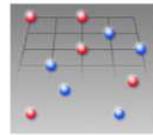


Tuning the Quantum Phase Transition of Bosons in Optical Lattices

Axel Pelster



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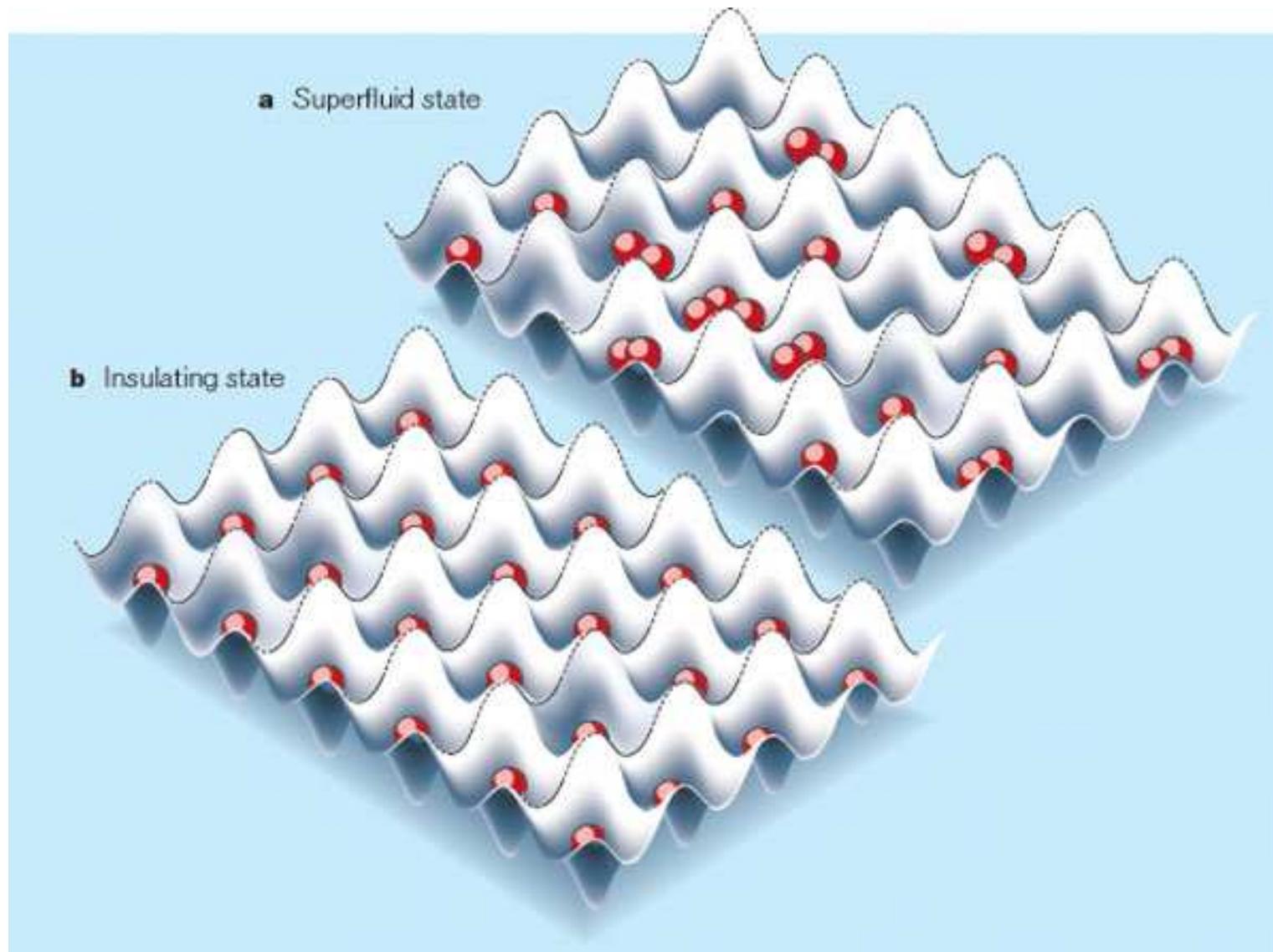
Frankfurt – Kaiserslautern – Mainz

Condensed matter systems with variable
many-body interactions



- 1. Introduction**
- 2. Spinor Bose Gases**
- 3. Periodically Modulated Interaction**
- 4. Kagome Superlattice**

