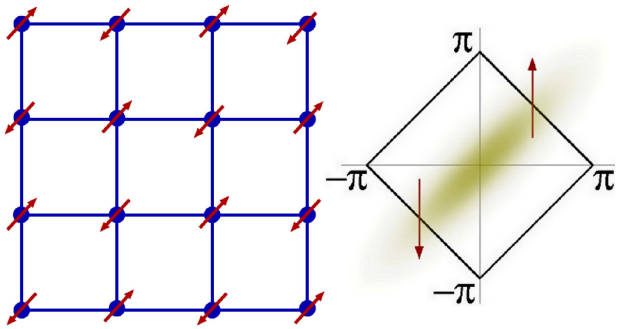
# Fortaleza-Ceará




# Competing Magnetic and Superfluid Phases in the Two-Channel Fermi-Hubbard Model

**Victor Bezerra**
Freie Universität Berlin

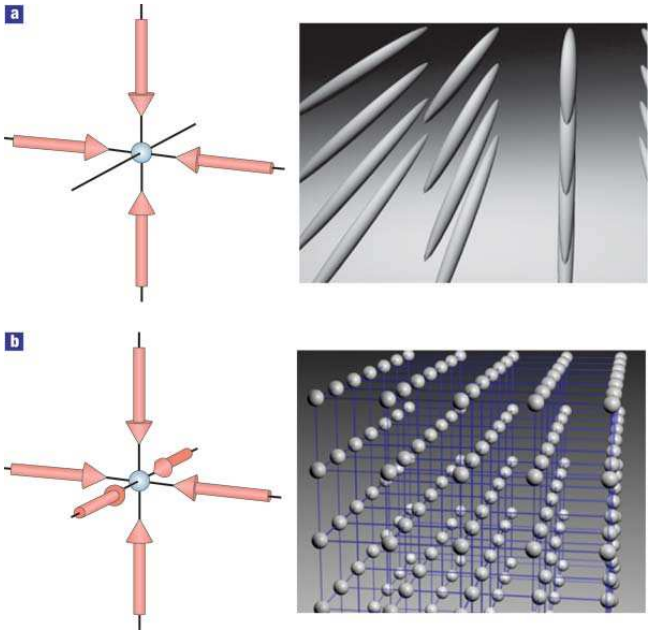
Technische Universität Kaiserslautern - Kaiserslautern, April 24, 2014

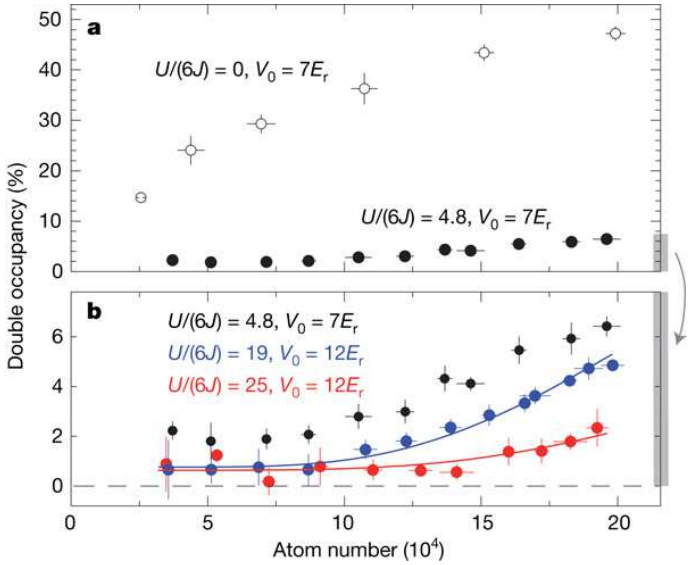
# Outline

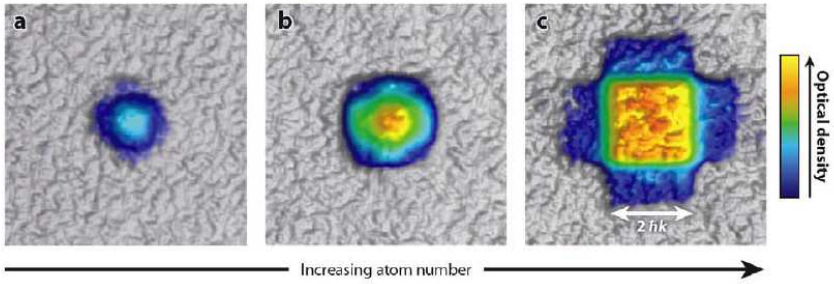
# Outline

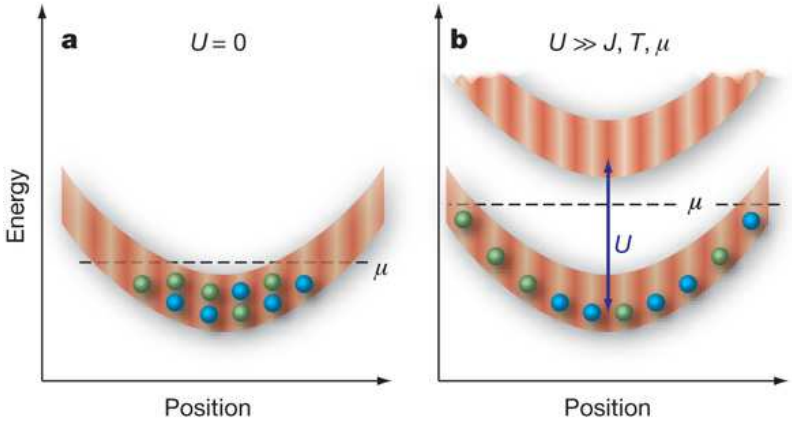
# Mott insulator regime of fermionic atoms in an optical lattice






M. Köhl et al., Phys. Rev. Lett. **94**, 080403 (2005)

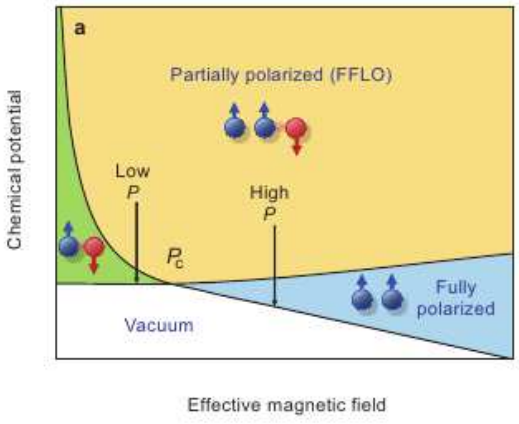
R. Jördens et al., Nature **455**, 204 (2008)

