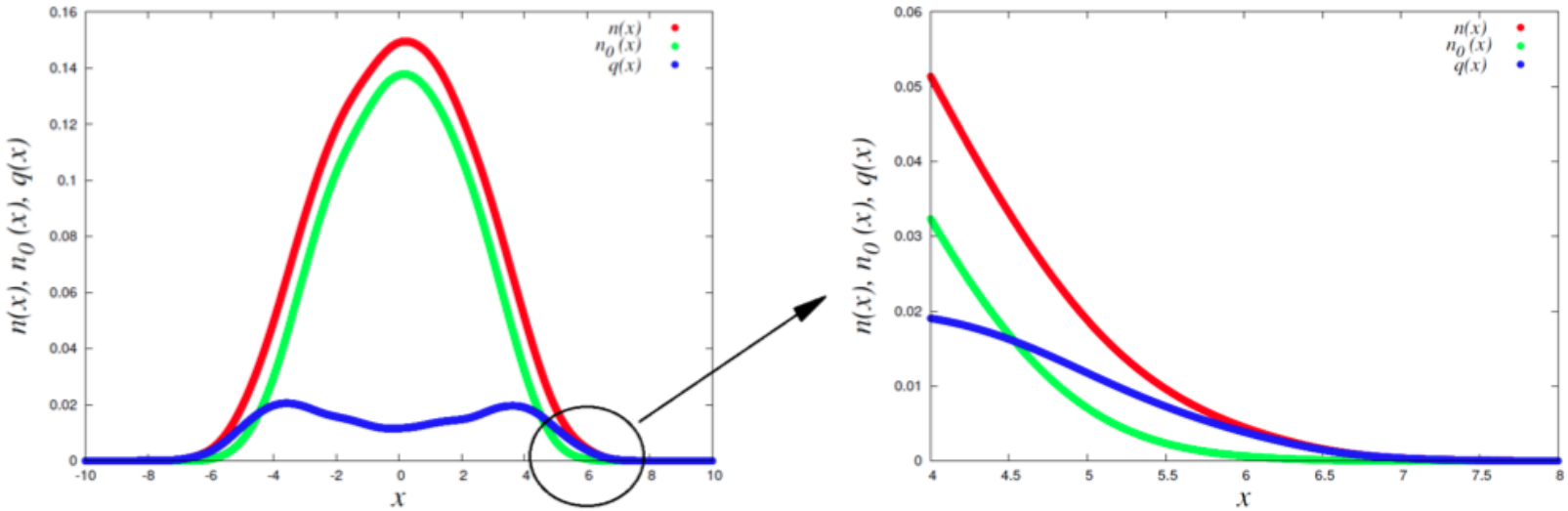
$$\left\{-\frac{1}{\theta}\Delta + V - \mu + 3gn - 2gn_0\right\}\psi = \frac{2R\psi}{\xi + \xi^2\sqrt{V - \mu + 3gn}}$$

$$n = n_0 + \frac{R}{\sqrt{V - \mu + 3gn}}\,\frac{n_0}{\left(1 + \xi\sqrt{V - \mu + 3gn}\right)^2}$$

$$n_0 = \psi^2\,, \quad 1 = \int 4\pi r^2 n\, dr\,, \quad N_0/N = \int 4\pi r^2 n_0\, dr\,, \quad V = r^2$$

- For ^{87}Rb, with $N = 2 \cdot 10^5$, $a_s = 100.4\, a_0$, $\Omega = 100 \cdot 2\pi$ Hz:
  - $L_{\rm ho} = 1.08\,\mu$m, $R_{\rm TF} = 7.35\,\mu$m
  - $\theta = (R_{\rm TF}/L_{\rm ho})^4 \sim 2100$
- Strongly suppressed kinetic term hides the Bose-glass phase
- Its extent depends not only on θ, but also on disorder strength R and correlation length ξ

Trapped case: preliminary numerical results



Particle density (red), condensate density (green), and condensate depletion (blue) in $D = 1$ for a Gaussian-correlated disorder $R(k) = R\,e^{-\xi^2 k^2/2}$, $R = 100$, $\xi = 1$.

- Numerical data suggest that the Bose-glass phase appears at the boundary of the condensate

Conclusions and outlook

- We have studied the emergence of a Bose-glass phase in trapped BECs
- Non-perturbative approach for strong disroder, Gaussian approximation
- Lorentzian-correlated disorder in $D = 1, 2, 3$
  - No Bose-glass phase in the homogeneous case
  - In the trapped case, TF approximation is not sufficient to study the effect
  - Numerical data suggest that the full approach captures the emergence of a Bose-glass phase at the boundary
- Outlook:
  - Variational and numerical solution of the full system of equations
  - $D = 2$ relevant for photonic BEC experiments with disorder - Weitz group