On the Theoretical Description of Photon BECs

Axel Pelster

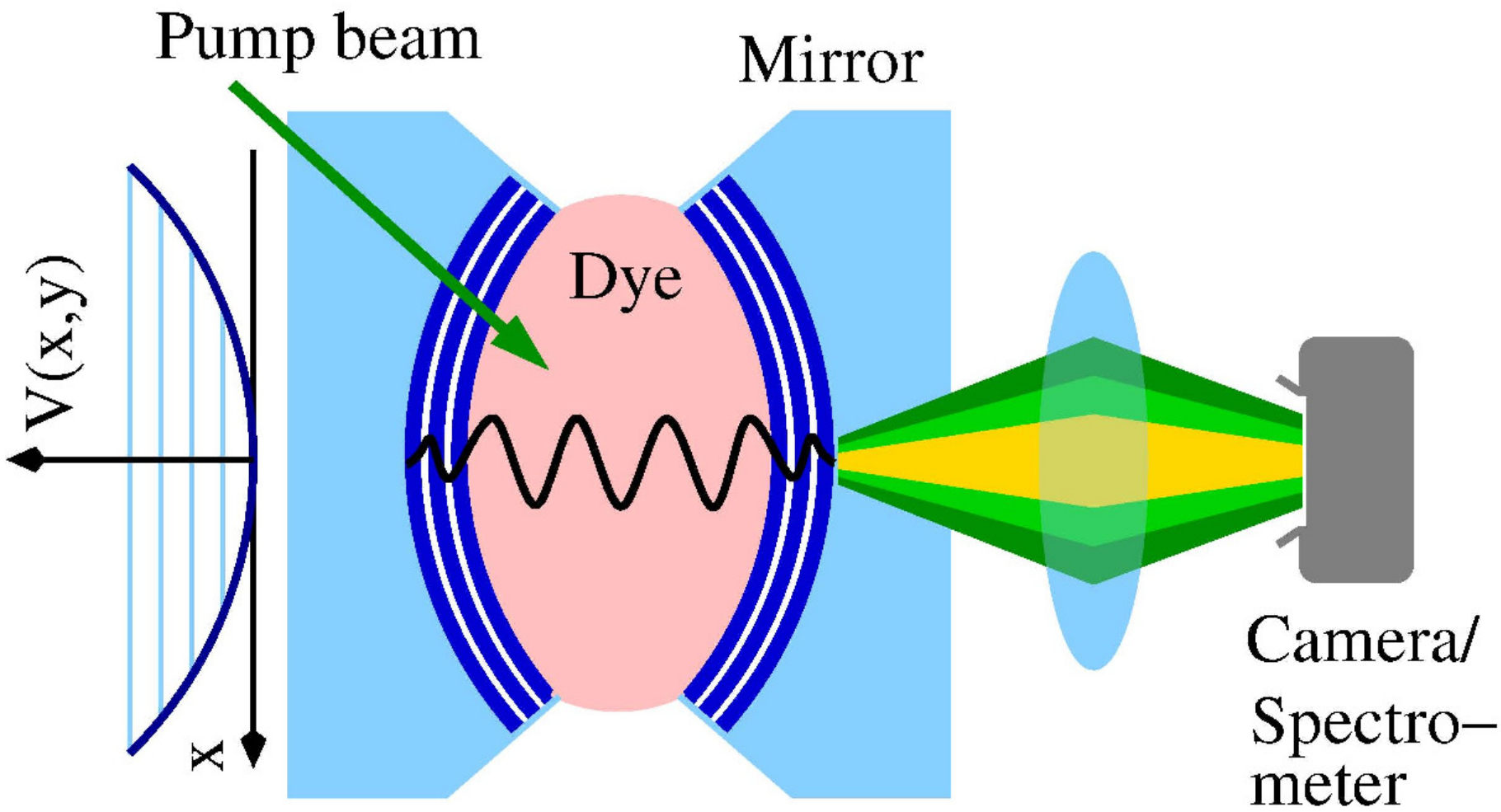
1. Introduction

2. Dimensional Crossover: Ideal Bose Gas

3. Dimensional Crossover: Interactions

4. Outlook

1.1 Set-Up of Bonn Experiment

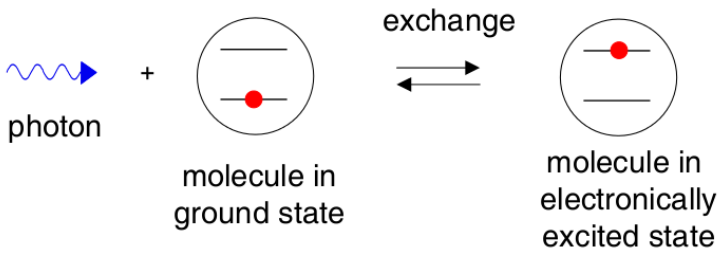


Klaers, Vewinger, and Weitz, Nature Phys. **6**, 512 (2010)

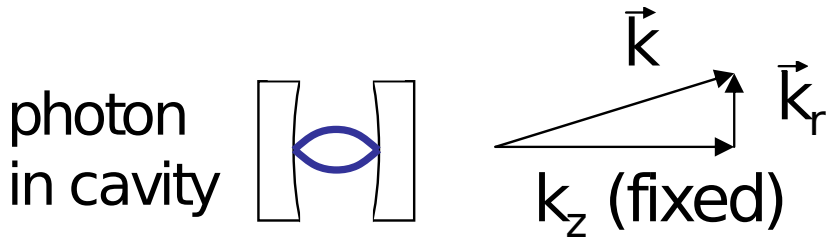
Klaers, Schmitt, Vewinger, and Weitz, Nature **468**, 545 (2010)

1.2 Basic Ingredients

- **Multiple absorption and emission of photons by dye molecules:**



- **Quadratic photon dispersion due to paraxial approximation:**



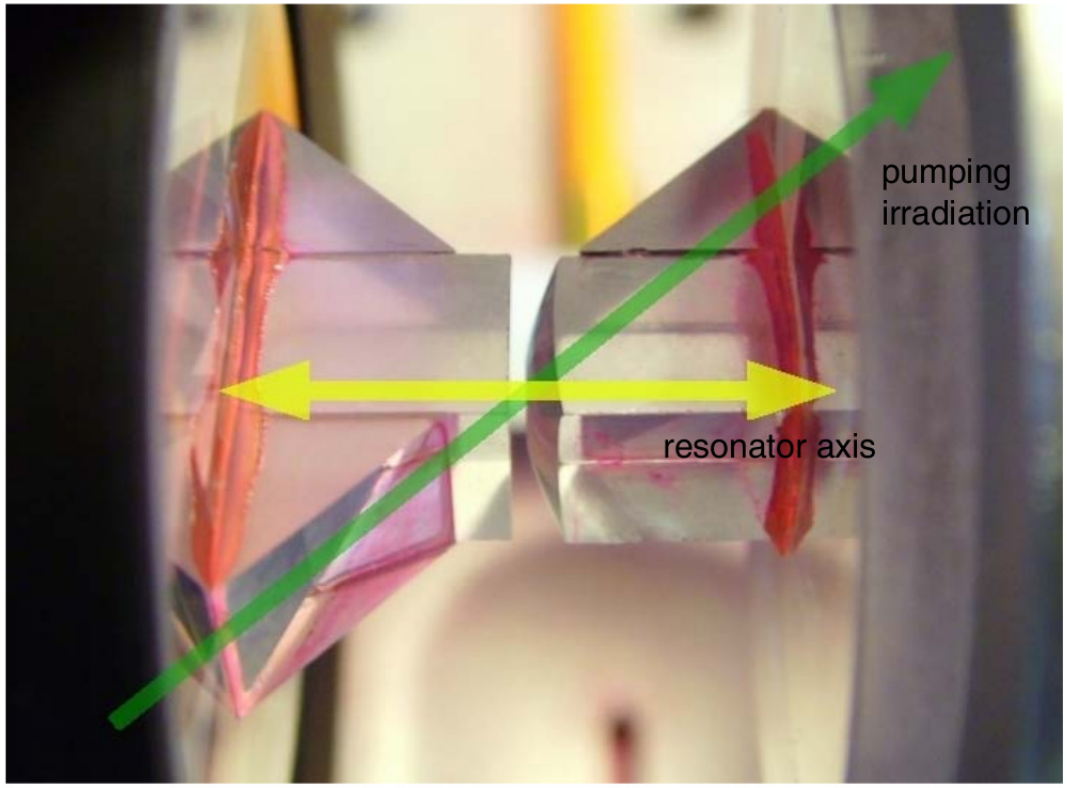
$$E_{\text{kin}} = \hbar c\sqrt{k_z^2 + k_r^2} \approx m_{\text{eff}}c^2 + \frac{\hbar^2 k_r^2}{2m_{\text{eff}}}\,, \qquad m_{\text{eff}} = \frac{\hbar k_z}{c}$$

- **Harmonic potential from mirror curvature:**

$$E_{\text{pot}} = \frac{1}{2}m_{\text{eff}}\Omega^2 r^2\,, \qquad \Omega = c\sqrt{\frac{2}{LR}}$$

- **c: light velocity in dye solution**

1.3 Experiment



Bonn: Klaers, Schmitt, Vewinger, and Weitz, Nature **468**, 545 (2010)