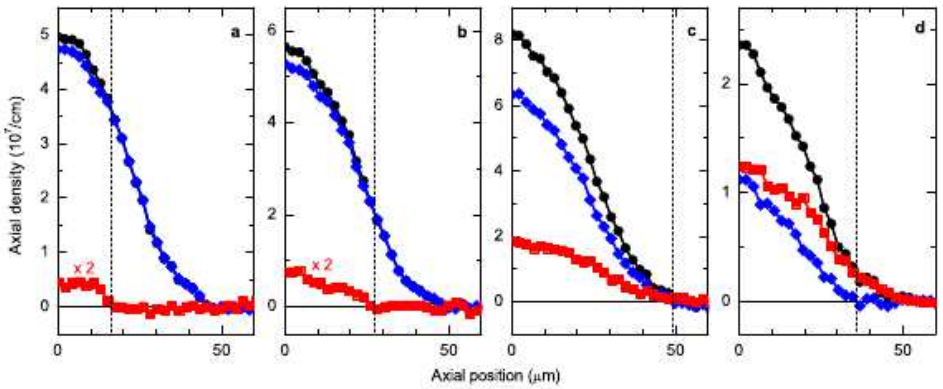
# Outline

# Polarized superfluid-FFLO in 1-d optical lattice

Cooper pairs with total momentum different from zero:
P. Fulde and R. Ferrel, Phys. Rev. **135**, 550 (1964)
A. Larkin and Y. Ovichnikov Sov. Phys. JETP. **20**, 762 (1965)

Hint in observing the FFLO phase.



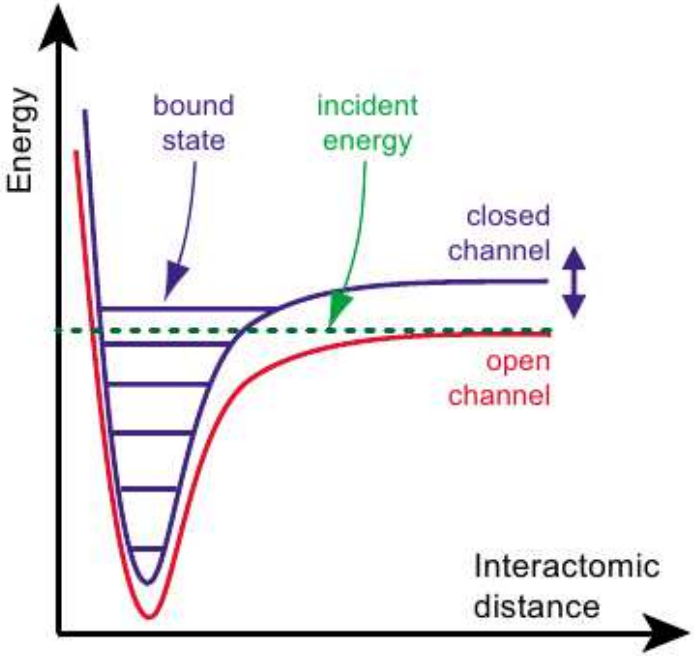

Y.-A. Liao et al., Nature **467**, 567 (2010)

# Outline

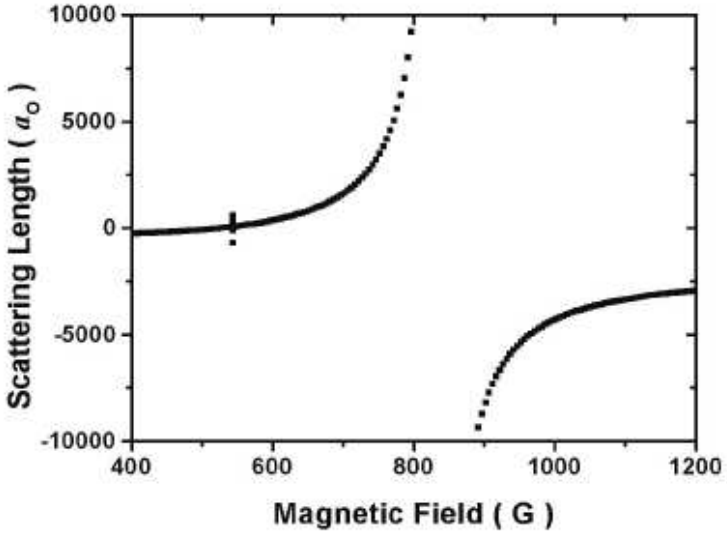
# Feshbach resonance

- Possibility of controlling the interaction of ultra cold atomic systems
- Realization of BEC-BCS crossover with cold atoms
- Near resonance $a_s$ depends only on magnetic field $B$:

