Rotating Bose-Einstein condensates

S. Kling
Institut für Theoretische Physik,
Freie Universität Berlin, 14195 Berlin, Germany

A. Pelster
Fachbereich Physik,
Universität Duisburg-Essen, 45117 Essen, Germany

1. Theoretical description of the dynamics at $T = 0$
2. BEC in anharmonic trap
3. Outlook

1.1 Hydrodynamic

Gross-Pitaevskii equation:

$$\boxed{\left\{ i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2M}\Delta - V(\vec{r},t) - g|\psi|^2 \right\}\psi = 0}$$

Ansatz: $\psi = \sqrt{n}e^{i\varphi}$, $\quad n = |\psi|^2$ particle density

Continuity equation: $$\boxed{\frac{\partial}{\partial t}n + \vec{\nabla}(n\vec{v}) = 0}$$

$$\vec{v} = \frac{\hbar}{M}\vec{\nabla}\varphi \quad \text{velocity field} \quad \Rightarrow \quad \vec{\nabla}\times\vec{\nabla}\varphi = \vec{0}$$

Euler equation: $$\boxed{\frac{\partial\vec{v}}{\partial t} = -\vec{\nabla}\left[\frac{\vec{v}^{\,2}}{2} + \frac{V+gn}{M} - \frac{\hbar^2\vec{\nabla}n}{2M^2n}\right]}$$

Thomas-Fermi (TF) approximation:

$$\frac{\hbar^2}{2M}\Delta \longrightarrow 0 \ , \qquad n_{\mathrm{eq}} = \frac{\mu - V}{g} > 0$$

1.2 Collective modes in harmonic traps

Ansatz: $n = n_{\mathrm{eq}} + \delta n, \quad \delta n \propto e^{i\omega t}, \qquad \vec{v} = \delta\vec{v}$

Hydrodynamic collective modes:

$$\boxed{\omega^2 \delta n = \frac{1}{M}\left[\left(\vec{\nabla} V\right)\vec{\nabla} - (\mu - V)\vec{\nabla}^2\right]\delta n}$$

Anisotropic trap: $V = \frac{M\omega_z^2}{2}\left(\lambda^2 r_\perp^2 + z^2\right), \quad \lambda^2 = \frac{\omega_\perp^2}{\omega_z^2}$

Quadrupole surface modes:

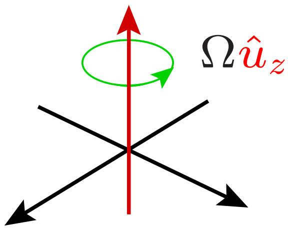
n_r	l	m	isotrop $(\lambda^2 = 1)$	anisotrop
0	2	± 2	$\omega^2 = 2\omega_z^2$	$\omega_Q^2 = 2\lambda^2\omega_z^2$
0	2	± 1	$\omega^2 = 2\omega_z^2$	$\omega^2 = \left(1+\lambda^2\right)\omega_z^2$

Mode coupling: $(1,0,0) + (0,2,0)$

$$\omega_\pm^2 = 2\omega_z^2\left(\frac{3}{4} + \lambda^2 \pm \sqrt{\lambda^4 - \lambda^2 + \frac{9}{16}}\right)$$

2.1 Anharmonic trap

Corotating frame: $H \longrightarrow H - \vec{\Omega}\vec{L}$

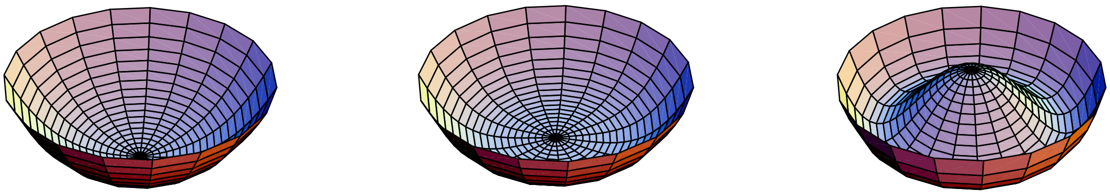


$$V(\vec{r},\Omega) = V(\vec{r}) - \frac{M\Omega^2}{2} r_\perp^2$$

Experiments of Dalibard group (ENS, Paris, 2004):