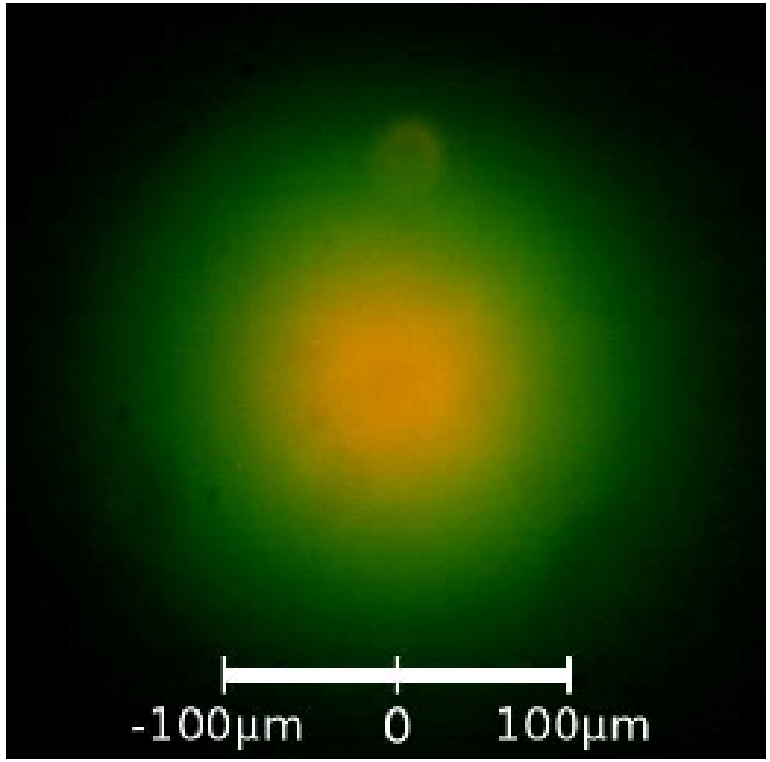
London: Marelic and Nyman, PRA **91**, 033813 (2015)

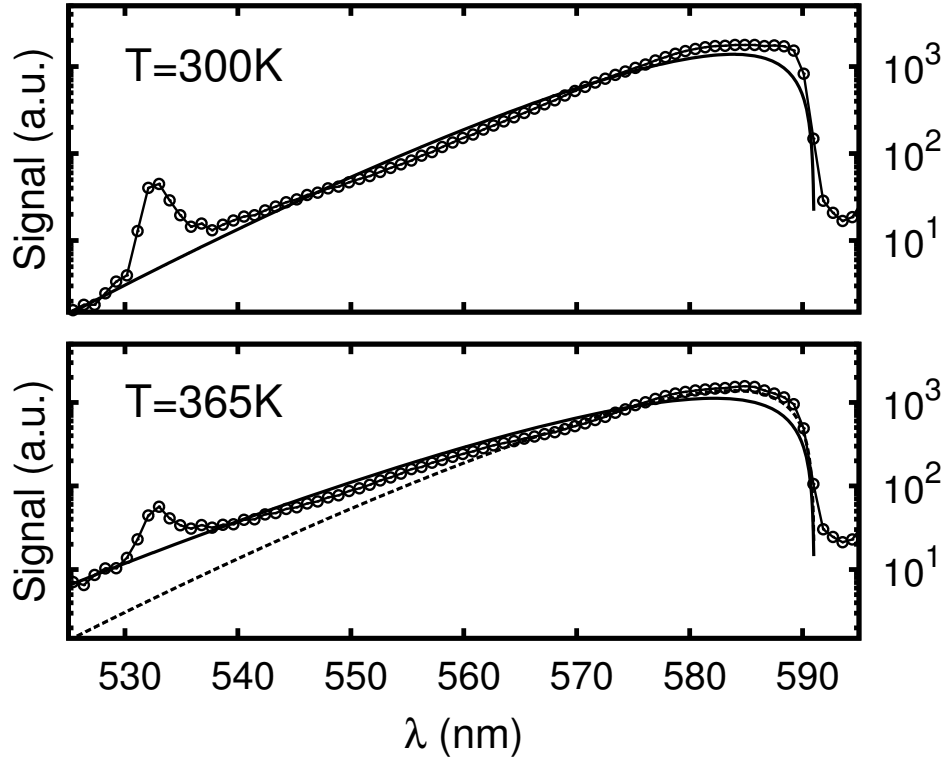
Utrecht: Greveling, Perrier, and van Oosten, PRA **98**, 013810 (2018)

Twente: Vretenar, Kassenberg, Bissesar, Toebes, and Klaers, PRR **3**, 023167 (2021)

[**Haifa:** Weill, Bekker, and Fischer, NC **10**, 747 (2019) **(1D fiber cavity)**]

1.4 Thermal Photon Gas





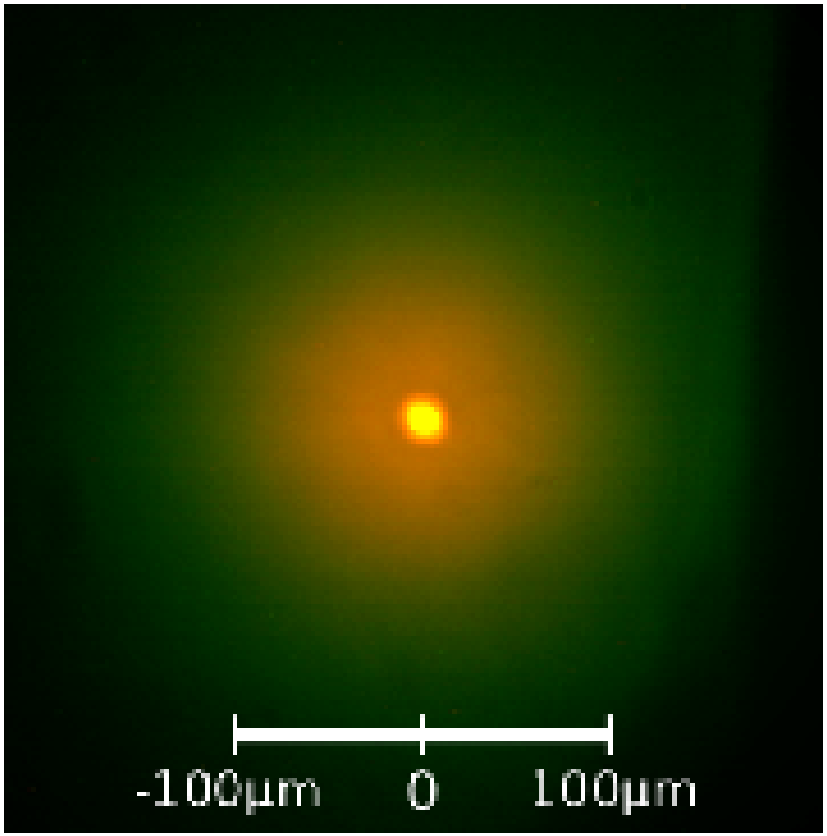
Below BEC transition:

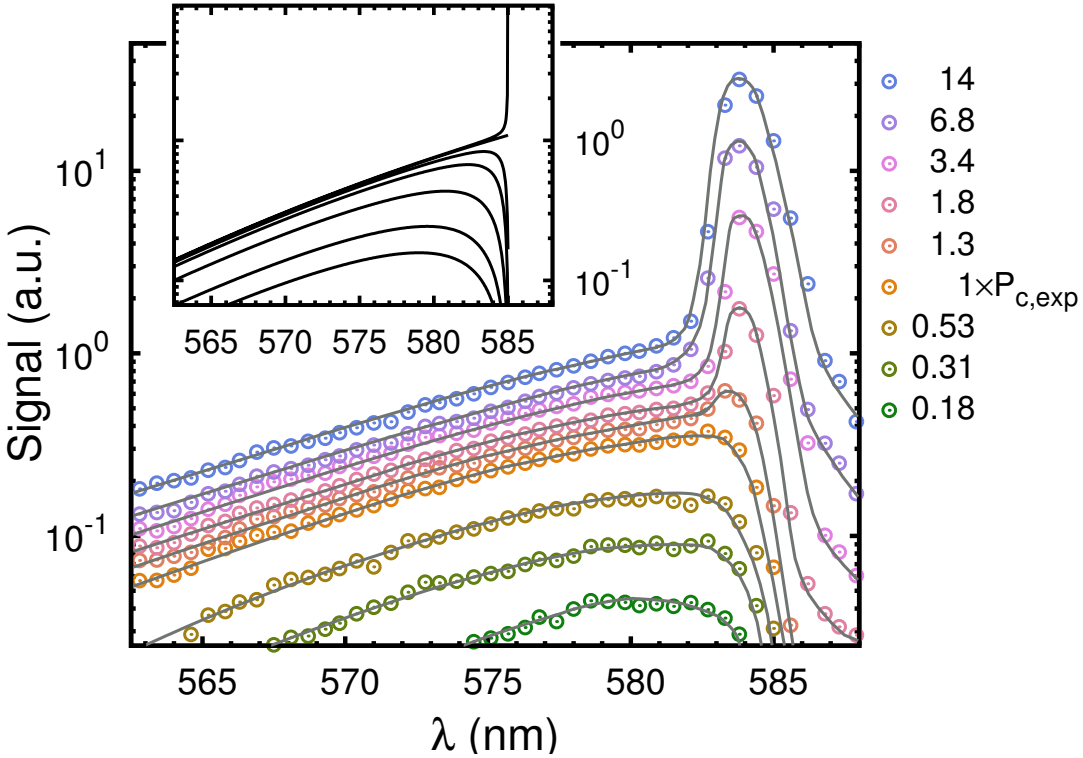
$$N < N_\mathrm{c} = \frac{\pi^2}{3}\left(\frac{k_\mathrm{B}T}{\hbar\Omega}\right)^2 \approx 77\,000\,, \qquad N_\mathrm{c}^\mathrm{exp} = (6.3 \pm 2.4) \times 10^4$$