$$a_s(B) = a_0\left(1 - \frac{\Delta B}{B - B_0}\right)$$




I. Bloch et al., Rev. Mod. Phys. **80**, 885 (2008)

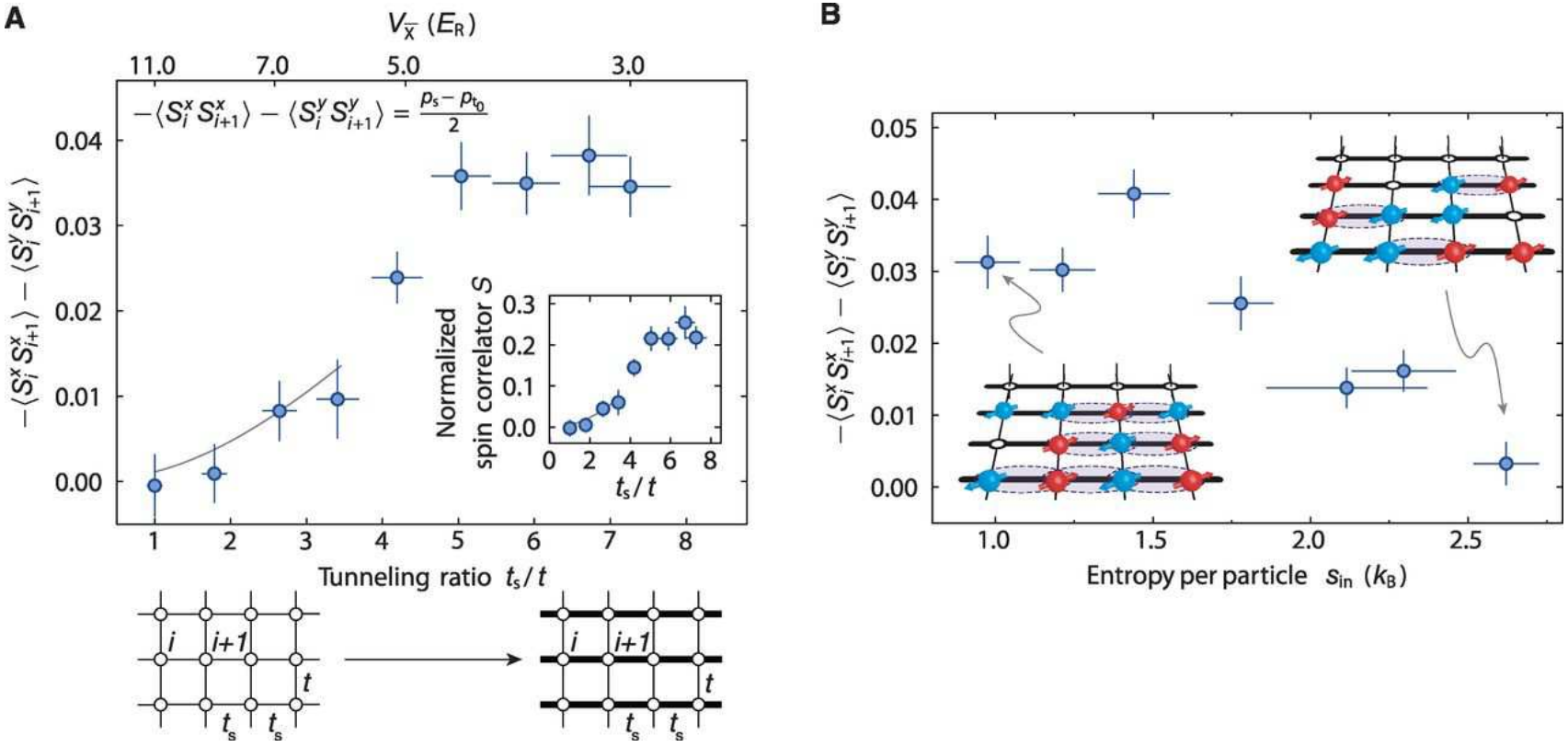



E. Partridge et al., Phys. Rev. Lett. **91**, 80406 (2003)

# Outline

# Short range quantum magnetism in 3-d optical lattices



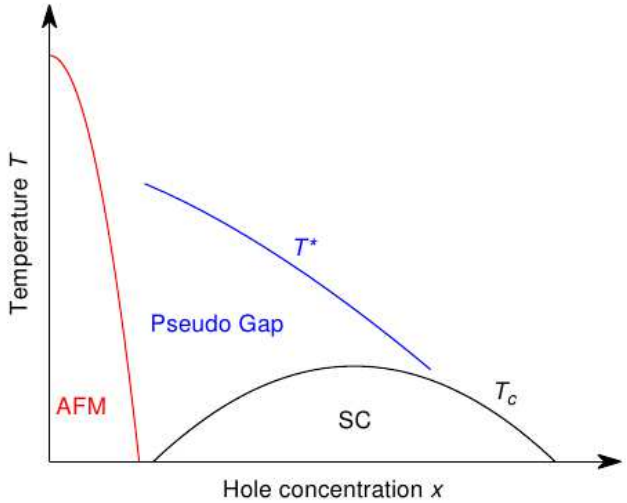



D. Greif et al., Science **340**, 1307 (2013)

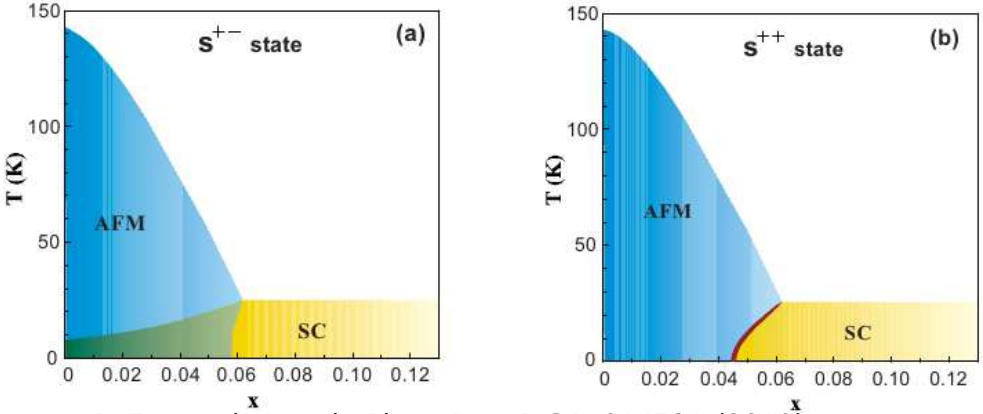
# Outline

# Simulator of high temperature superconductors?

- ★ Way to understand high $T_C$ SC?
- Cuprates (d-wave SC):




- Pnictides (d-wave SC and s-wave SC):




R. Fernandes et al., Phys. Rev. B **81**, 014521 (2010)

- Quantum simulator: R. Feynman, Int. J. Theor. Phys. **21**, 467 (1982)

# Outline

# Quantum field Hamiltonian

- Effective quantum field theoretical model

$$\hat{H} = \sum_{\sigma=\uparrow,\downarrow} \int \hat{\Psi}^{\dagger}_{\sigma}(\mathbf{r}) \left( -\frac{\hbar^2 \nabla^2}{2m} - \mu'_{\sigma} + V_{\mathrm{op}}(\mathbf{r}) \right) \hat{\Psi}_{\sigma}(\mathbf{r}) d^3\mathbf{r}$$
$$+ \int \int \hat{\Psi}^{\dagger}_{\uparrow}(\mathbf{r}_1) \hat{\Psi}^{\dagger}_{\downarrow}(\mathbf{r}_2) V_{\mathrm{int}}(\mathbf{r}_1, \mathbf{r}_2) \hat{\Psi}_{\downarrow}(\mathbf{r}_2) \hat{\Psi}_{\uparrow}(\mathbf{r}_1) d^3\mathbf{r}_1 d^3\mathbf{r}_2$$

$$\left\{ \hat{\Psi}^{\dagger}_{\sigma}(\mathbf{r}_1), \hat{\Psi}^{\dagger}_{\alpha}(\mathbf{r}_2) \right\} = \left\{ \hat{\Psi}_{\sigma}(\mathbf{r}_1), \hat{\Psi}_{\alpha}(\mathbf{r}_2) \right\} = 0, \quad \left\{ \hat{\Psi}^{\dagger}_{\sigma}(\mathbf{r}_1), \hat{\Psi}_{\alpha}(\mathbf{r}_2) \right\} = \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta_{\sigma\alpha}$$

- Periodic Potential

$$V_{\mathrm{op}}(\mathbf{r}) = V_0 \left[ \sin^2(kx) + \sin^2(ky) + \sin^2(kz) \right]$$