$$V(\vec{r},\Omega) = \frac{M\omega_z^2}{2}\left[\lambda^2\eta(\Omega)r_\perp^2 + z^2\right] + \frac{K}{4}r_\perp^4$$

$$\eta(\Omega) = 1 - \frac{\Omega^2}{\omega_\perp^2}$$ shape parameter



$\Omega < \omega_\perp$ $\qquad$ $\Omega = \omega_\perp$ $\qquad$ $\Omega > \omega_\perp$

Datas for $^{87}\mathrm{Rb}$:

$$M = 1.45 \cdot 10^{-25}\,\mathrm{kg}\ , \qquad N = 3 \cdot 10^5\ ,$$

$$\omega_z = 2\pi \cdot 11\,\mathrm{Hz}\ , \qquad \lambda = \omega_\perp/\omega_z = 6\ ,$$

$$-0.04 \leq \eta(\Omega) \leq 1\ , \qquad k = K\hbar/(M^2\omega_z^3) = 0.4$$

2.2 Collective modes in anharmonic trap

Variational approach:

$$\psi = \frac{\pi^{-3/4}}{\sqrt{\alpha_1\alpha_2\alpha_3}} \exp\left\{ -\sum_{k=1}^{3} \left[\frac{1}{2\alpha_k(t)^2} + i\beta_k(t) \right] x_k^2 \right\}$$

Variation: $L = \int \mathrm{d}^3x\, \mathcal{L}(\psi^\star, \psi) \longrightarrow$ Euler-Lagrange equations

Equations of motion: $\ddot{\alpha}_k = -\dfrac{\partial V_{\text{eff}}(\alpha_1, \alpha_2, \alpha_3)}{\partial \alpha_k}$

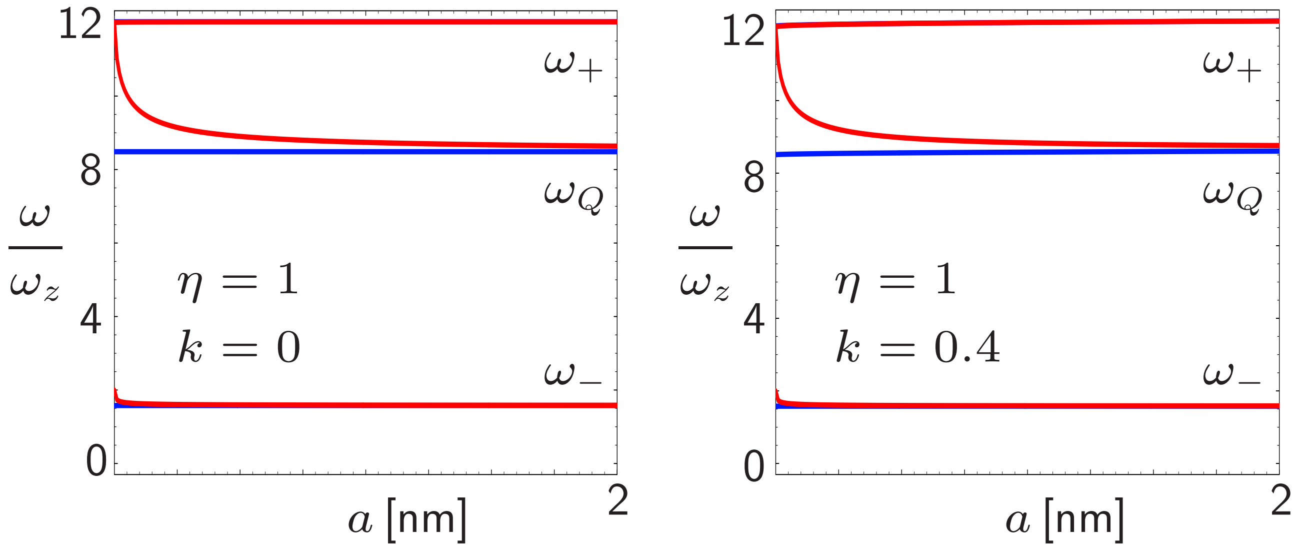
Low energy excitation: $\alpha_k = \alpha_{k0} + \delta\alpha_k$