$T = 300$ K, $\Omega = 2\pi \cdot 4 \times 10^{10}$ Hz, $m_\mathrm{eff} = 6.7 \times 10^{-36}$ kg $\approx 10^{-10}\, m_\mathrm{Rb}$

Klärs, Schmitt, Vewinger, and Weitz, Nature **468**, 545 (2010)

1.5 Photon Bose-Einstein Condensate



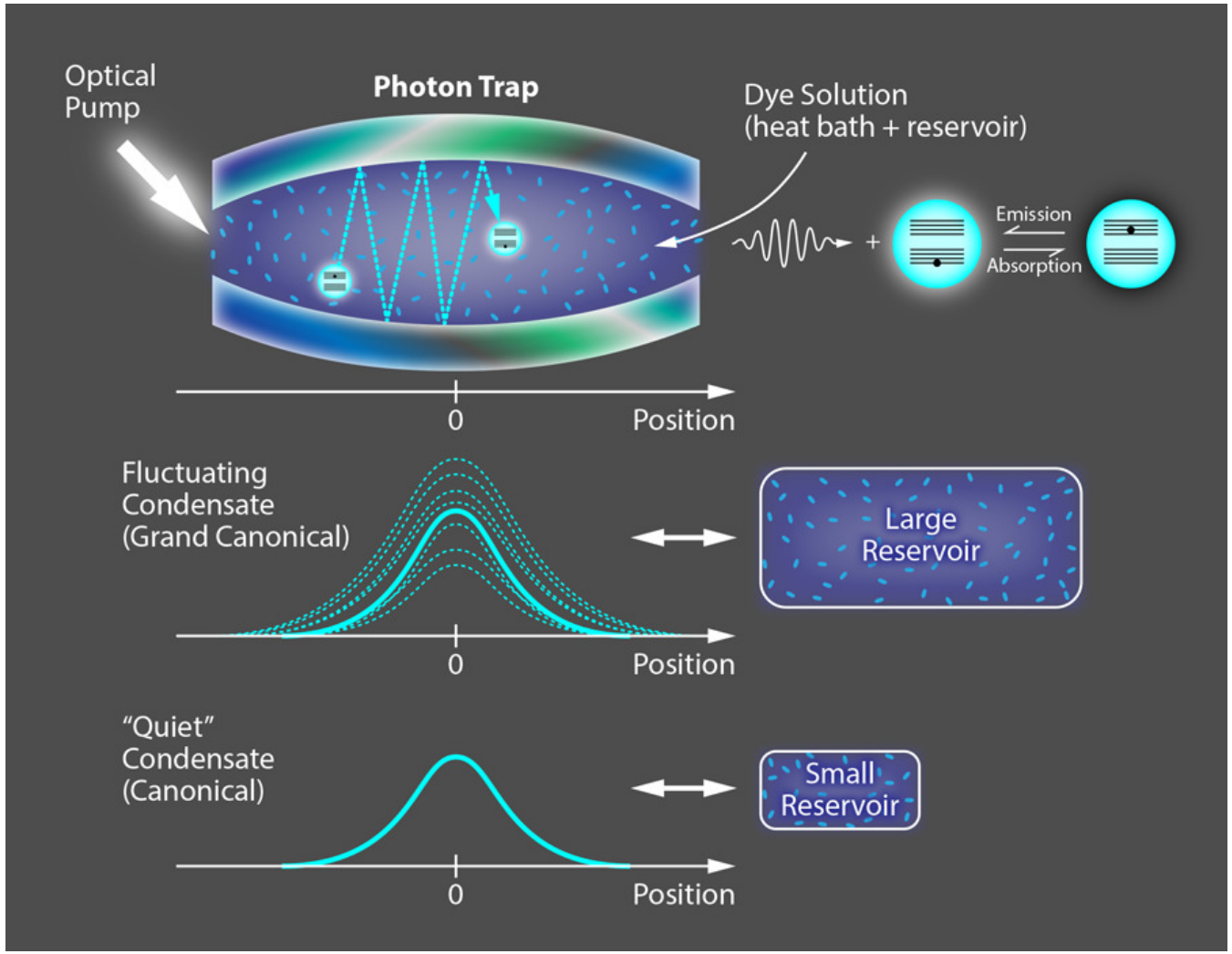


Klärs, Schmitt, Vewinger, and Weitz, Nature **468**, 545 (2010)

Overview: Pelster, Physik-Journal **10**, Nr. 1, 20 (2011)

Pelster, Physik-Journal **13**, Nr. 3, 20 (2014)

1.6 Canonical Versus Grand-Canonical Ensemble



Klaers, Schmitt, Damm, Vewinger, and Weitz, PRL **108**, 160403 (2012)

Schmitt, Damm, Dung, Vewinger, Klaers, and Weitz, PRL **112**, 030401 (2014)

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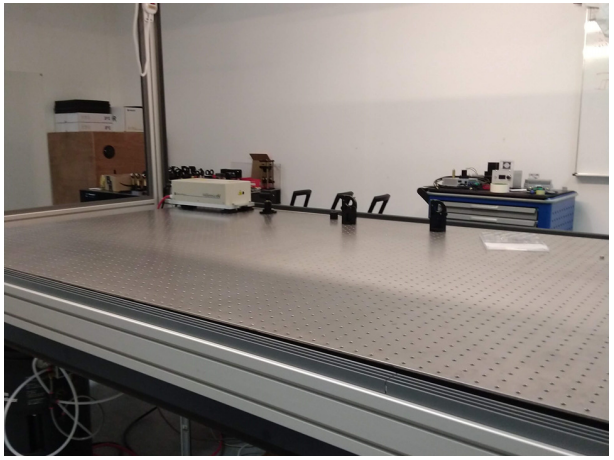
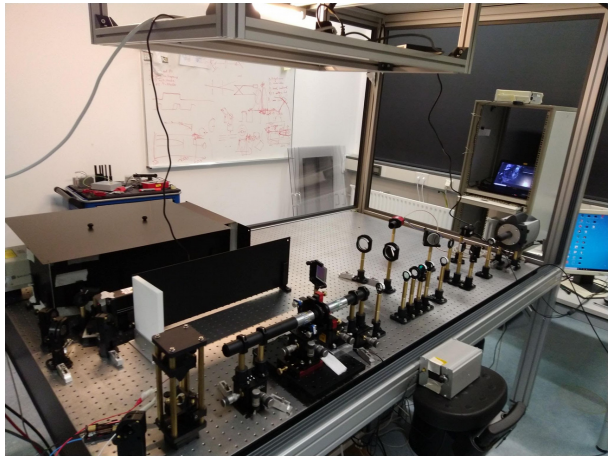
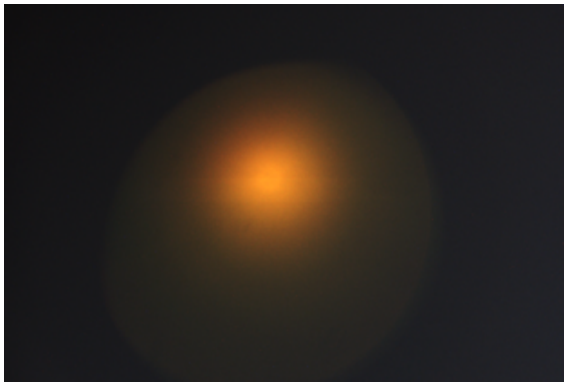
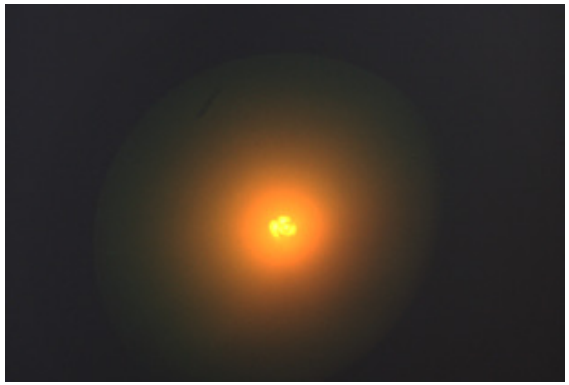
2.1 New Photon BEC Set Up in Kaiserslautern







$N < N_c$

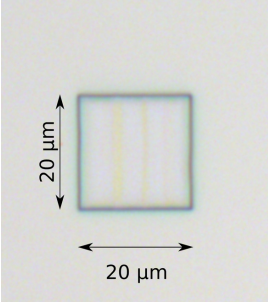


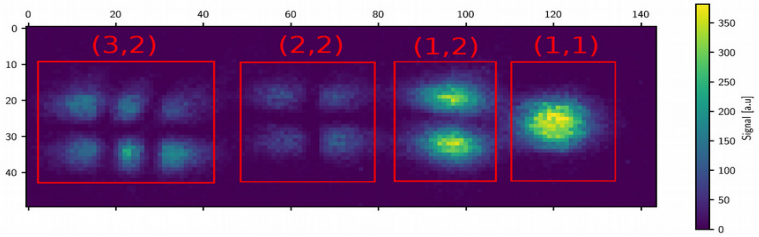
$N > N_c$

Schulz (von Freymann, Kaiserslautern) with help of Umesh (Vewinger, Bonn)