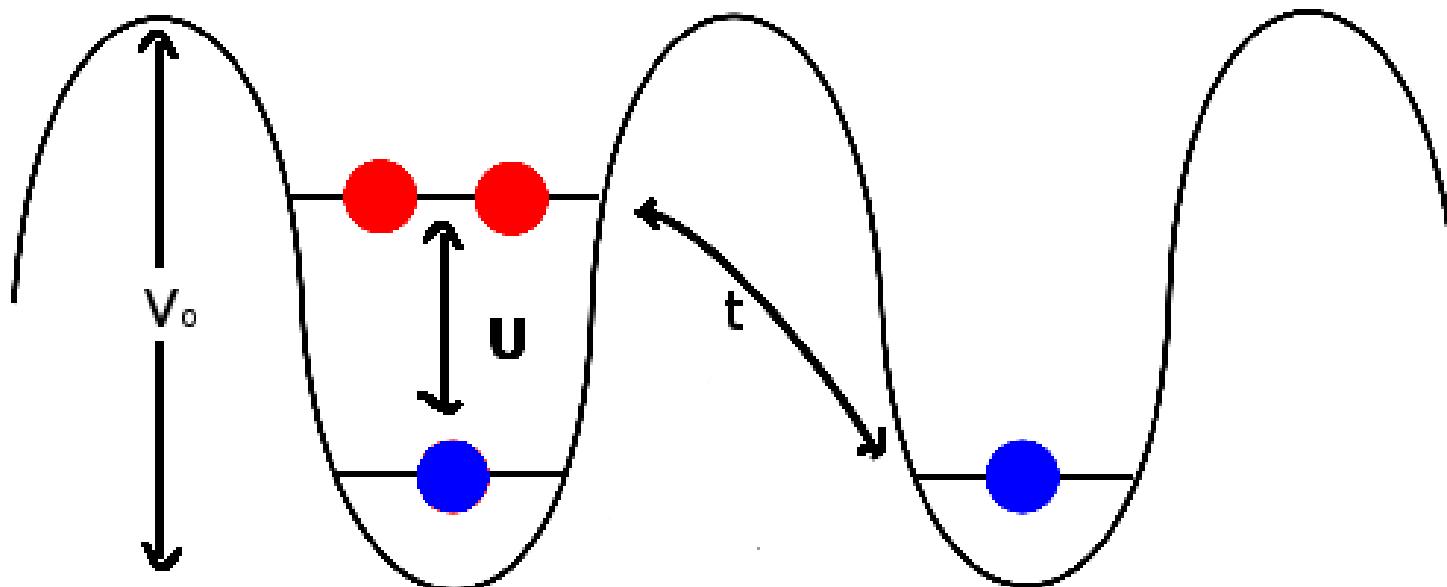
1.1 Quantum Phase Transition



1.2 Theoretical Description

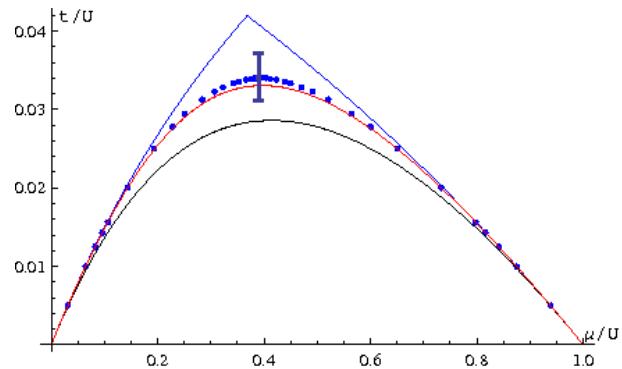
Bose-Hubbard Hamiltonian:

$$\hat{H}_{\text{BH}} = -t \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \left[\frac{U}{2} \hat{n}_i (\hat{n}_i - 1) - \mu \hat{n}_i \right], \quad \hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$$



1.3 Ginzburg-Landau Theory

Quantum Phase Diagram:



Error bar: Extrapolated strong-coupling series

Black line: Mean-field

Blue line: 3rd strong-coupling order

Red line: Landau theory

Blue dots: Monte-Carlo data

Santos and Pelster, PRA **79**, 013614 (2009)

Extension to higher orders:

Teichmann, Hinrichs, Holthaus, and Eckardt, PRB **79**, 100503(R) (2009)

Hinrichs, Pelster, and Holthaus, Appl. Phys. B **113**, 57 (2013)

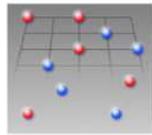
Excitation Spectra:

Bradlyn, Santos, and Pelster, PRA **79**, 013615 (2009)

Graß, Santos, and Pelster, PRA **84**, 013613 (2011)

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1. Introduction

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Mobarak and Pelster, LPL **10**, 115501 (2013)

3. Periodically Modulated Interaction

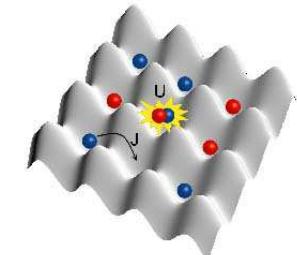
4. Kagome Superlattice

2.1 Bose-Hubbard Model for Spin-1 Boson

$$\hat{H}_{\text{BH}} = \hat{H}^{(0)} + \hat{H}^{(1)}$$

$$\hat{H}^{(0)} = \sum_i \left[\frac{U_0}{2} \hat{n}_i (\hat{n}_i - 1) + \frac{U_2}{2} (\hat{\mathbf{S}}_i^2 - 2\hat{n}_i) - \mu \hat{n}_i - \eta \hat{S}_{iz} \right]$$

$$\hat{H}^{(1)} = - \sum_{ij} J_{ij} \sum_{\alpha} \hat{a}_{i\alpha}^\dagger \hat{a}_{j\alpha}$$



- $\hat{n}_i = \sum_{\alpha} \hat{n}_{i\alpha} = \sum_{\alpha} \hat{a}_{i\alpha}^\dagger \hat{a}_{i\alpha}$

- $\begin{bmatrix} \hat{a}_{i\alpha}, \hat{a}_{j\beta} \end{bmatrix} = \begin{bmatrix} \hat{a}_{i\alpha}^\dagger, \hat{a}_{j\beta}^\dagger \end{bmatrix} = 0$
 $\begin{bmatrix} \hat{a}_{i\alpha}, \hat{a}_{j\beta}^\dagger \end{bmatrix} = \delta_{\alpha,\beta} \delta_{i,j}$

- $\hat{\mathbf{S}}_i = \sum_{\alpha,\beta} \hat{a}_{i\alpha}^\dagger \mathbf{F}_{\alpha\beta} \hat{a}_{i\beta}$

- $U_2 = \begin{cases} > 0, & \text{Anti-ferromagnetic, e.g. } {}^{23}\text{Na} \\ < 0, & \text{Ferromagnetic, e.g. } {}^{87}\text{Rb} \end{cases}$

- $J_{ij} = \begin{cases} J, & \text{if } i; j \text{ are next neighbors} \\ 0, & \text{otherwise} \end{cases}$

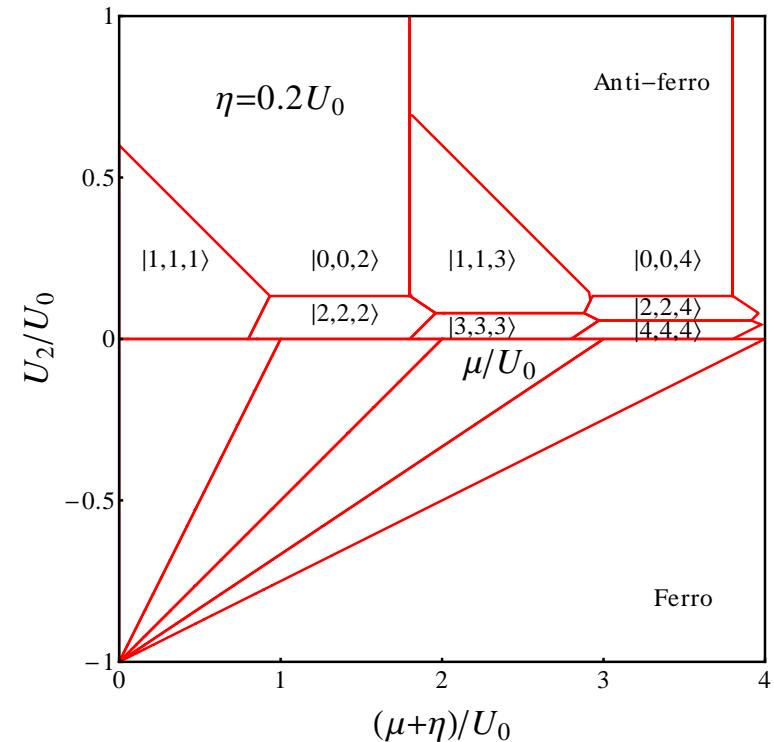
2.2 Atomic Limit: Mott Phases

Site-diagonal: $\hat{H}^{(0)} = \sum_i \hat{H}_i^{(0)}$

$$\hat{H}_i^{(0)} |S_i, m_i, n_i\rangle = E_{S_i, m_i, n_i}^{(0)} |S_i, m_i, n_i\rangle$$