- Contact interaction

$$V_{\mathrm{int}}(\mathbf{r}_1, \mathbf{r}_2) = \frac{4\pi\hbar^2}{m} a_0 \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

# Outline

# Wannier functions

- Expanding field operators in orthonormal basis:

$$\hat{\Psi}_\sigma(\mathbf{r}) = \sum_{i,\mathbf{n}} \hat{f}_{i\sigma,\mathbf{n}} w^{(\mathbf{n})}(\mathbf{r}-\mathbf{r}_i), \quad \hat{\Psi}^\dagger_\sigma(\mathbf{r}) = \sum_{i,\mathbf{n}} \hat{f}^\dagger_{i\sigma,\mathbf{n}} w^{(\mathbf{n})*}(\mathbf{r}-\mathbf{r}_i)$$

- Restriction to the lowest energy band
- Discarding overlap of Wannier functions at different sites for calculating matrix elements

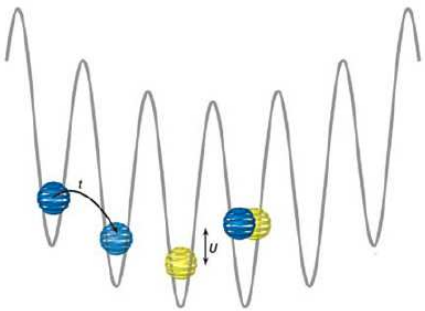
$$\begin{aligned}
t_{i,j} &= -\int w^*(\mathbf{r}-\mathbf{r}_i)\left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{op}}(\mathbf{r})\right] w(\mathbf{r}-\mathbf{r}_j) d^3\mathbf{r} \\
U &= \frac{4\pi\hbar^2}{m} a_0 \int |w(\mathbf{r})|^4 d^3\mathbf{r} \\
\mu_\sigma &= \mu'_\sigma - \int w^*(\mathbf{r}-\mathbf{r}_i)\left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{op}}(\mathbf{r})\right] w(\mathbf{r}-\mathbf{r}_i) d^3\mathbf{r}
\end{aligned}$$

# Outline

# Fermi-Hubbard model

Lattice Hamiltonian:

$$\hat{H} = -t \sum_{\langle i,j \rangle, \sigma} \hat{f}^{\dagger}_{i\sigma} \hat{f}_{j\sigma} + U \sum_{i} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} - \mu \sum_{i,\sigma} \hat{n}_{i\sigma}, \quad \text{with} \quad \hat{n}_{i\sigma} = \hat{f}^{\dagger}_{i\sigma} \hat{f}_{i\sigma}$$




- Introduced in 1963 by John Hubbard to study transitions metals and alloys.
- Exact solution in $1-d$:

  E. Lieb and F. Wu, Phys. Rev. Lett. **20**, 1445 (1968)
- Review of analytic results and open problems:

  E. Lieb, XI Int. Cong. MP, Int. Press, 392 (1995)

# Outline

# Two-channel Fermi-Hubbard model

- Periodic potential
- Feshbach resonance results in Feshbach molecules: $\hat{\Psi}^{(m)}$

$$H = \sum_{\sigma=\uparrow,\downarrow} \int \hat{\Psi}_\sigma^{(a)\dagger}(\mathbf{r})(T_a + V_a)\hat{\Psi}_\sigma^{(a)}(\mathbf{r})d^3\mathbf{r} + \int \hat{\Psi}^{(m)\dagger}(\mathbf{r})(T_m + V_m + \delta')\hat{\Psi}^{(m)}(\mathbf{r})d^3\mathbf{r}$$
$$+ \frac{4\pi\hbar^2 a_0}{m} \int \hat{\Psi}_\uparrow^{(a)\dagger}(\mathbf{r})\hat{\Psi}_\downarrow^{(a)\dagger}(\mathbf{r})\hat{\Psi}_\downarrow^{(a)}(\mathbf{r})\hat{\Psi}_\uparrow^{(a)}(\mathbf{r})d^3\mathbf{r}$$
$$+ g \int \left[\hat{\Psi}^{(m)\dagger}(\mathbf{r})\hat{\Psi}_\downarrow^{(a)}(\mathbf{r})\hat{\Psi}_\uparrow^{(a)}(\mathbf{r}) + \hat{\Psi}_\uparrow^{(a)\dagger}(\mathbf{r})\hat{\Psi}_\downarrow^{(a)\dagger}(\mathbf{r})\hat{\Psi}^{(m)}(\mathbf{r})\right]d^3\mathbf{r}$$

- Detuning parameter $\delta'$ depends on magnetic field $B$
- $g = \hbar\sqrt{4\pi\Delta\mu_B\Delta B|a_0|/m}$
- Lattice Hamiltonian or two channel Fermi-Hubbard model:

$$\hat{H} = -t_\mathrm{f} \sum_{\langle i,j\rangle,\sigma} \hat{f}_{i\sigma}^\dagger \hat{f}_{j\sigma} - t_\mathrm{b} \sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j - \mu \sum_i \hat{n}_i + (\delta - 2\mu) \sum_i \hat{b}_i^\dagger \hat{b}_i$$
$$+ \sum_i \left[g(\hat{b}_i^\dagger \hat{f}_{i\downarrow}\hat{f}_{i\uparrow} + \hat{f}_{i\uparrow}^\dagger \hat{f}_{i\downarrow}^\dagger \hat{b}_i) + U\hat{n}_{i\uparrow}\hat{n}_{i\downarrow}\right]$$

D. Dickerscheid et al., Phys. Rev. A **71**, 43604 (2005); L. Carr and M. Holland, Phys. Rev. A **72**, 31604 (2005)

# Outline

# Hubbard interaction in the spin representation



$$\hat{H} = -t_{\mathrm{f}} \sum_{\langle i,j\rangle,\sigma} \hat{f}^{\dagger}_{i\sigma}\hat{f}_{j\sigma} - t_{\mathrm{b}} \sum_{\langle i,j\rangle} \hat{b}^{\dagger}_{i}\hat{b}_{j} - \mu \sum_{i} \hat{n}_i + (\delta - 2\mu) \sum_{i} \hat{b}^{\dagger}_{i}\hat{b}_{i}$$
$$+ \sum_{i} \left[ g(\hat{b}^{\dagger}_{i}\hat{f}_{i\downarrow}\hat{f}_{i\uparrow} + \hat{f}^{\dagger}_{i\uparrow}\hat{f}^{\dagger}_{i\downarrow}\hat{b}_{i}) + U\hat{n}_{i\uparrow}\hat{n}_{i\downarrow} \right]$$