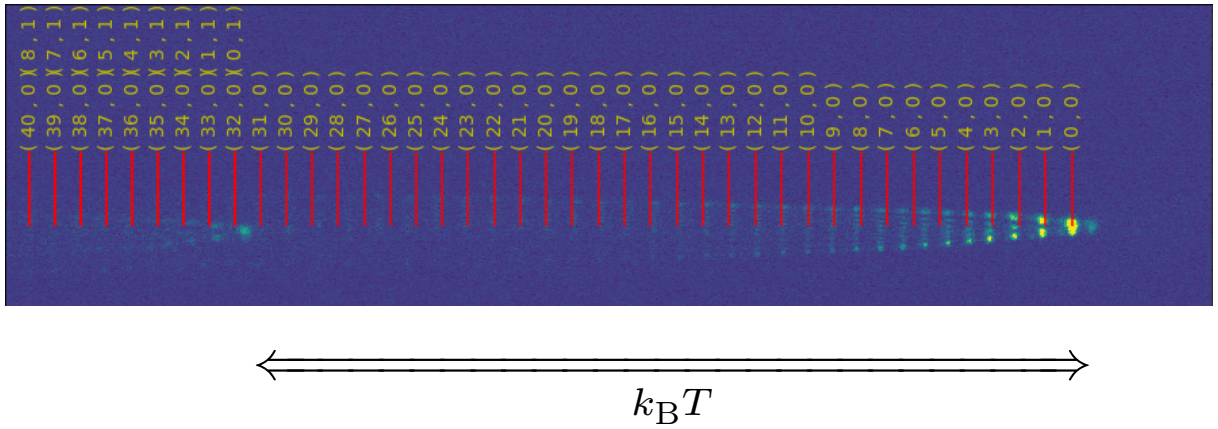
2.2 Direct Laser Writing: Photon BECs at D $\leq$ 2

- **Direct laser writing of structures on mirrors:**
  - 3D printed box potential ✓
  - Structural stability ✓
  - Thermodynamics investigated ✓





- **Parabolic structures for dimensional crossover study:**



$k_\mathrm{B}T$

mode spectrum

dimple trap: depth $k_\mathrm{B}T$

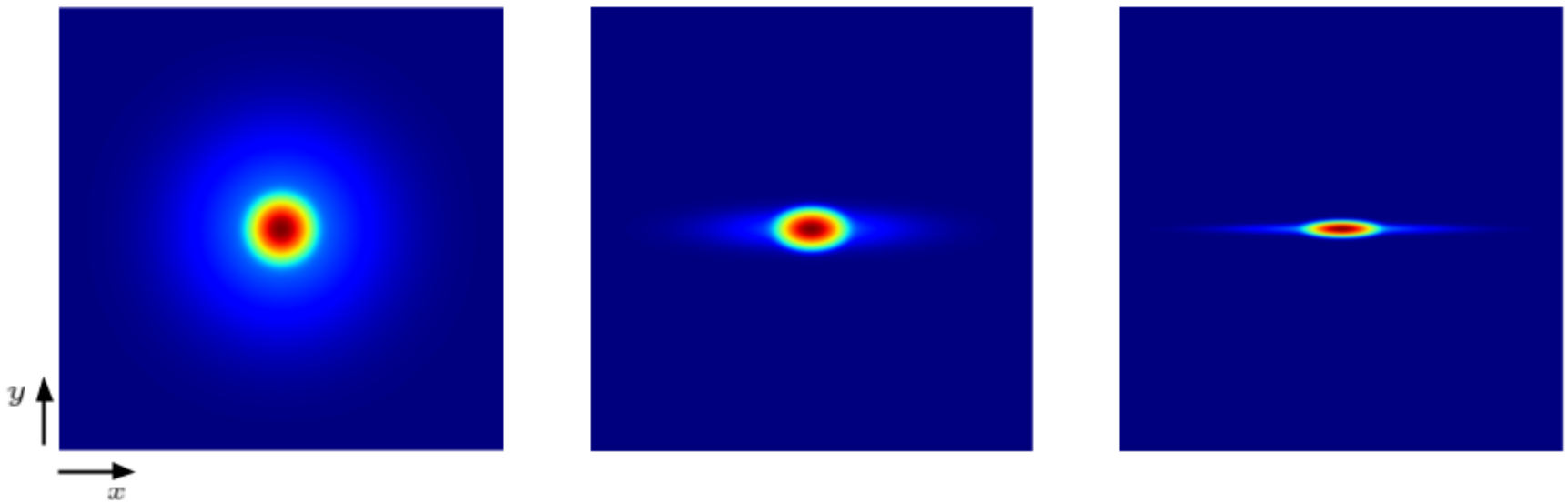
$\omega_x = 2\pi \times 0.21$ THz

$\omega_y = 32 \times \omega_x$

Umesh (Vewinger, Bonn) with structures from Schulz (von Freymann, Kaiserslautern)

2.3 Dimensional Crossover: 2D $\Longleftrightarrow$ 1D

$$V(x,y) = \frac{M}{2}\left(\Omega_x^2 x^2 + \Omega_y^2 y^2\right) , \qquad \lambda = \frac{\Omega_y}{\Omega_x} , \qquad \Omega_x \text{ fixed}$$



2D $\longrightarrow$ 1D

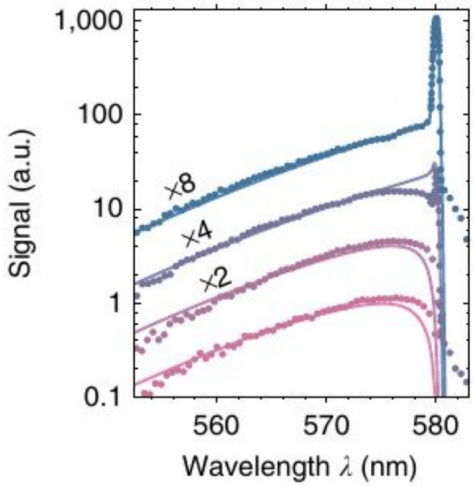
$\lambda = 1$ $\qquad$ $\lambda \to \infty$

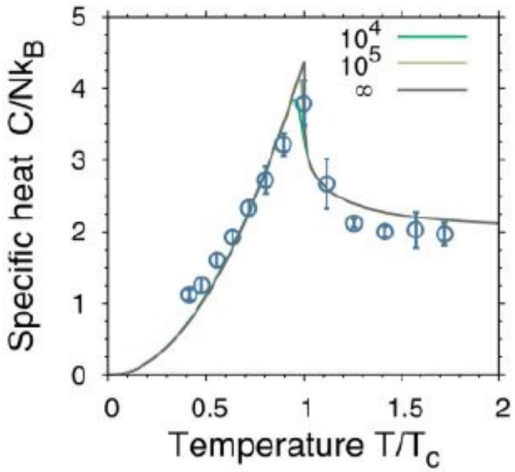
Quasi 1D: $\quad \lambda > \lambda_{1\mathrm{D}} = \dfrac{k_\mathrm{B} T}{\hbar \Omega_x}$