$$E_{S_i, m_i, n_i}^{(0)} = -\mu n_i + \frac{U_0}{2} n_i(n_i - 1)$$

$$+ \frac{U_2}{2} [S_i(S_i + 1) - 2n_i] - \eta m_i$$



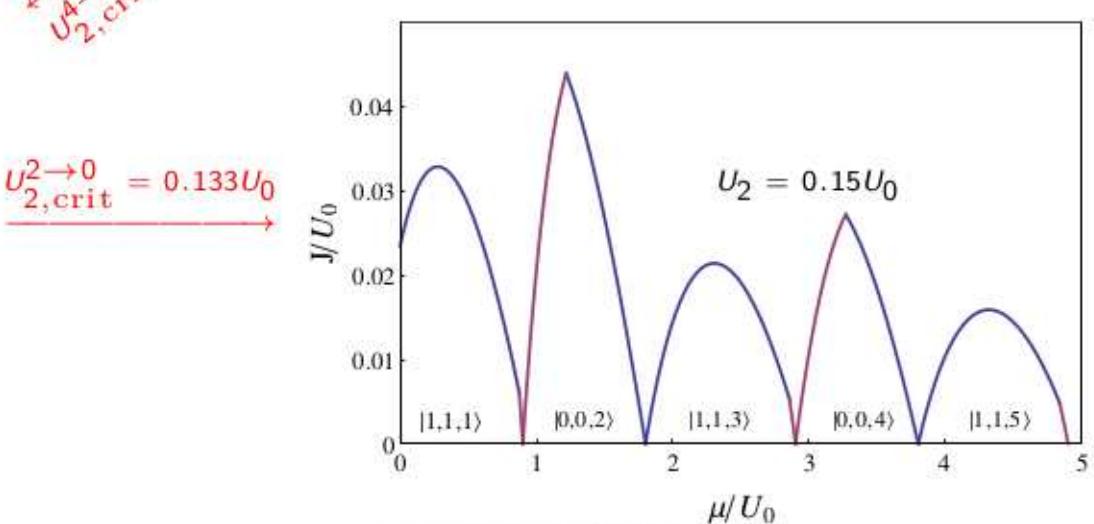
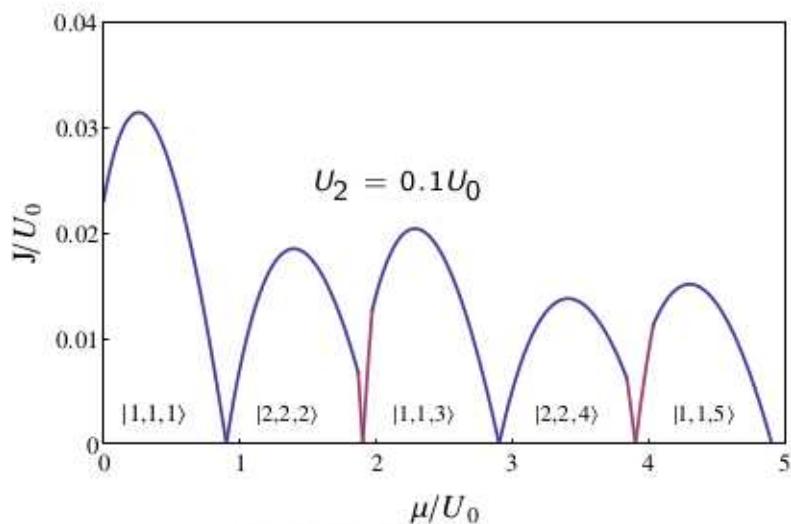
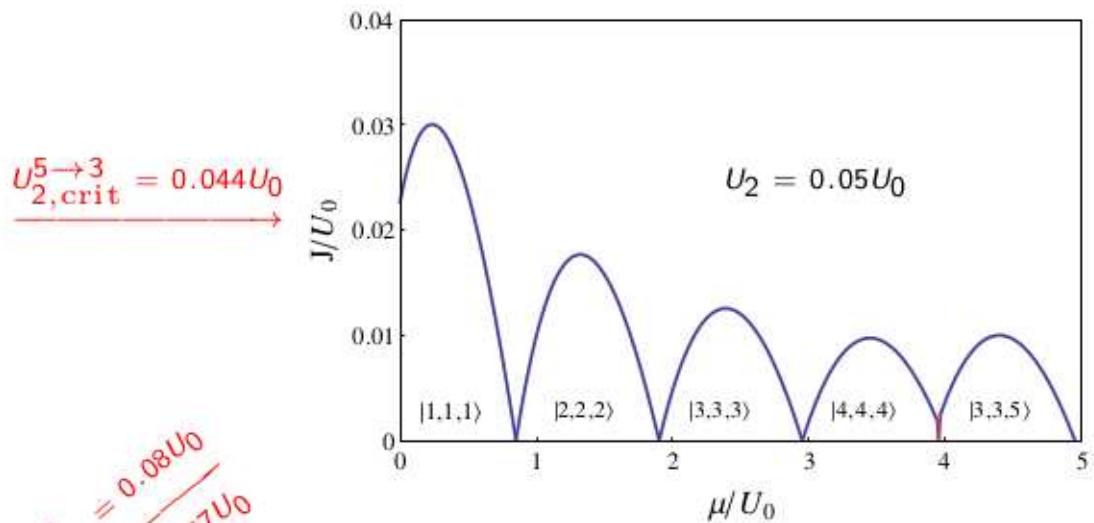
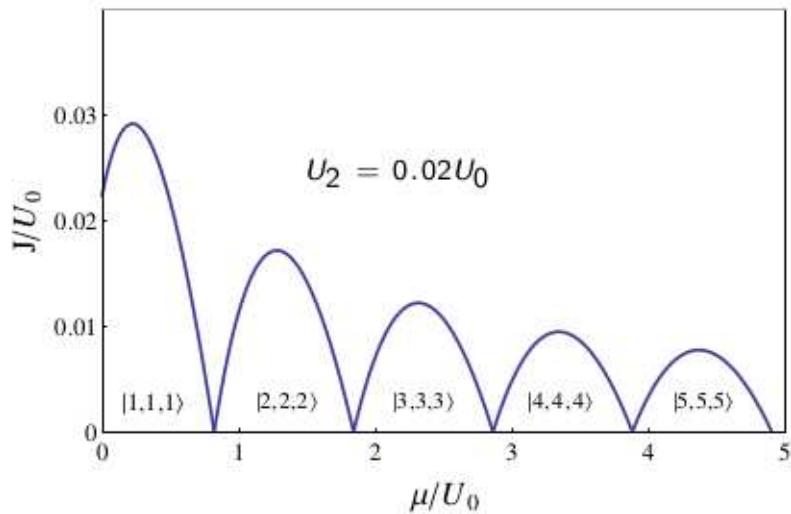
Minimization of energy →