$$U\hat{n}_{i\uparrow}\hat{n}_{i\downarrow} = U\frac{\hat{n}_{i\uparrow} + \hat{n}_{i\downarrow}}{2} - U\frac{2}{3}\hat{\mathbf{s}}^2_i, \quad \hat{S}^a_i = \frac{1}{2}\sum_{\alpha,\beta} \hat{f}^{\dagger}_{i\alpha}\sigma^a_{\alpha\beta}\hat{f}_{i\beta}$$

$$\hat{H} = -t_{\mathrm{f}} \sum_{\langle i,j\rangle,\sigma} \hat{f}^{\dagger}_{i\sigma}\hat{f}_{j\sigma} - t_{\mathrm{b}} \sum_{\langle i,j\rangle} \hat{b}^{\dagger}_{i}\hat{b}_{j} - \left(\mu - \frac{U}{2}\right) \sum_{i} \hat{n}_i + (\delta - 2\mu) \sum_{i} \hat{b}^{\dagger}_{i}\hat{b}_{i}$$
$$+ \sum_{i} \left[ g(\hat{b}^{\dagger}_{i}\hat{f}_{i\downarrow}\hat{f}_{i\uparrow} + \hat{f}^{\dagger}_{i\uparrow}\hat{f}^{\dagger}_{i\downarrow}\hat{b}_{i}) - \frac{2U}{3}\hat{\mathbf{s}}^2_i \right]$$

- $U > 0$ prerequisite for magnetic order

# Outline

# General mean-field approach

- Mean-field approximation to field $\hat{\mathbf{S}}$:

$$\hat{\mathbf{S}}_i = \langle \hat{\mathbf{S}}_i \rangle + \delta \hat{\mathbf{S}}_i$$

$$\delta \hat{\mathbf{S}}_i^2 = \hat{\mathbf{S}}_i^2 - 2\hat{\mathbf{S}}_i \cdot \langle \hat{\mathbf{S}}_i \rangle + \langle \hat{\mathbf{S}}_i \rangle^2$$

- Discarding second order fluctuations:

$$\frac{2}{3}\hat{\mathbf{S}}_i^2 \sim \frac{2}{3}\left(2\hat{\mathbf{S}}_i \cdot \langle \hat{\mathbf{S}}_i \rangle - \langle \hat{\mathbf{S}}_i \rangle^2\right)$$

- Introducing $\mathbf{m}_i = \frac{4}{3}\langle \hat{\mathbf{S}}_i \rangle$:

$$\begin{aligned} \hat{H} &= -t_{\mathrm{f}} \sum_{\langle i,j \rangle, \sigma} \hat{f}_{i\sigma}^{\dagger} \hat{f}_{j\sigma} - t_{\mathrm{b}} \sum_{\langle i,j \rangle} \hat{b}_i^{\dagger} \hat{b}_j - \left(\mu - \frac{U}{2}\right) \sum_i \hat{n}_i + (\delta - 2\mu) \sum_i \hat{b}_i^{\dagger} \hat{b}_i \\ &+ \sum_i \left[ g(\hat{b}_i^{\dagger} \hat{f}_{i\downarrow} \hat{f}_{i\uparrow} + \hat{f}_{i\uparrow}^{\dagger} \hat{f}_{i\downarrow}^{\dagger} \hat{b}_i) - U \mathbf{m}_i \cdot \hat{\mathbf{S}}_i + \frac{3U}{8} \mathbf{m}_i^2 \right] \end{aligned}$$

- $m \neq 0$ lowers system energy. Magnetic orders are favored.

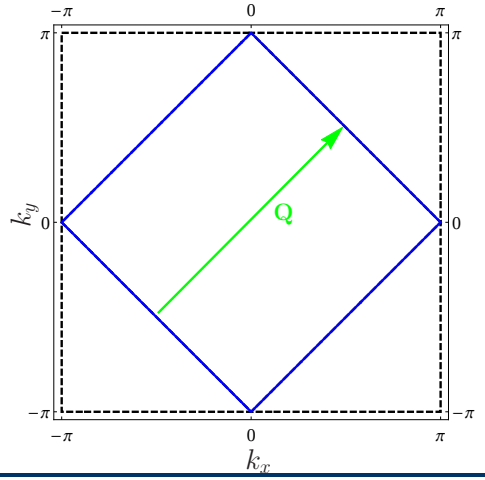
# General mean-field approach

- Mean-Field to boson field: $\hat{b}_i \sim \langle \hat{b}_i \rangle / g$
- Wave solutions for both order parameters:
  $\mathbf{m}_i = m\cos(\mathbf{q}\cdot\mathbf{R}_i)\mathbf{e}_z$ and $\langle \hat{b}_i \rangle = \Delta e^{i\mathbf{p}\cdot\mathbf{R}_i}$ ($L$ lattice volume)
- General MF Hamiltonian

$$\hat{H}_{\mathrm{MF}} = -t_{\mathrm{f}} \sum_{\langle i,j\rangle,\sigma} \hat{f}^{\dagger}_{i\sigma}\hat{f}_{j\sigma} - \left(\mu - \frac{U}{2}\right)\sum_i \hat{n}_i + \left(-t_{\mathrm{b}} \sum_{\langle i,j\rangle} e^{-i\mathbf{p}\cdot(\mathbf{R}_j-\mathbf{R}_i)} + \delta - 2\mu\right)\frac{|\Delta|^2}{g^2}L$$
$$+ \sum_i \left[\Delta^* e^{-i\mathbf{p}\cdot\mathbf{R}_i}\hat{f}_{i\downarrow}\hat{f}_{i\uparrow} + \hat{f}^{\dagger}_{i\uparrow}\hat{f}^{\dagger}_{i\downarrow}\Delta e^{i\mathbf{p}\cdot\mathbf{R}_i} - \frac{mU}{2}(n_{i\uparrow}-n_{i\downarrow})\cos(\mathbf{q}\cdot\mathbf{R_i}) + \frac{3U}{8}m^2\right]$$

| General | FFLO ($\mathbf{q}=\mathbf{0}$) | AF+BCS ($\mathbf{p}=\mathbf{0}, \mathbf{q}=\mathbf{Q}$) | BEC-BCS |
|---|---|---|---|
| $\mathbf{m}_i = m\cos(\mathbf{q}\cdot\mathbf{R}_i)\mathbf{e}_z$<br>$\langle \hat{b}_i \rangle = \Delta e^{i\mathbf{p}\cdot\mathbf{R}_i}$ | $\mathbf{m}_i = m\mathbf{e}_z$<br>$\langle \hat{b}_i \rangle = \Delta e^{i\mathbf{p}\cdot\mathbf{R}_i}$ | $\mathbf{m}_i = m\cos(\mathbf{Q}\cdot\mathbf{R}_i)\mathbf{e}_z$<br>$\langle \hat{b}_i \rangle = \Delta$ | $\mathbf{m}_i = \mathbf{0}$<br>$\langle \hat{b}_i \rangle = \Delta$ |