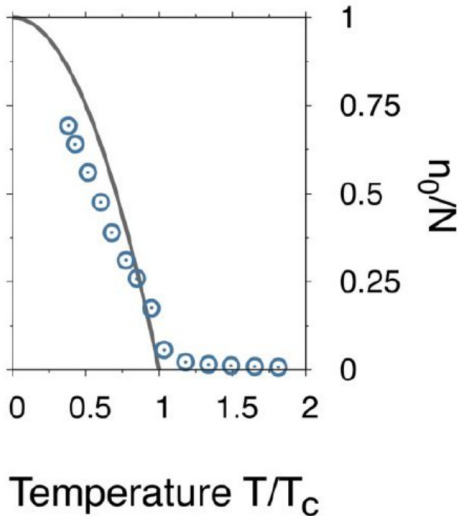
Stein: Simulation of ideal Bose gas

2.4 Current Experimental Status

- **Calorimetry in 2D:**
  Spectrum yields thermodynamic properties





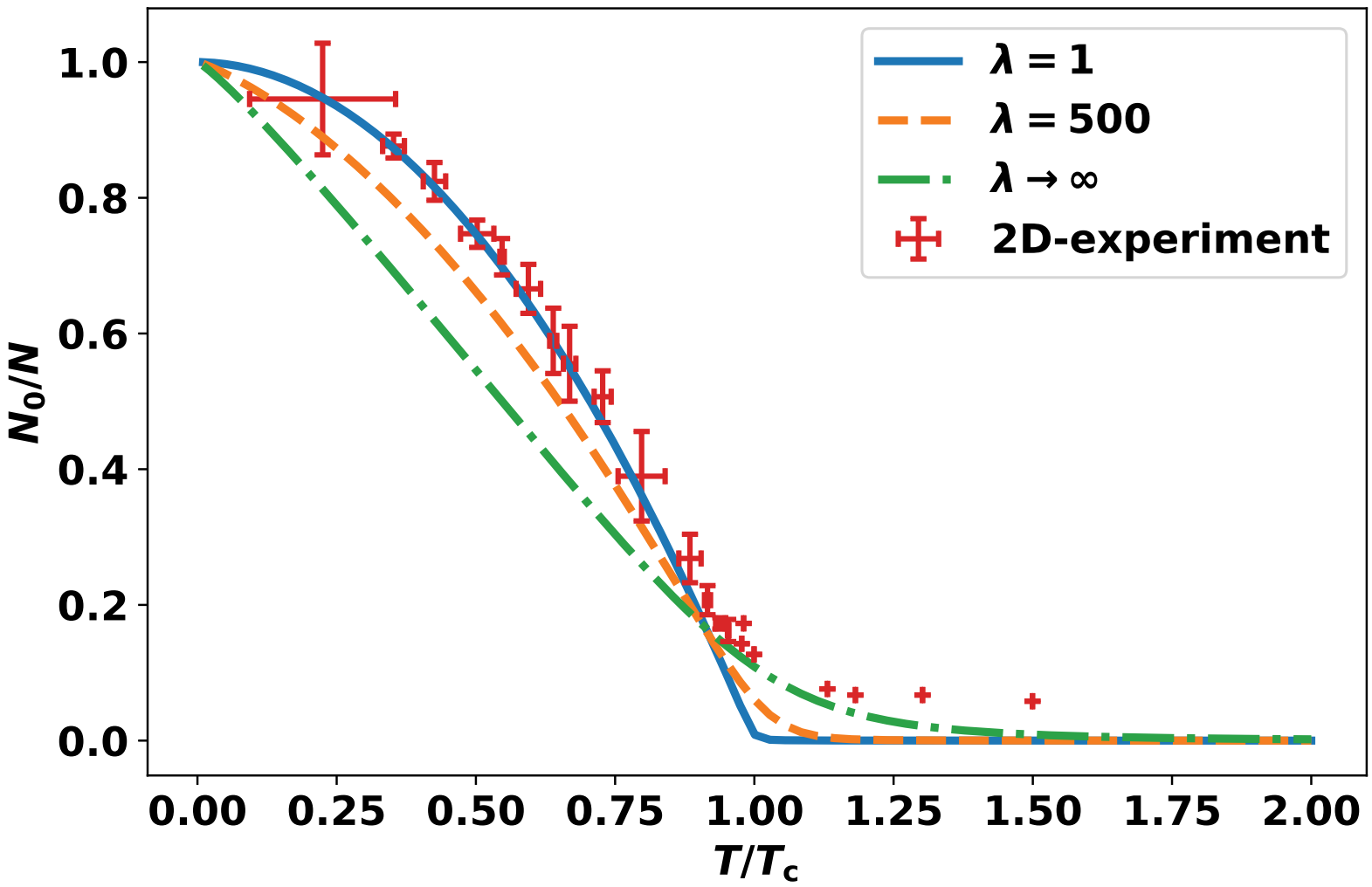


T fixed, N varied $\quad\Longrightarrow\quad \sqrt{\dfrac{N_c}{N}} = \dfrac{T}{T_c} \quad\Longrightarrow\quad$ T varied, N fixed

- **Result: Photon BEC $\widehat{=}$ ideal Bose gas in 2D**

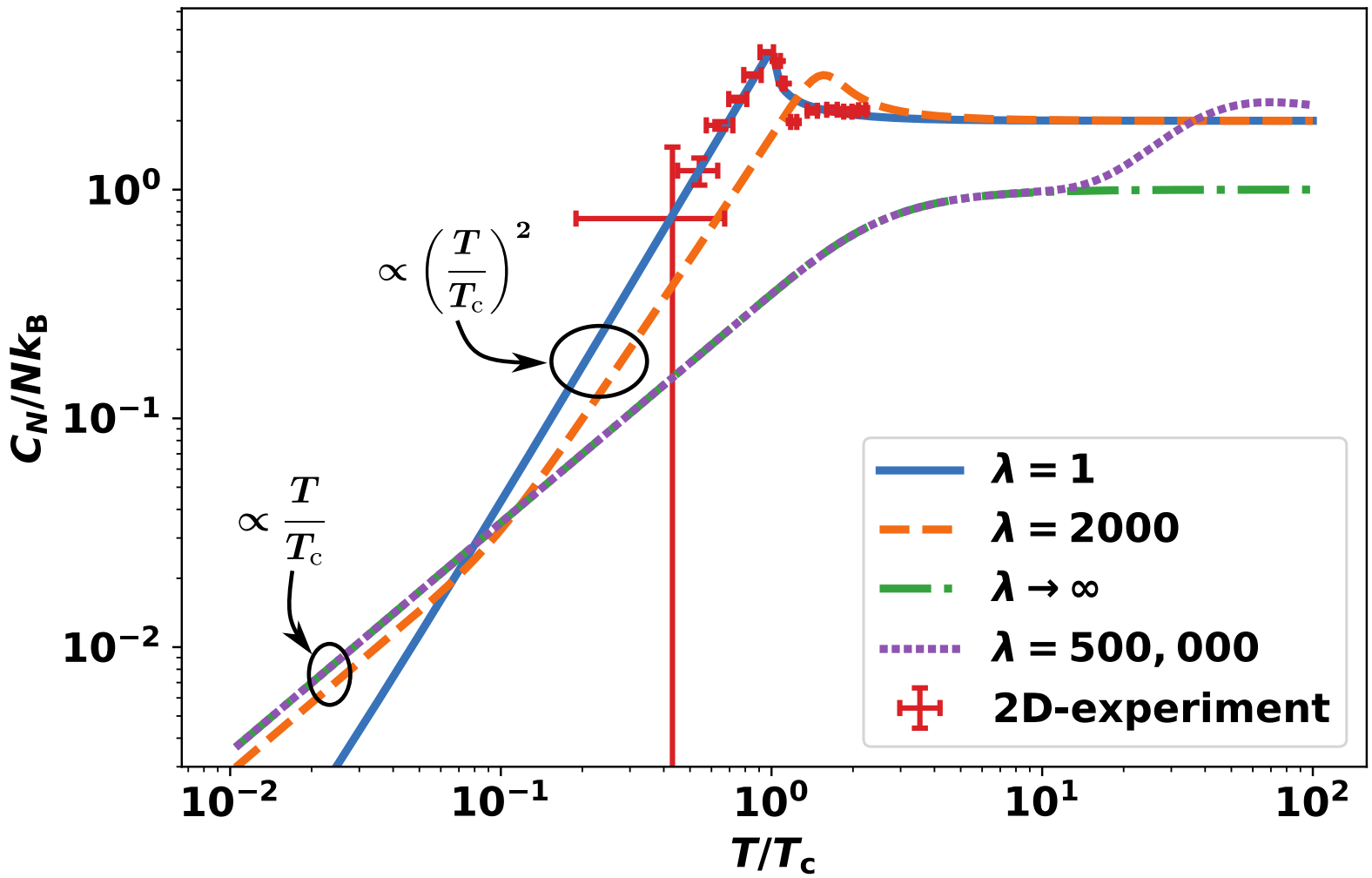
Damm, et al., Nature Comm. **7**, 11340 (2016)

2.5 Condensate Fraction: $N = 100,000$



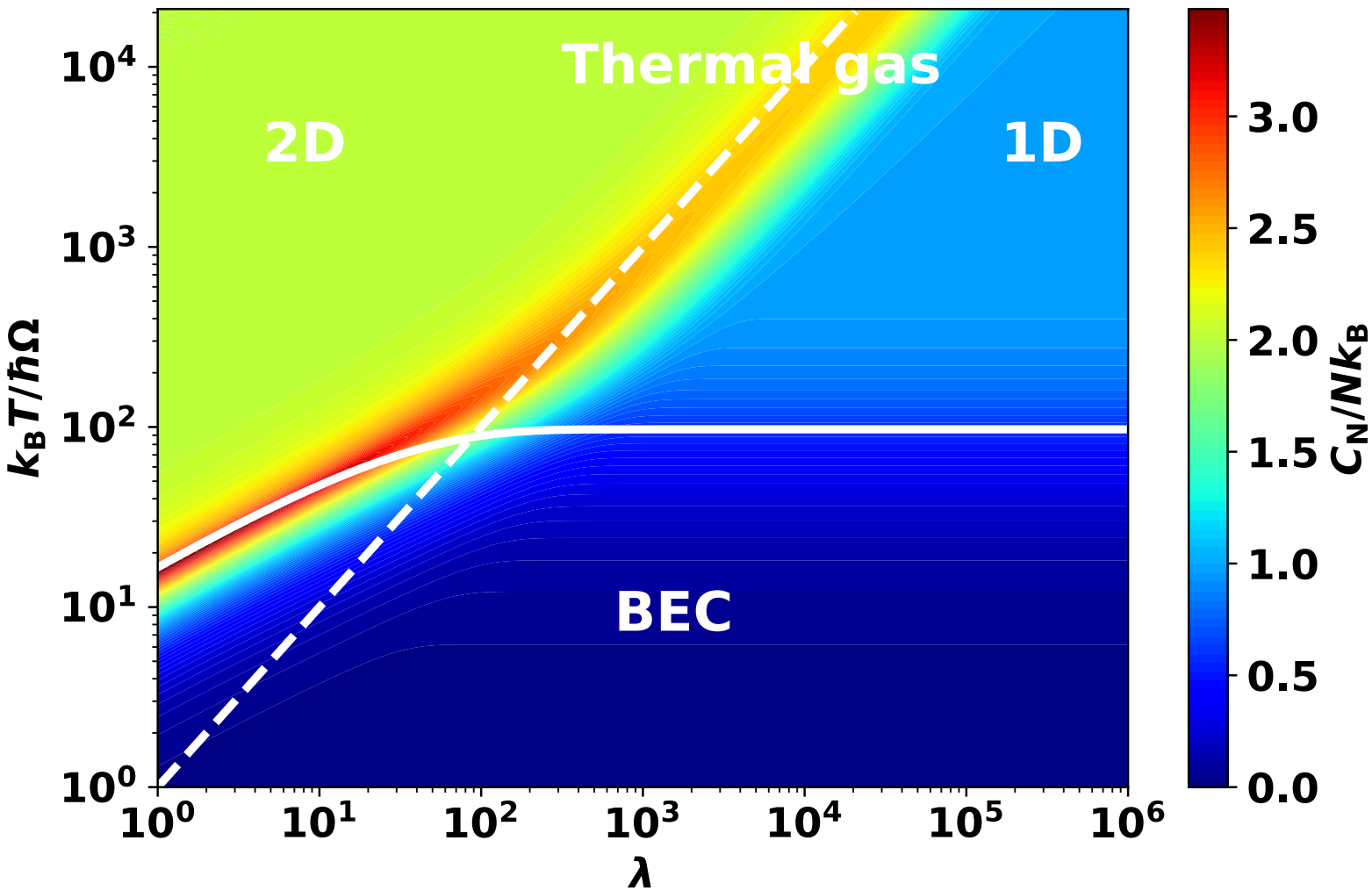
E. Stein and A. Pelster, New J. Phys. **24**, 023013 (2022)