1. $\eta > 0 \implies m_i = S_i$

2. Two competing effects:

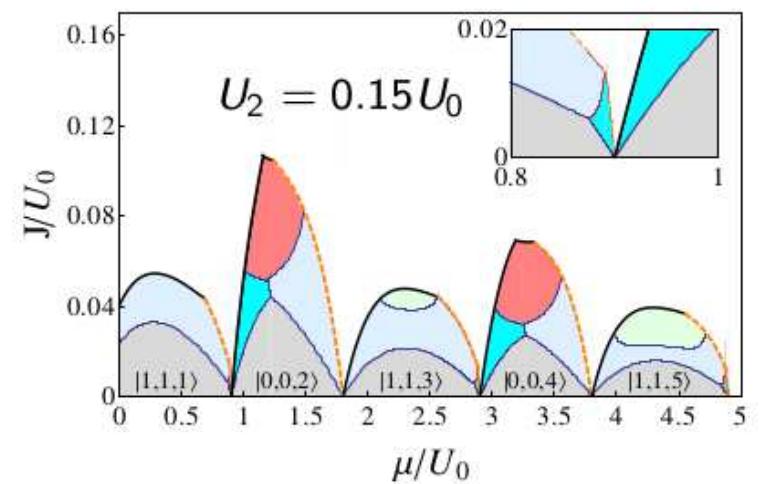
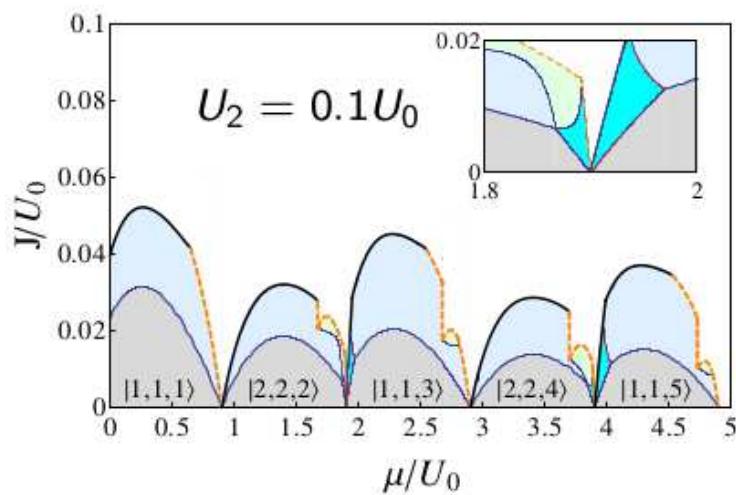
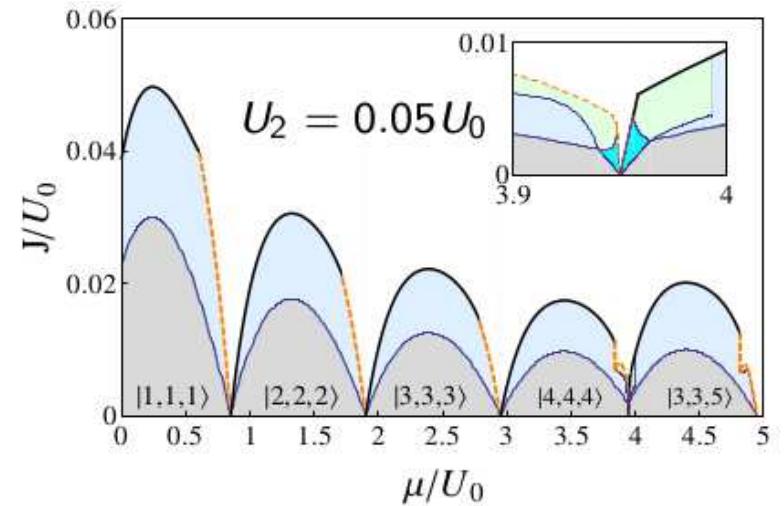
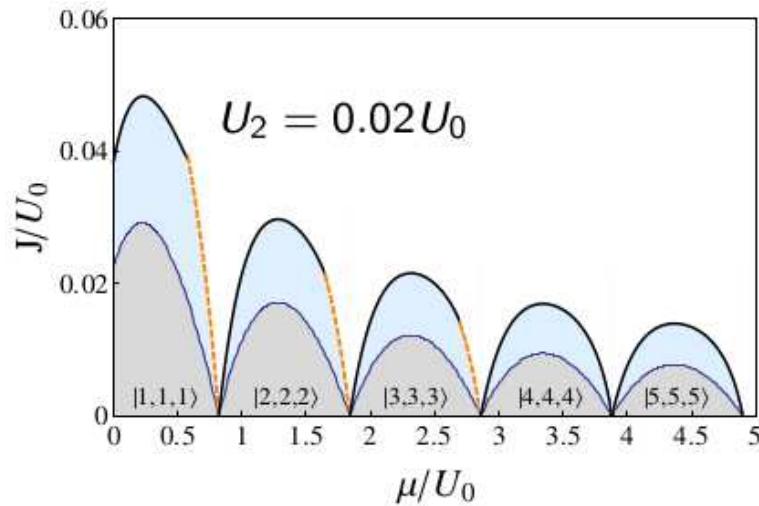
$U_2 > 0 \hat{=} \text{minimize } S_i \quad \longleftrightarrow \quad \eta > 0 \hat{=} \text{maximize } S_i$

2.3 Quantum Phase Boundary ($\eta = 0.2U_0$)



Blue (red) line corresponds to emerging non-vanishing spin-1 (spin-(-1)) component

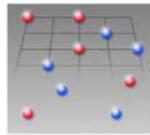
2.4 Magnetic Superfluid Phases ($\eta = 0.2U_0$)