$$V_{\text{eff}}(\vec{\alpha}) = V_{\text{eff}}(\vec{\alpha}_0) + \frac{1}{2}\delta\vec{\alpha} M \delta\vec{\alpha}$$

Hydrodynamic	$\leftrightarrow$	Variation
All frequencies	$\leftrightarrow$	Only three
Only valid in (TF)	$\leftrightarrow$	All regimes, (TF) limit coincides
Only harmonic traps	$\leftrightarrow$	arbitrary traps

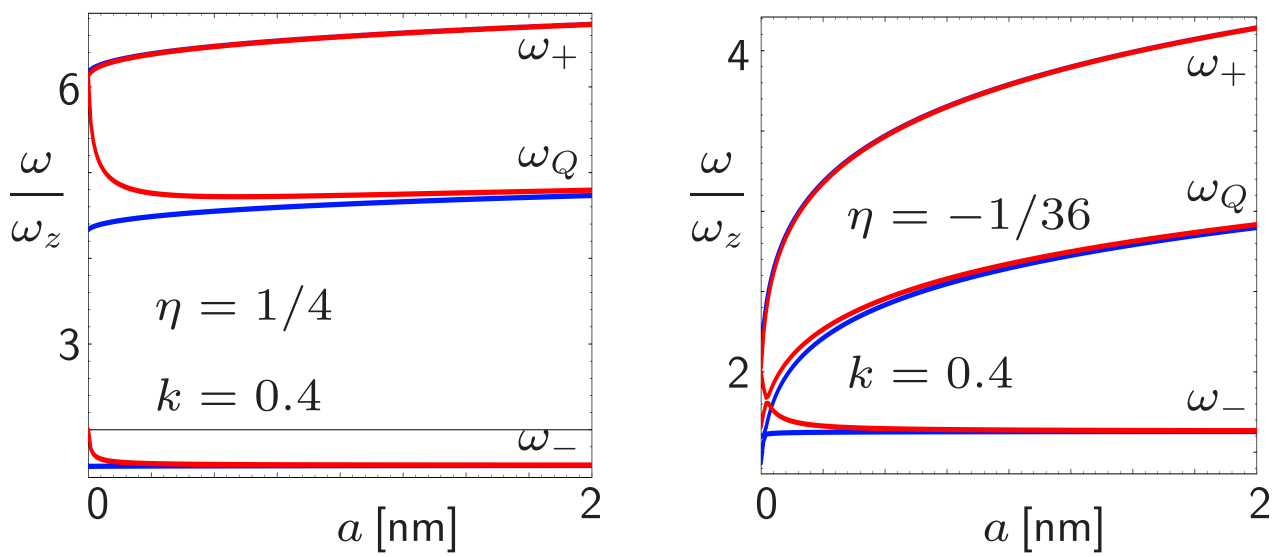
2.3 Results

Collective modes for values of Dalibard group $(k = 0.4)$:



— Variational approach , — TF-limit

scattering length $(^{87}\mathrm{Rb})$: $a = \dfrac{gM}{4\pi\hbar^2} = 5.2\,\mathrm{nm}$