2.6 Heat Capacity $N = 100,000$



E. Stein and A. Pelster, New J. Phys. **24**, 023013 (2022)

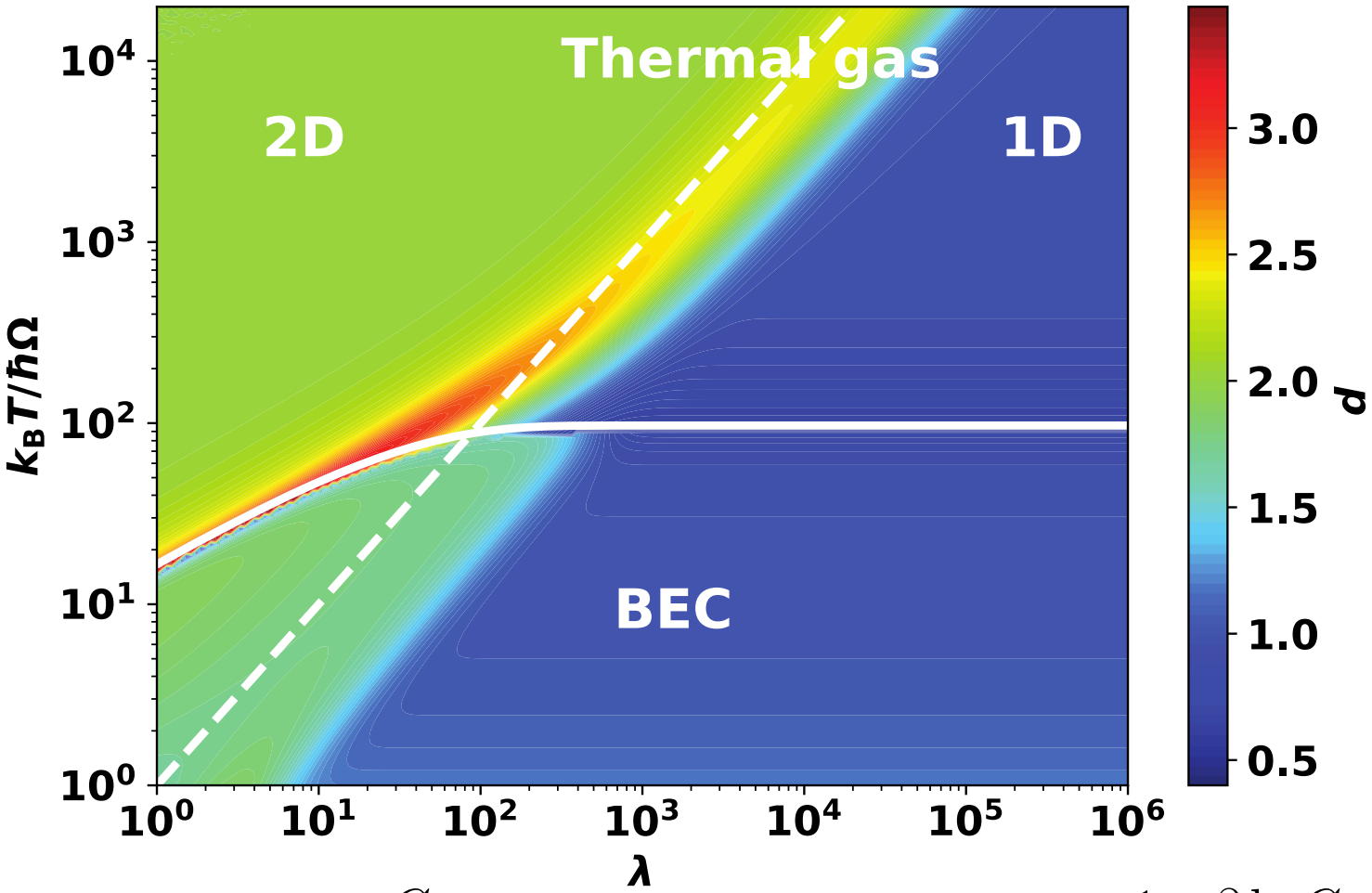
2.7 Phase Diagram: $N = 1,000$



solid white line: T_c , dashed white line: $\lambda_{1\mathrm{D}} = \dfrac{k_\mathrm{B}T}{\hbar\Omega_x}$

E. Stein and A. Pelster, New J. Phys. **24**, 023013 (2022)

2.8 Effective Dimension: $N = 1,000$



Thermal phase: $d = \dfrac{C_N}{k_B N}$, BEC phase: $d = \dfrac{1}{k_B N} \dfrac{\partial \ln C_N}{\partial \ln T}$

E. Stein and A. Pelster, New J. Phys. **24**, 023013 (2022)

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3.1 Photon-Photon Interaction

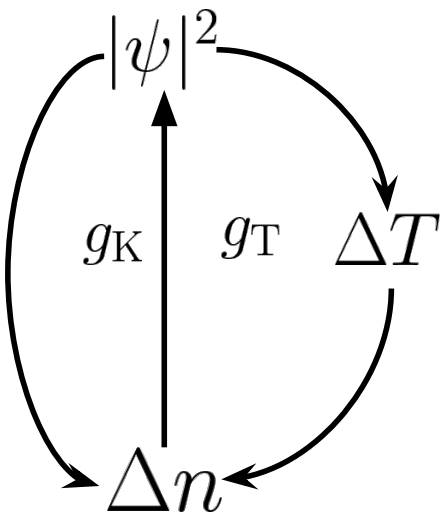
Change of photon energy: $\Delta E(t) \sim \int d^2x \big[\Delta n_{\mathrm{K}} + \Delta n_{\mathrm{T}}(t)\big] |\psi|^2$

Kerr effect:

- Change of refractive index due to intensity:

  $\Delta n_{\mathrm{K}} \sim |\psi|^2$

- Local, instantaneous contact interaction
- $\tilde{g}_{\mathrm{K}} = M g_{\mathrm{K}}/\hbar^2 \sim 10^{-8}$
- **Negligible**



Thermo-optic effect:

- Heating of dye medium due to intensity: heat diffusion

$\Delta n_{\mathrm{T}}(t) \sim \int_0^t dt' \int d^2x' G(\mathbf{x}-\mathbf{x}', t-t') |\psi(\mathbf{x}', t')|^2$