$\mathbf{Q} = (\pi, \ldots, \pi)$ Nesting Vector



# General mean-field approach

- In Fourier space

$$\hat{H}_{\mathrm{MF}} = \sum_{\mathbf{k}} \epsilon^{\mathrm{f}}_{\mathbf{k}} (\hat{f}^{\dagger}_{\mathbf{k}\uparrow}\hat{f}_{\mathbf{k}\uparrow} + \hat{f}^{\dagger}_{-\mathbf{k}\downarrow}\hat{f}_{-\mathbf{k}\downarrow}) - \left(\mu - \frac{U}{2}\right)\sum_{\mathbf{k}} (\hat{f}^{\dagger}_{\mathbf{k}\uparrow}\hat{f}_{\mathbf{k}\uparrow} + \hat{f}^{\dagger}_{-\mathbf{k}\downarrow}\hat{f}_{-\mathbf{k}\downarrow}) + \frac{\Delta^2}{g^2}(\epsilon^{\mathrm{b}}_{\mathbf{p}} + \delta - 2\mu)L$$
$$+ \quad \frac{3U}{8}m^2 L + \sum_{\mathbf{k}} \left[ \Delta \hat{f}^{\dagger}_{\mathbf{k}+\frac{\mathbf{p}}{2}\uparrow} \hat{f}^{\dagger}_{-\mathbf{k}+\frac{\mathbf{p}}{2}\downarrow} - \frac{mU}{4}(\hat{f}^{\dagger}_{\mathbf{k}\uparrow}\hat{f}_{\mathbf{k}+\mathbf{q}\uparrow} - \hat{f}^{\dagger}_{\mathbf{k}\downarrow}\hat{f}_{\mathbf{k}+\mathbf{q}\downarrow}) + \mathrm{h.c} \right]$$

- Free particle lattice dispersion: $\epsilon^{\mathrm{f,b}}_{\mathbf{k}} = -2t_{\mathrm{f,b}} \sum_a \cos k_a$
- $k_a$ dimensionless, lattice spacing is set equal to unity
- Nambu spinors as:

$$\hat{\Psi}_{\mathbf{k}} \equiv \begin{pmatrix} \hat{f}_{\mathbf{k}+\frac{\mathbf{p}}{2}\uparrow} \\ \hat{f}_{\mathbf{k}-\mathbf{q}+\frac{\mathbf{p}}{2}\uparrow} \\ \hat{f}^{\dagger}_{-\mathbf{k}+\frac{\mathbf{p}}{2}\downarrow} \\ \hat{f}^{\dagger}_{-\mathbf{k}+\mathbf{q}+\frac{\mathbf{p}}{2}\downarrow} \end{pmatrix}$$

$\hat{H}_{\mathrm{MF}} = \sum'_{\mathbf{k}} \hat{\Psi}^{\dagger}_{\mathbf{k}} M_{\mathbf{k}}(\mathbf{p},\mathbf{q}) \hat{\Psi}_{\mathbf{k}} + C$ where

$C = \sum'_{\mathbf{k}} \left( \xi_{-\mathbf{k}+\frac{\mathbf{p}}{2}} + \xi_{-\mathbf{k}+\mathbf{q}+\frac{\mathbf{p}}{2}} \right) + \frac{\Delta^2}{g^2}(\epsilon^{\mathrm{b}}_{\mathbf{p}} + \delta - 2\mu)L + \frac{3U}{8}m^2 L, \quad \xi_{\mathbf{k}} = \epsilon^{\mathrm{f}}_{\mathbf{k}} - (\mu - \frac{U}{2}), \quad \sum'_{\mathbf{k}}$ AF-BZ

- $\mathrm{Det}\left[M_{\mathbf{k}}(\mathbf{p},\mathbf{q}) - E_{\mathbf{k},\mathbf{p},\mathbf{q}} I\right] = 0$, $M_{\mathbf{k}}(\mathbf{p},\mathbf{q})$ and $I$ are 4 x 4 matrices

# Outline

# Néel Versus BCS order

- $\mathbf{q} = \mathbf{Q}$ and $\mathbf{p} = \mathbf{0}$ $\quad (a, b = \pm)$

$$E_{\mathbf{k}}^{(a,b)} = a\sqrt{\left(\frac{mU}{4}\right)^2 + \Delta^2 + \left(\mu - \frac{U}{2}\right)^2 + \epsilon_{\mathbf{k}}^2 + 2b\sqrt{\left(\frac{mU}{4}\right)^2 \Delta^2 + \left(\mu - \frac{U}{2}\right)^2 \left[\left(\frac{mU}{4}\right)^2 + \epsilon_{\mathbf{k}}^2\right]}}$$

- Thermodynamic Potential

$$\Omega_{\mathrm{MF}} = -\frac{1}{\beta}\sum_{\mathbf{k},a,b}' \ln\left(1 + e^{-\beta E_{\mathbf{k},\mathbf{q}}^{(a,b)}}\right) + C_{\mathrm{AF}}, \qquad C_{\mathrm{AF}} = \sum_{\mathbf{k}}'\left(\xi_{-\mathbf{k}} + \xi_{-\mathbf{k}+\mathbf{q}}\right) + \frac{\Delta^2}{g^2}(-zt_{\mathrm{b}} + \delta - 2\mu)L + \frac{3U}{8}m^2 L$$

- Extremizing $\Omega$ with respect to $\Delta$ and $m$

$$\frac{1}{L}\sum_{\mathbf{k},a,b}' a\, N_{\mathrm{F}}\left(E_{\mathbf{k}}^{(a,b)}\right)\frac{\Delta}{2E_{\mathbf{k}}^{(b)}}\left(1 + b\frac{m^2U^2}{2\lambda_{\mathbf{k}}}\right) = \frac{\Delta}{g^2}\left(-zt_{\mathrm{b}} + \delta - 2\mu\right)$$

$$\frac{1}{L}\sum_{\mathbf{k},a,b}' a\, N_{\mathrm{F}}\left(E_{\mathbf{k}}^{(a,b)}\right)\frac{mU}{4E_{\mathbf{k}}^{(b)}}\left[1 + 2b\frac{(\mu - U/2)^2 + \Delta^2}{\lambda_{\mathbf{k}}}\right] = 3m$$