2.4 Critical temperature

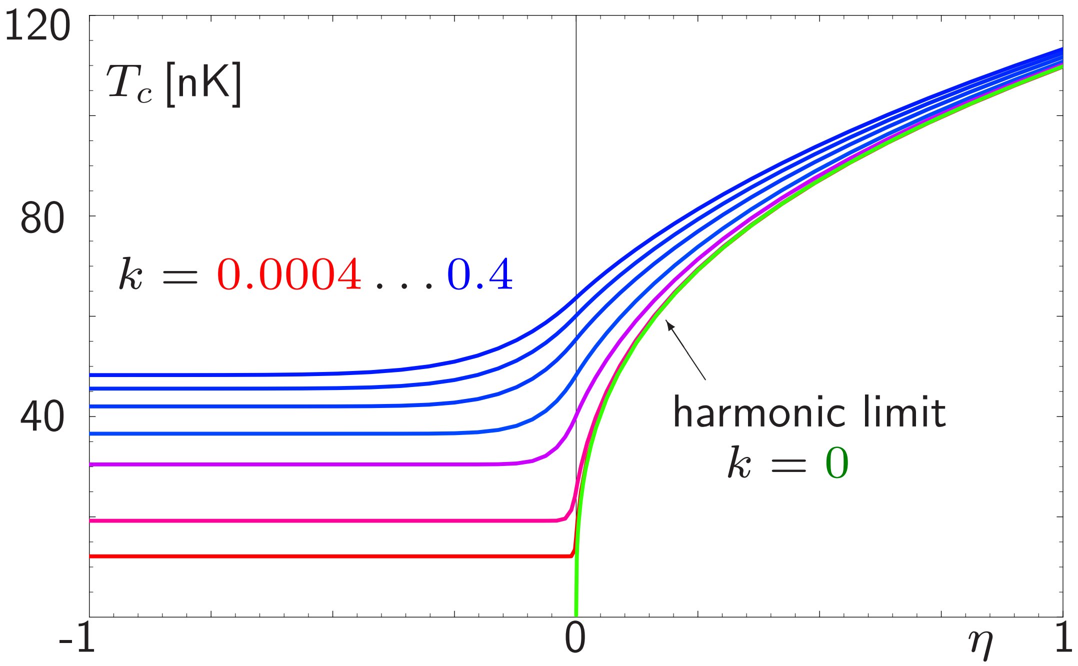
Grand-canonical ensemble: $$N = \sum_{\vec{n}} \frac{1}{e^{\beta(E_{\vec{n}}-\mu)} - 1}$$

Semiclassical approximation:

$$N = \int \frac{d^3p}{(2\pi\hbar)^3} \int d^3x \, \frac{1}{e^{\beta[H(\vec{x},\vec{p})-\mu]} - 1} \;, \qquad H = \frac{\vec{p}^{\,2}}{2M} + V$$

Critical point: $$\mu_c = \min_{\vec{x}} V(\vec{x})$$

Critical temperature: $$T_c(\eta) \;, \qquad \eta = 1 - \frac{\Omega^2}{\omega_\perp^2}$$



3. Outlook

Thermodynamics:

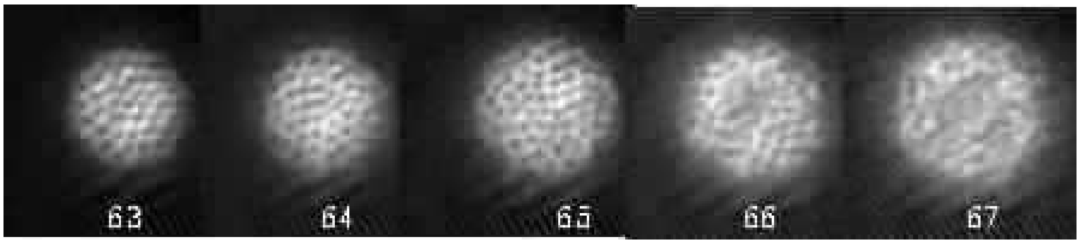
- Finite-size corrections
- Condensate density and thermodynamic quantities

Dynamics:

- Systematic pertubation theory
- Variational resummation

Vortices:

- Description of vortex arrays
- Melting of vortex lattices or fusion of vortex cores



V. Bretin, S. Stock, Y. Seurin, and J. Dalibard,

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