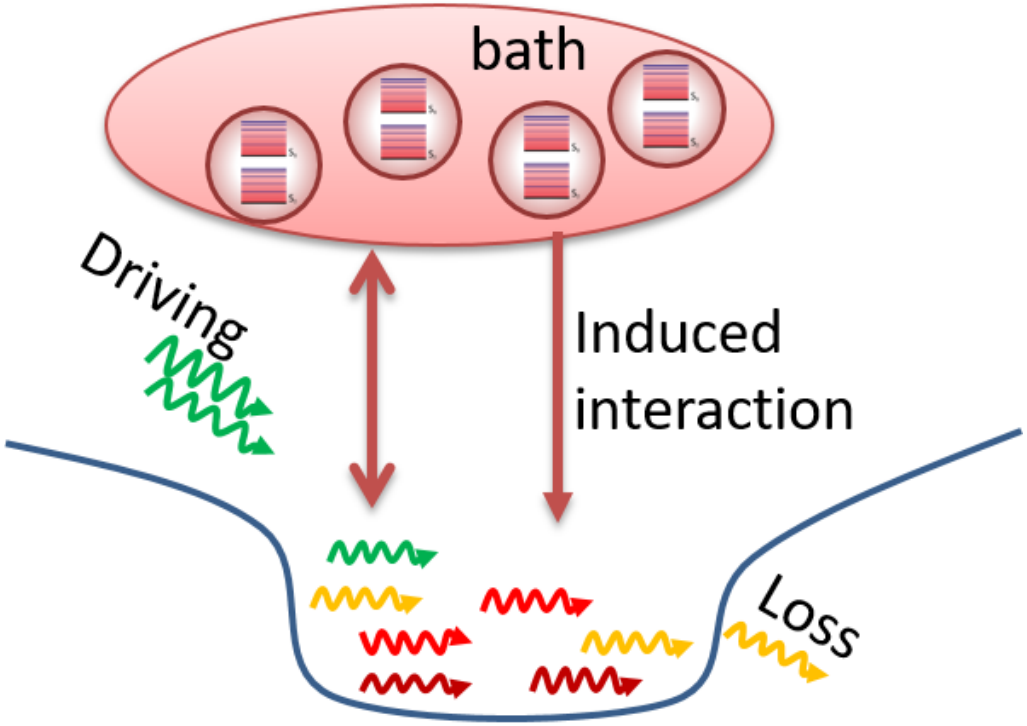
- Non-local, retarded interaction
- $\tilde{g}_{\mathrm{T}} = M g_{\mathrm{T}}/\hbar^2 \sim 10^{-4}$
- **Leading mechanism**

Klaers, Schmitt, Damm, Vewinger, and Weitz, Appl. Phys. B **105**, 17 (2011)

Stein, Vewinger, and Pelster, New J. Phys. **21**, 103044 (2019)

3.2 Kerr Effect: Microscopic Theory

- **Bath^2 model: Lindblad master equation**
- **Coherent and dissipative bath influences**
- **To be, or not to be BEC/LASER?**



Remark:
No solvent effect!

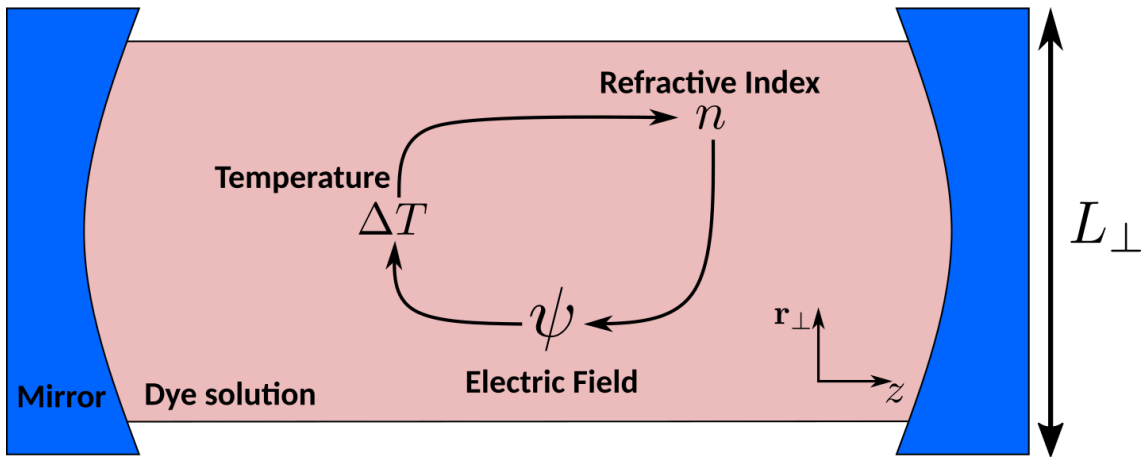
Kirton and Keeling, Phys. Rev. Lett. **111**, 100404 (2013)

Radonjić, Kopylov, Balaž, and Pelster, New J. Phys. **20**, 055014 (2018)

3.3 Thermo-Optic Effect: Mean-Field Theory

- **Linear stability analysis around steady state**
- **Lowest-lying collective modes:**
  - **Centre-of-mass motion: Kohn theorem violated**
  - **Damping due to openess of system**
- **Temperature diffusion in mirrors**

$$\tilde{g}_{\mathrm{T}} \sim \tau = \frac{L_\perp^2}{2\pi^2 D_1}$$



Stein, Vewinger, and Pelster, New J. Phys. **21**, 103044 (2019)

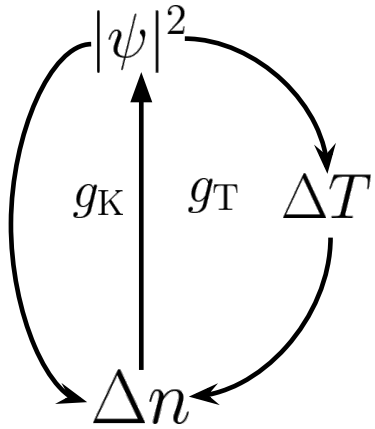
3.4 Interaction Strength @ Dimensional Crossover

- **Steady-state variational analysis: 2D $\Longleftrightarrow$ 1D**

- **Minimise steady-state energy functional:**

$$E\left[\psi,\psi^*\right] = \int d^2x \left\{ \psi^* \left[-\frac{\hbar^2\Delta}{2M} + \frac{M}{2}\Omega_x^2\left(x^2+\lambda^4 y^2\right)\right]\psi \right.$$

$$\left. + \frac{g_{\mathrm{k}}}{2}\left|\psi\right|^4 + \frac{g_{\mathrm{T}}}{2}\int_0^\infty dt \int d^2x'\, G(\mathbf{x}-\mathbf{x}',t)\left|\psi(\mathbf{x}')\right|^2\left|\psi(\mathbf{x})\right|^2 \right\}$$

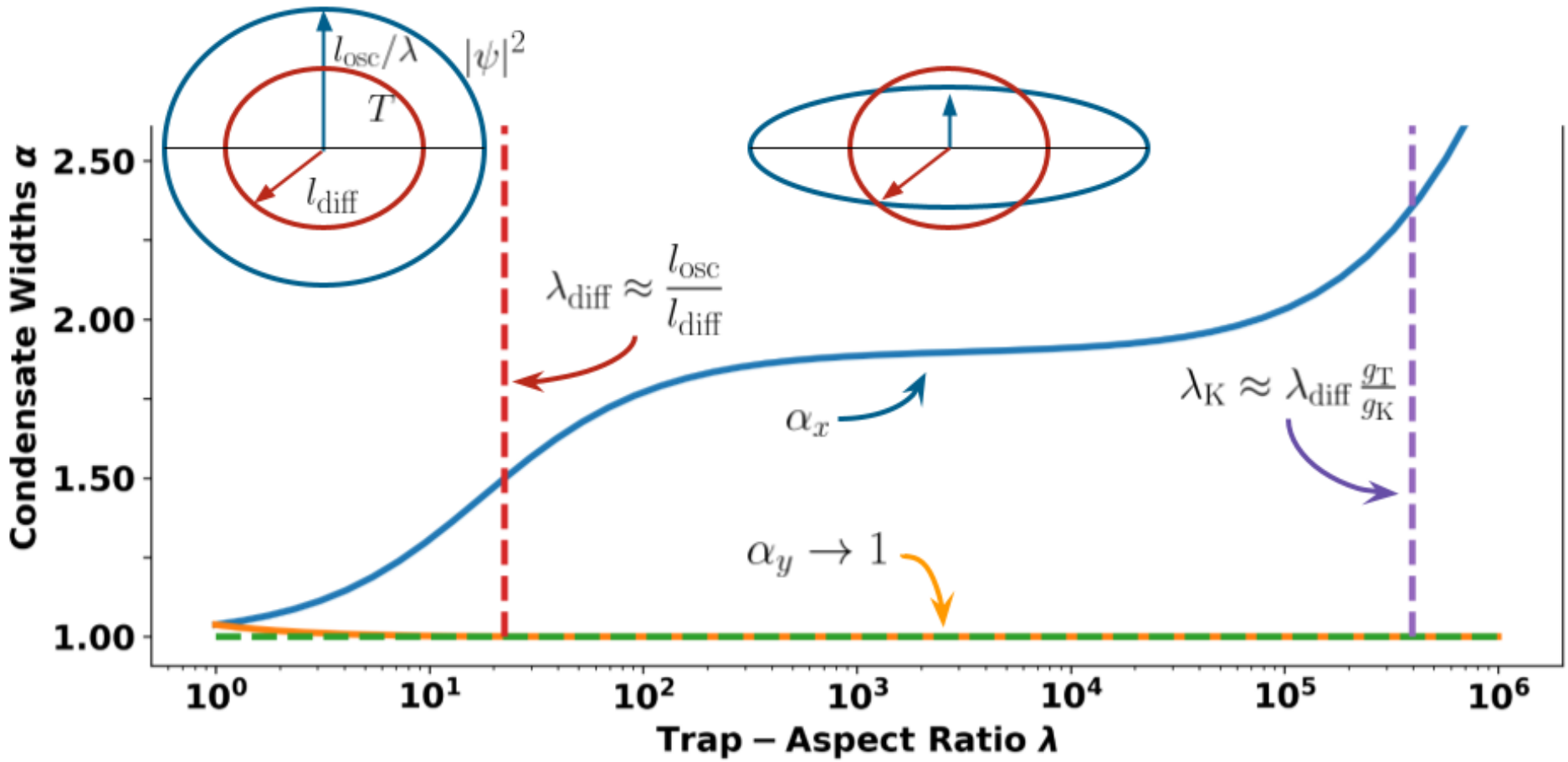


- **Gauß ansatz for ground-state BEC wave function:**

$$\psi(x,y) \sim \exp\left\{ -\frac{1}{2l_{\mathrm{osc}}^2}\left(\frac{x^2}{\alpha_x^2} + \lambda^2\frac{y^2}{\alpha_y^2}\right)\right\}, \qquad l_{\mathrm{osc}} = \sqrt{\frac{\hbar}{M\Omega_x}}$$