- Equation of state

$$n = 1 + 2\frac{\Delta^2}{g^2} - \frac{1}{L}\sum_{\mathbf{k},a,b}' a\, N_{\mathrm{F}}\left(E_{\mathbf{k}}^{(a,b)}\right)\frac{\mu - U/2}{E_{\mathbf{k}}^{(b)}}\left[1 + 2b\frac{\left(\frac{mU}{4}\right)^2 + \epsilon_{\mathbf{k}}^2}{\lambda_{\mathbf{k}}}\right]$$

- ★ Self consistent equations

with Fermi function: $\quad N_{\mathrm{F}}(E) = \dfrac{1}{e^{\beta E} + 1}$ $\quad$ and $\quad \lambda_{\mathbf{k}} = \sqrt{\left(\frac{mU}{4}\right)^2 \Delta^2 + \left(\mu - \frac{U}{2}\right)^2 \left[\left(\frac{mU}{4}\right)^2 + \epsilon_{\mathbf{k}}^2\right]}$

# Outline

# Half filling case -2 − $d$

- $\mu = U/2$ yields $n = 1 + 2n_{\mathrm{b}} = 2\frac{\Delta^2}{g^2} + 1 \quad \Rightarrow \quad \Delta = g\sqrt{\frac{n-1}{2}}$
- at $T = 0$

$$\frac{1}{L}\sum_{\mathbf{k},a=\pm}{}' \frac{1}{2E_{\mathbf{k}}^{(a)}}\left(1 + a\frac{4\Delta}{mU}\right) = \frac{1}{g^2}\left(-zt_{\mathrm{b}} + \delta - U\right)$$

$$\frac{1}{L}\sum_{\mathbf{k},a=\pm}{}' \frac{1}{E_{\mathbf{k}}^{(a)}}\left[1 + a\frac{4\Delta}{mU}\right] = \frac{12}{U}$$

- Ground-state energy

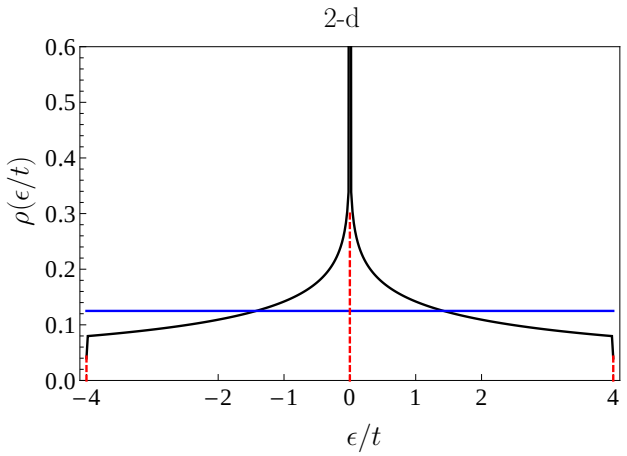
$$\Omega_{\mathrm{MF}}^{T\to 0} = -\sum_{\mathbf{k},a=\pm}{}' E_{\mathbf{k}}^{(a)} + \Delta^2\frac{\delta - t_{\mathrm{b}} - U}{g^2} + \frac{3m^2U}{8}$$

$E_{\mathbf{k}}^{(a)} = \sqrt{\epsilon_{\mathbf{k}}^2 + (\frac{mU}{4} + a\Delta)^2}$

- Some simplifications: Thermodynamic limit $1/L\sum_{\mathbf{k}} \to \int_{-4t}^{4t} d\epsilon\rho(\epsilon)$

Constant density of states

$$\rho(\epsilon) = \frac{1}{8t}\theta\left(4t - |\epsilon|\right)$$



# Half filling case -2 — $d$

$$\frac{1}{8t}\left[I^{(+)}(m,\Delta)\left(1+\frac{4\Delta}{mU}\right)+I^{(-)}(m,\Delta)\left(1-\frac{4\Delta}{mU}\right)\right]=\frac{12}{U}$$

$$\frac{1}{8t}\left[I^{(+)}(m,\Delta)\left(1+\frac{mU}{4\Delta}\right)+I^{(-)}(m,\Delta)\left(1-\frac{mU}{4\Delta}\right)\right]=\frac{2}{U_{\text{eff}}}$$

$$I^{(\pm)}=\ln\left(\frac{4t+\sqrt{(4t)^2+\left(\frac{mU}{4}\pm\Delta\right)^2}}{\left|\frac{mU}{4}\pm\Delta\right|}\right)$$

and we have defined

$$\frac{1}{U_{\text{eff}}}=\frac{\delta-zt_{\text{b}}-U}{g^2}$$

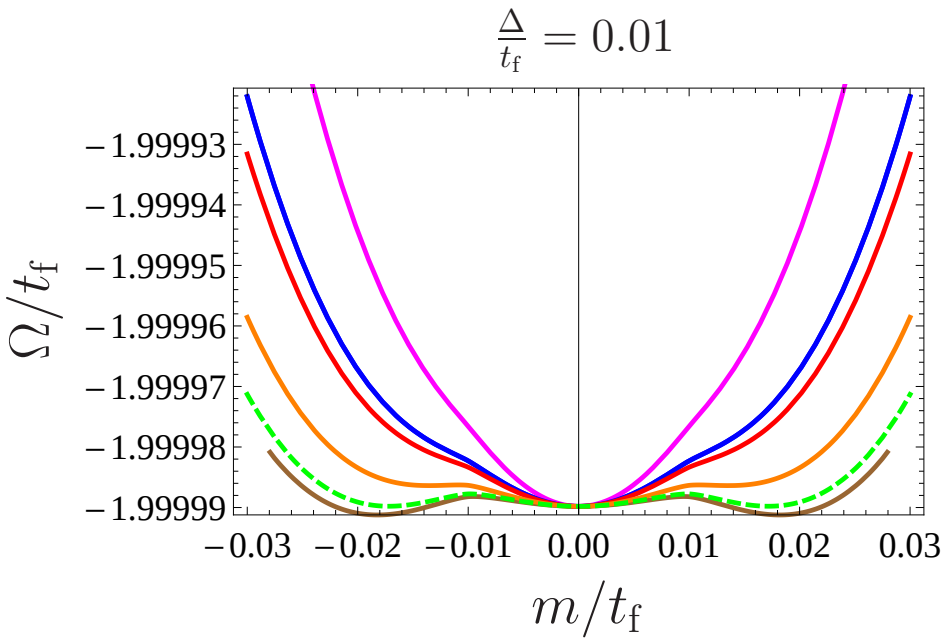
$$\Omega_{\text{MF}}^{T\to 0}=-\frac{1}{16t}\sum_{a=\pm}\left[\left(\frac{mU}{4}+a\Delta\right)^2 I^{(a)}+4tJ^{(a)}\right]+\frac{\Delta^2}{U_{\text{eff}}}+\frac{3m^2U}{2}$$