$\Psi_1 \neq 0, \Psi_0 = \Psi_{-1} = 0 ; \Psi_{-1} \neq 0, \Psi_0 = \Psi_1 = 0 ; \Psi_1 \neq 0, \Psi_{-1} \neq 0, \Psi_0 = 0$; and $\Psi_1 \neq 0, \Psi_{-1} \neq 0, \Psi_0 \neq 0$

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Wang, Zhang, Santos, Eggert, and Pelster,
PRA **90**, 013633 (2014)

4. Kagome Superlattice

3.1 Floquet Theory

$$\begin{aligned}\hat{H}(t) &= \hat{H}_{\text{BH}} + A \cos(\omega t) \sum_i g(\hat{n}_i), \quad \hat{n}_i = \hat{a}_i^\dagger \hat{a}_i \\ \hat{H}_{\text{BH}} &= \sum_i f(\hat{n}_i) - t \sum_{\langle ij \rangle} \hat{a}_i^\dagger \hat{a}_j, \quad f(\hat{n}_i) = \frac{U}{2} (\hat{n}_i^2 - \hat{n}_i) - \mu \hat{n}_i\end{aligned}$$



$U, t \ll \hbar\omega \ll \Delta$:
time average over driving period

Arimondo et al., Adv. Atom. Mol. Opt. Adv. **61**, 515 (2012)

$$\begin{aligned}\hat{H}_{\text{eff}} &= \sum_i f(\hat{n}_i) - t \sum_{\langle ij \rangle} \hat{a}_i^\dagger J_0(G(\hat{n}_i, \hat{n}_j)) \hat{a}_j \\ G(\hat{n}_i, \hat{n}_j) &= \frac{g(\hat{n}_j) - g(\hat{n}_j - 1) + g(\hat{n}_i) - g(\hat{n}_i + 1)}{\hbar\omega}\end{aligned}$$

3.2 Examples

Shaken lattice:

$$\hat{H}(t) = \hat{H}_{\text{BH}} + A \cos(\omega t) \sum_i \hat{n}_i$$



$$\hat{H}_{\text{eff}} = \sum_i f(\hat{n}_i) - t J_0 \left(\frac{A}{\hbar \omega} \right) \sum_{\langle ij \rangle} \hat{a}_i^\dagger \hat{a}_j$$

⇒ **renormalized hopping**

Eckardt et al., PRL **95**, 260404 (2005)

Lignier et al., PRL **99**, 220403 (2007)

Zenesini et al., PRL **102**, 100403 (2009)

Modulated interaction:

$$\hat{H}(t) = \hat{H}_{\text{BH}} + \frac{A}{2} \cos(\omega t) \sum_i (\hat{n}_i^2 - \hat{n}_i)$$



$$\hat{H}_{\text{eff}} = \sum_i f(\hat{n}_i) - t \sum_{\langle ij \rangle} \hat{a}_i^\dagger J_0 \left(\frac{A}{\hbar \omega} (\hat{n}_j - \hat{n}_i) \right) \hat{a}_j$$

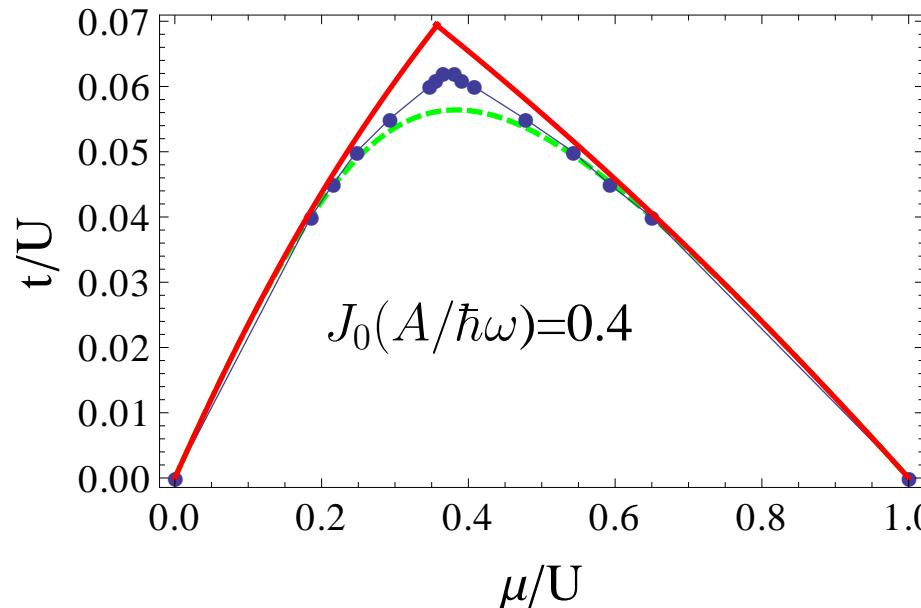
⇒ **conditional hopping**

Rapp et al., PRL **109**, 203005 (2012)

Hubbard, PRSA **276**, 238 (1963)

Experimental verification: still open

3.3 Second Order (2D)



strong-coupling method:

Freericks and Monien, PRB **53**, 2691 (1996)

Quantum Monte Carlo:

Sandvik, PRB **59**, R14157 (1999)

second-order Landau theory:

Wang, Zhang, Eggert, and Pelster, PRA **87**, 063615 (2013)

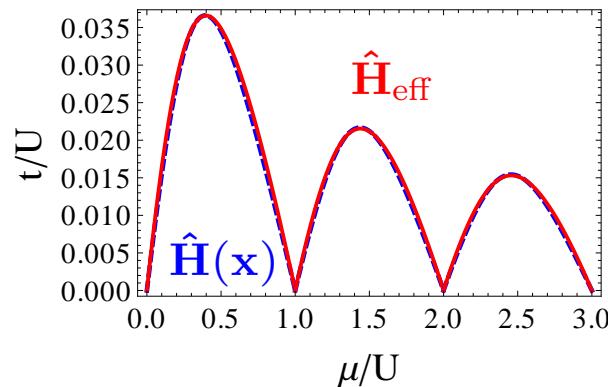
⇒ error less than 6 % for $0 < \frac{A}{\hbar\omega} < x_2 \approx 2.4$, $J_0(x_2) = 0$