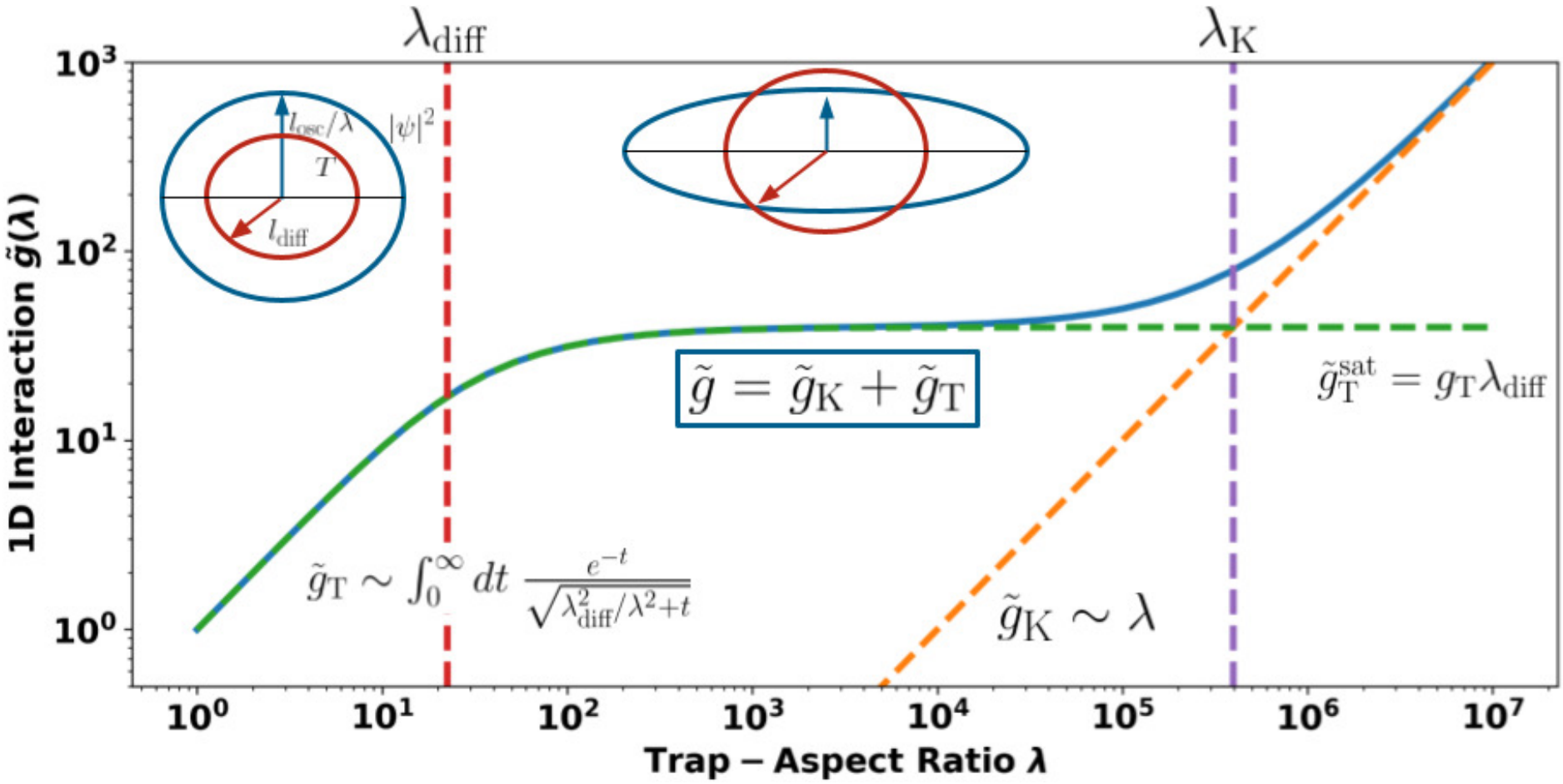
Stein and Pelster, New J. Phys. **24**, 023032 (2022)

3.5 Condensate Widths



Stein and Pelster, New J. Phys. **24**, 023032 (2022)

3.6 Interaction Strengths



Stein and Pelster, New J. Phys. **24**, 023032 (2022)

3.7 Experimental Signature

- **Photon-photon interaction measurable via condensate widths:**
  - **Real space images of light leaking out the cavity**
  - **Possible but inaccurate**

    Klaers, Schmitt, Vewinger, and Weitz, Nature **468**, 545 (2010)

- **More precise results expected by spectroscopic measurements**
  $\Longrightarrow$ **Energy shift of mode l: $\Delta E_l \sim \tilde{g}$**

- **Experimentally hard to reach steady-state region:**
  $\Longrightarrow$ **Dye bleaching**

- **Lifetime $10^{-7}\mathrm{s} \ll$ temperature relaxation time $10^{-1}\mathrm{s}$:**
  - **Consider short-time evolution**
  - **Time-dependent interaction strength: $\tilde{g} \sim t$**
  - **Crossover 2D $\Longleftrightarrow$ 1D: thermal cloud important**

Stein and Pelster, NJP **25**, 033025 and 033026 (2023)

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Homepage

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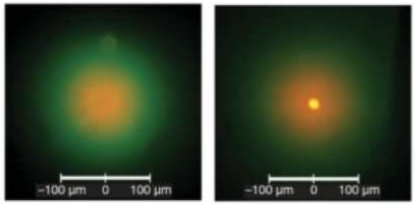
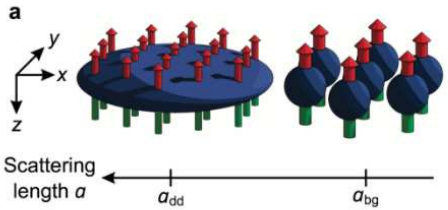
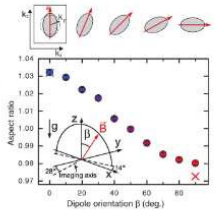
Theory of Ultracold Quantum Gases:



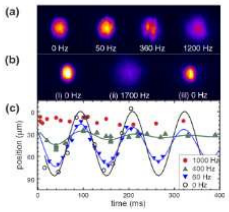
Photon Bose-Einstein Condensate



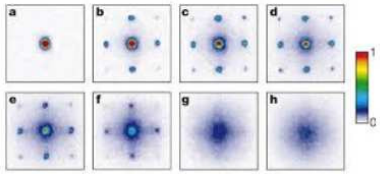
Dipolar Quantum Droplets



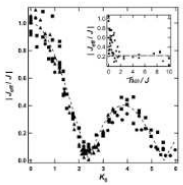
Fermi Surface Deformation



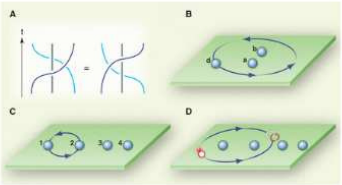
Dirty Boson Problem



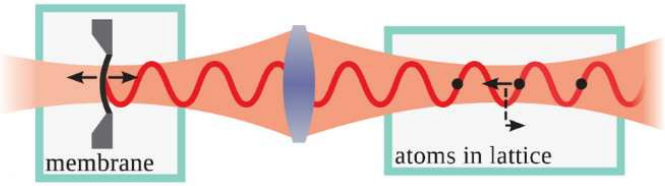
Superfluid Mott Quantum Phase Transition



Floquet Physics

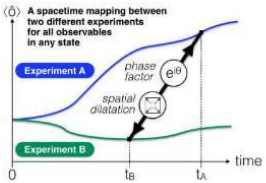


Anyonic Quantum Statistics



Hybrid Atom-Optomechanical Systems

Quantum Gases on Curved Manifolds



Quantum Gas Mappings

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Bad Honnef Physics School on Ultracold Atoms and Molecules

organized by Carlos Sá de Melo and Axel Pelster

Bad Honnef (Germany); August 6 – 12, 2023

- **Lauriane Chomaz:** *Quantum gases of magnetic atoms*
- **André Eckardt:** *Floquet engineering*
- **Tilman Esslinger:** *Topological pumping*
- **Nathan Lundblad:** *Ultracold atomic physics in microgravity*
- **Tilman Pfau:** *Ultralong-range Rydberg molecules*
- **Thomas Pohl:** *Rydberg-atom physics and technology*
- **Philipp Preiss:** *Pairing and superfluidity in ultracold Fermi gases*
- **Helmut Ritsch:** *Quantum gas cavity QED*
- **Jacques Tempere:** *Fermi superfluids in BEC-BCS crossover*
- **Päivi Törmä:** *Topological photonics with plasmonic lattices*

`https://www.dpg-physik.de/veranstaltungen/2023`