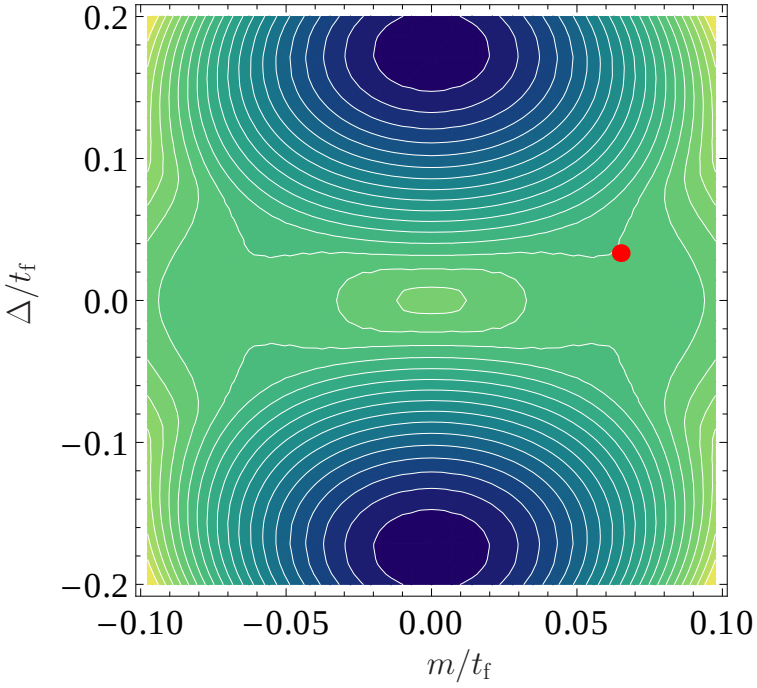
where

$$J^{(\pm)}=\sqrt{\left(\frac{mU}{4}\pm\Delta\right)^2+(4t)^2}$$

# Half filling case -2 – $d$

- Results

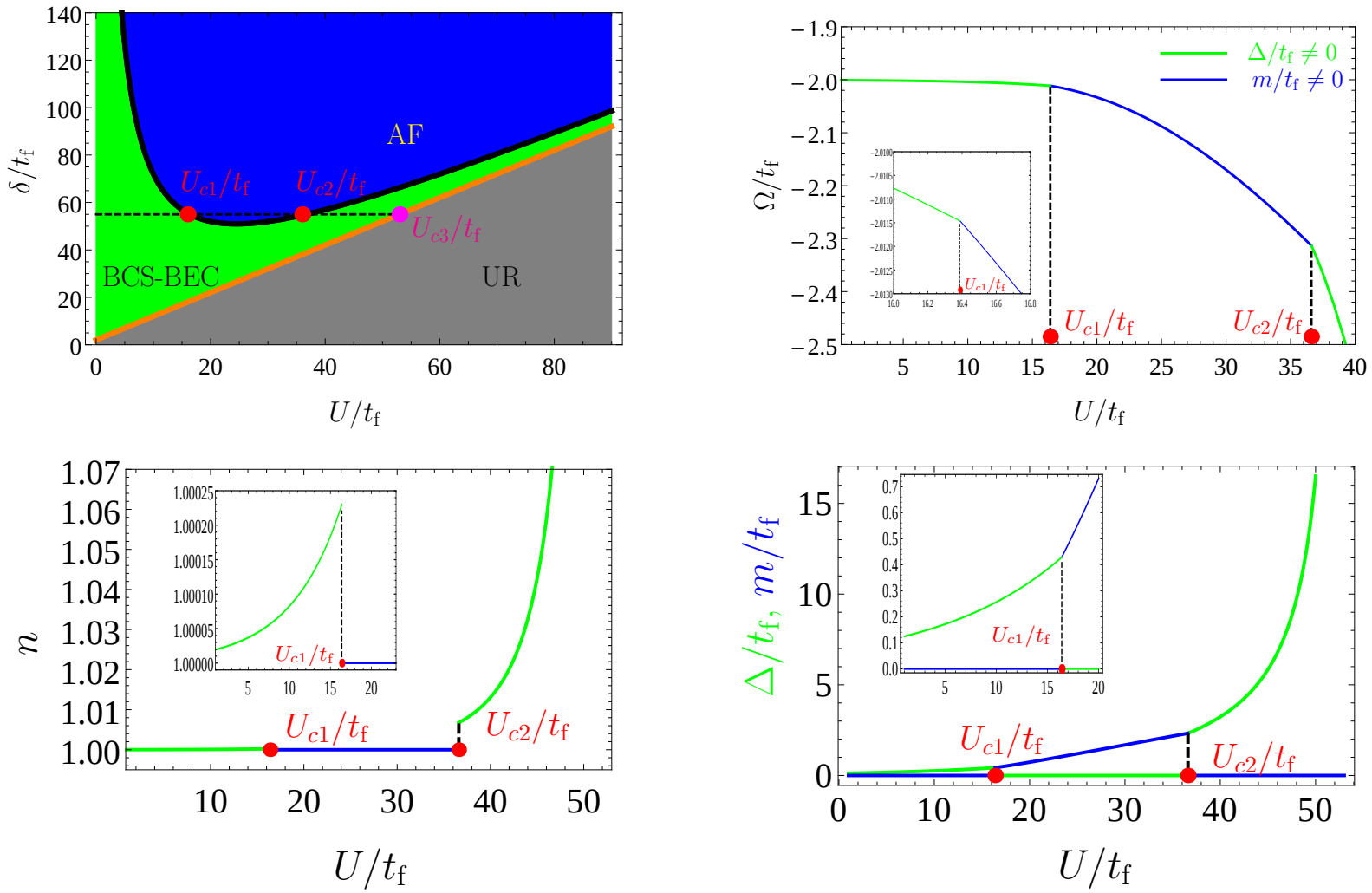



- Results for $U/t_f = 7$, $g/t_f = 1$ und $\delta/t_f = 7$
- Half-filling case $\equiv \mu = U/2$
- Conclusion: no coexistence of AF and BCS-BEC order

# Half filling case -2 — $d$

- Phase diagrams ($g/t_f = 10$):



# Summary

- Non-coexisting AF and BCS phase at half-filling
- Interplay between AF phase and BCS-BEC
- First order phase transition between AF and superfluid phase

Outlook

- ⋆ Extend calculations for general filling case
- ⋆ Ginzburg-Landau-Wilson theory
- ⋆ Include quantum fluctuations to GLW-theory

# Thank you for the attention!