3.4 Effective Bose-Hubbard Model

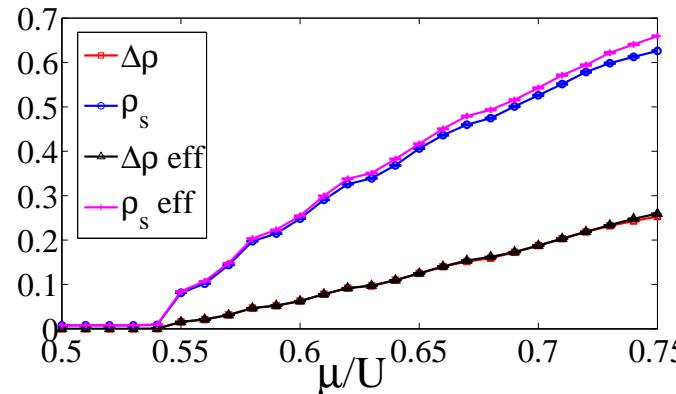
$$\hat{H}_{\text{eff}} = \sum_i \left[\frac{U}{2} \hat{n}_i (\hat{n}_i - 1) - \mu \hat{n}_i \right] - t \sum_{\langle ij \rangle} \hat{a}_i^\dagger J_0 \left(\frac{A}{\hbar\omega} (\hat{n}_j - \hat{n}_i) \right) \hat{a}_j$$

↓ **Approximation due to uniform renormalization
of critical hopping with driving amplitude**

$$\begin{aligned}\hat{H}(x) &= \sum_i \left[\frac{U}{2} \hat{n}_i (\hat{n}_i - 1) - \mu \hat{n}_i \right] - t \lambda(x) \sum_{\langle ij \rangle} \hat{a}_i^\dagger \hat{a}_j \\ \lambda(x) &= 1 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots, \quad x = \frac{A}{\hbar\omega}\end{aligned}$$



3D: $A = \hbar\omega$



QMC, 2D: $t = 0.05U$, $A = 0.4\hbar\omega$

3.5 Summary

Bose-Hubbard model with periodic modulated interaction:

$$\hat{H}_{\text{eff}} = \sum_i \left\{ \frac{1}{2} [U + A \cos(\omega t)] \hat{n}_i (\hat{n}_i - 1) - \mu \hat{n}_i \right\} - t \sum_{\langle ij \rangle} \hat{a}_i^\dagger \hat{a}_j$$



Approximation due to renormalization
of hopping with driving amplitude

Bose-Hubbard model with renormalized hopping:

$$\hat{H}(x) = \sum_i \left[\frac{U}{2} \hat{n}_i (\hat{n}_i - 1) - \mu \hat{n}_i \right] - t \lambda \left(\frac{A}{\hbar \omega} \right) \sum_{\langle ij \rangle} \hat{a}_i^\dagger \hat{a}_j$$

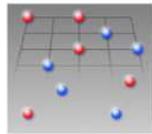
⇒ same universality class

⇒ same critical exponents

Hinrichs, Pelster, and Holthaus, Appl. Phys. B 113, 57 (2013)

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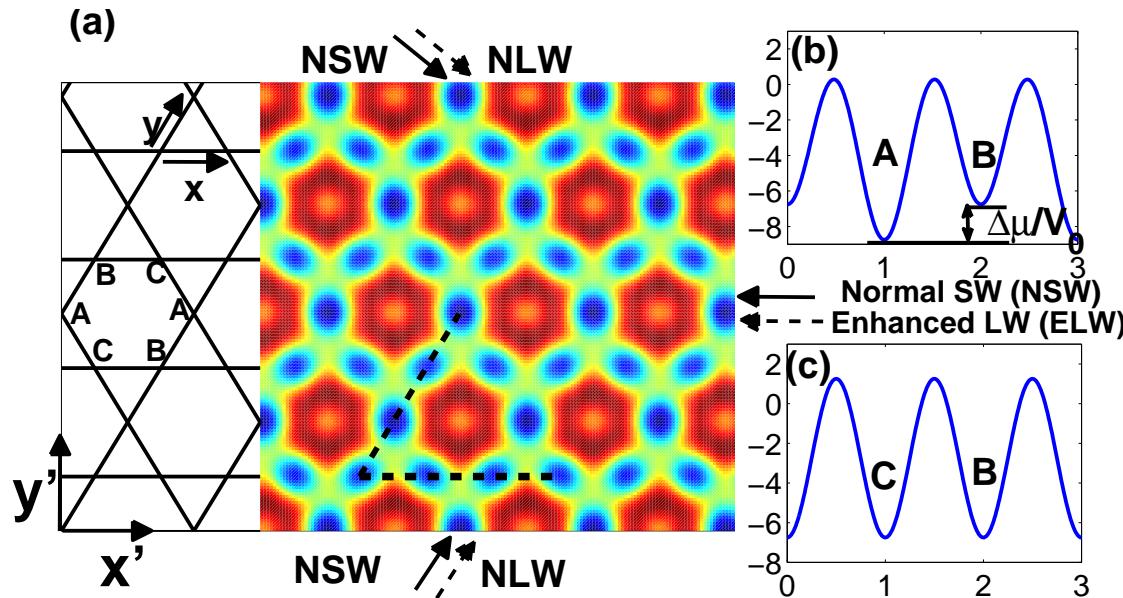
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 - 4. Kagome Superlattice**
- Zhang, Wang, Eggert, and Pelster,
PRB **92**, 014512 (2015)

4.1 Proposed Kagome Superlattice



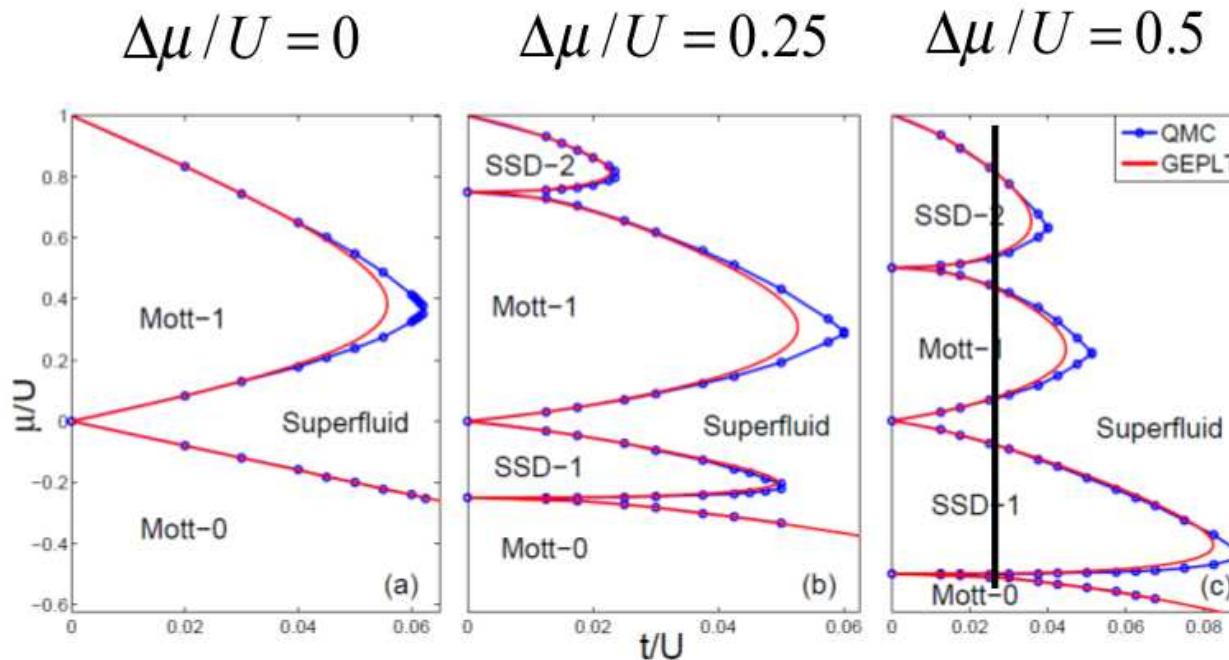
- **Optical potential** with $k = \sqrt{3}\pi/2\lambda_{\text{LW}}$, $\lambda_{\text{LW}} = 1064 \text{ nm}$, $\gamma = V_{\text{E}}/V_0$:

$$\begin{aligned} V_c/V_0 &= \gamma^2 - 1 + 4\gamma \cos(\sqrt{3}kx) \cos(ky) + 2 \cos(2ky) \\ &\quad - 2 \cos(4ky) - 4 \cos(2\sqrt{3}kx) \cos(2ky) \end{aligned}$$

- **Bose-Hubbard model** with $\Delta\mu = 4(\gamma - 1)V_0 > 0$:

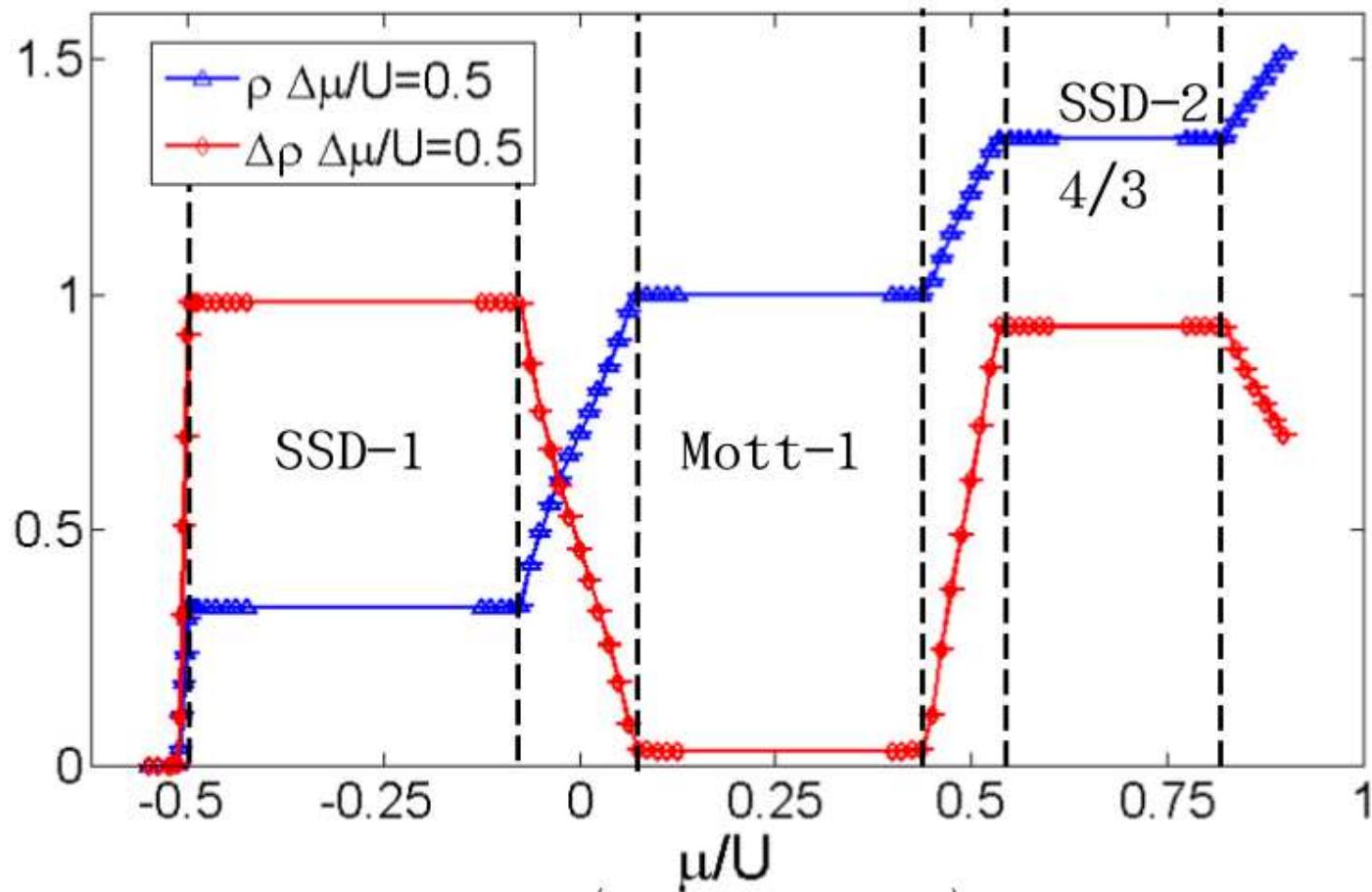
$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{a}_i^\dagger \hat{a}_j + \hat{a}_i \hat{a}_j^\dagger) + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) - \mu \sum_i \hat{n}_i - \Delta\mu \sum_{i \in A} \hat{n}_i$$

4.2 Quantum Phase Diagram



- **QMC: stochastic cluster series expansion**
Sandvik, PRB **59**, 14157(R) (1999)
- **Landau theory**
Santos and Pelster, PRA **79**, 013614 (2009)
Wang, Zhang, Eggert, Pelster, PRA **87**, 063615 (2013)
- **SSD (striped solid) phase:** fractional filling of 1/3 and 4/3

4.3 Insulating Phases



- **QMC:** $\beta U = 300$, $L = 9$, $t/U = 0.025$
- **Total density:** $\rho = (\rho_A + \rho_B + \rho_C) / 3$
- **Density difference:** $\Delta\rho = \rho_A - (\rho_B + \rho_C)/2$

4.4 Anisotropic Superfluidity

- **Superfluid density via winding number**

$$\rho_s^{x/y} = \langle W_{x/y}^2 / 4\beta t \rangle$$

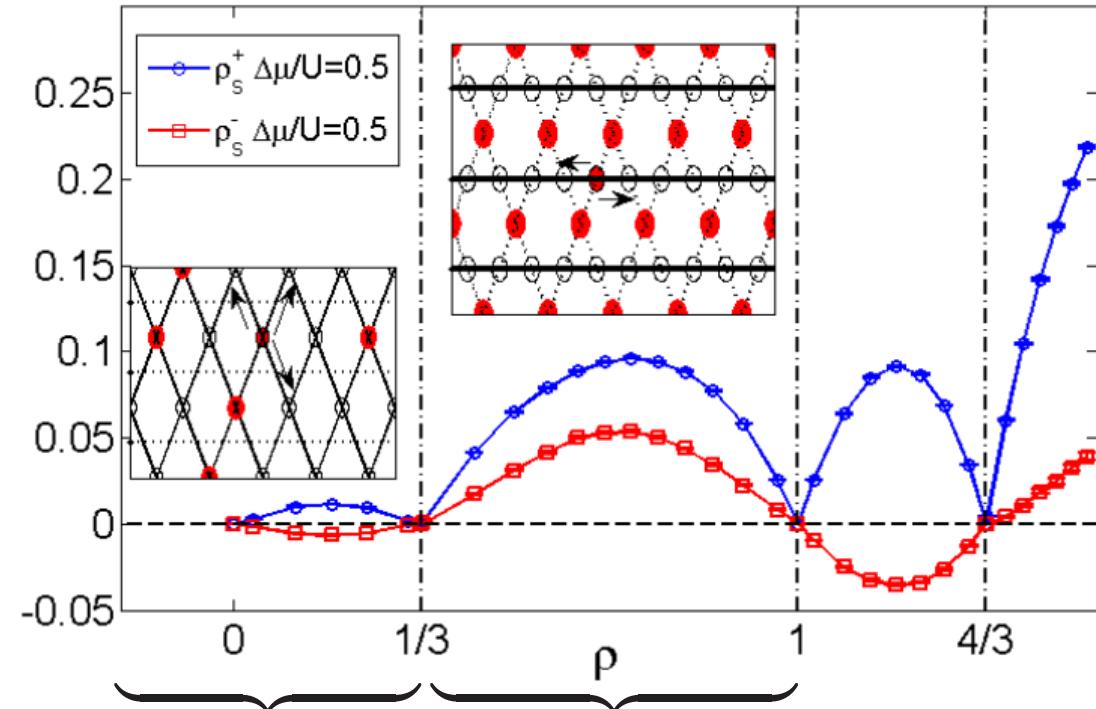
Pollock and Ceperley,
PRB 36, 8343 (1987)

- **Total superfluid density:**

$$\rho_s^+ = (\rho_s^x + \rho_s^y)/2$$

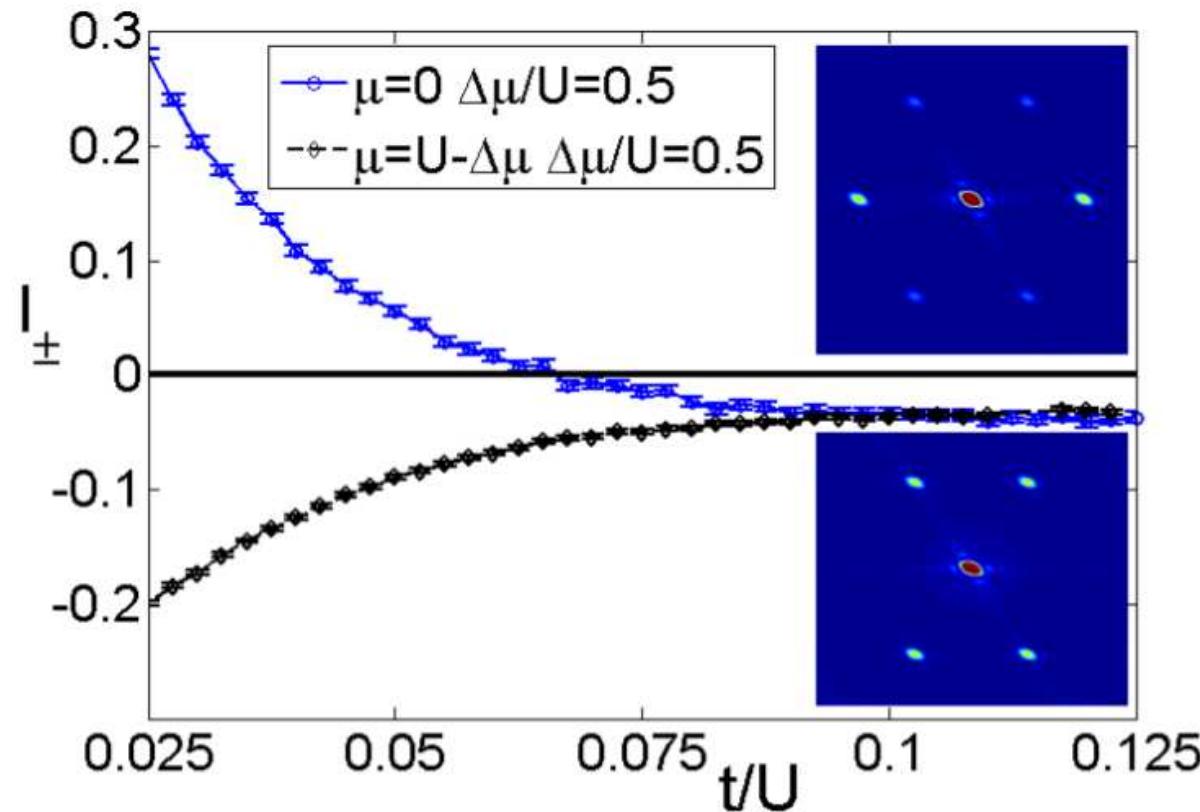
- **Superfluid density difference:**

$$\rho_s^- = \rho_s^x - \rho_s^y$$



- $\rho_s^x < \rho_s^y$
- A preferred square lattice
- Effective square lattice
- $\rho_s^x > \rho_s^y$
- A full, B/C preferred
- No supersolid due to artificial symmetry-breaking

4.5 Time-of-Flight Expansion



- **Anisotropic parameter:** $I_{\pm} = \frac{\rho_s^x - \rho_s^y}{\rho_s^x + \rho_s^y}$
- **Sign change indicates superfluid density is tensor**
- **Experimental detection by TOF absorption pictures**