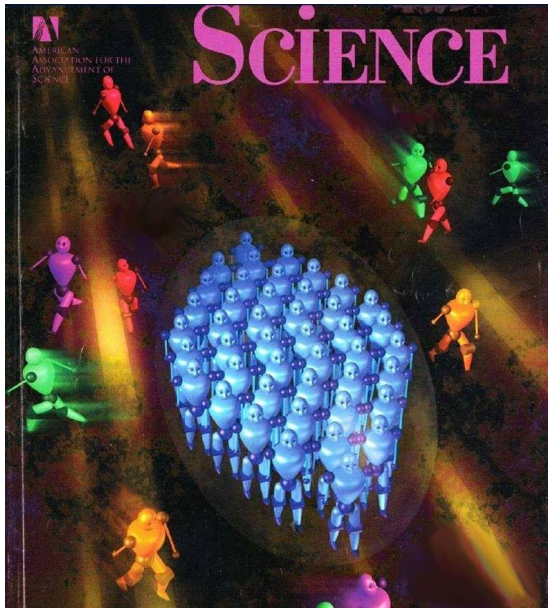
Bose-Einstein Condensation in Microgravity – Challenges and Perspectives

Axel Pelster

1. Ultracold Quantum Gases

2. BEC in Bubble Traps

3. Open Problems



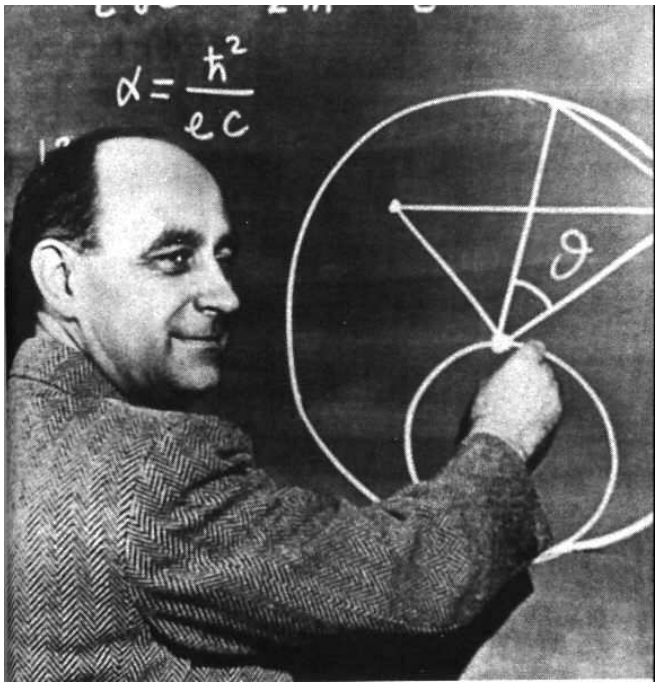
1.1 Identical Quantum Particles

Bosons:

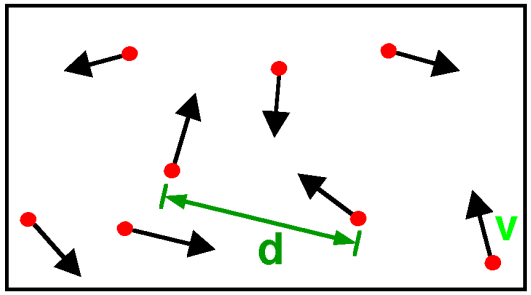
- Integer spin
- Symmetric wave function

Fermions:

- Half-integer spin
- Anti-symmetric wave function



1.2 What is Bose-Einstein Condensation?

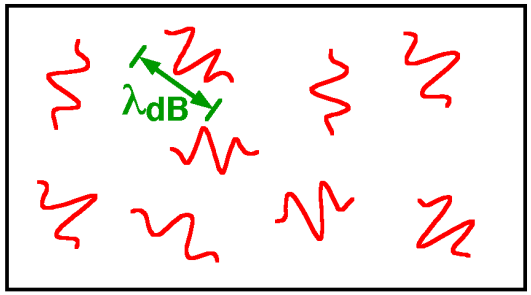


High Temperature T:

thermal velocity v

density d^{-3}

"Billiard balls"

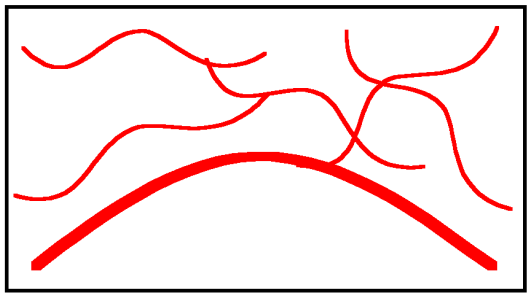


Low Temperature T:

De Broglie wavelength

$\lambda_{dB}=h/mv \propto T^{-1/2}$

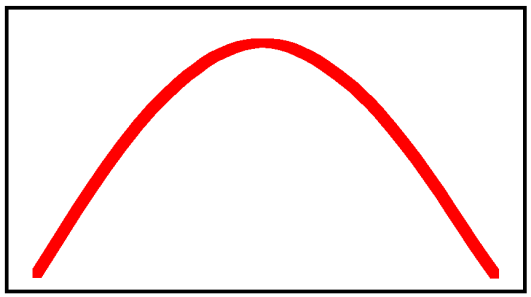
"Wave packets"



$T=T_{crit}$: Bose-Einstein Condensation

$\lambda_{dB} \approx d$

"Matter wave overlap"



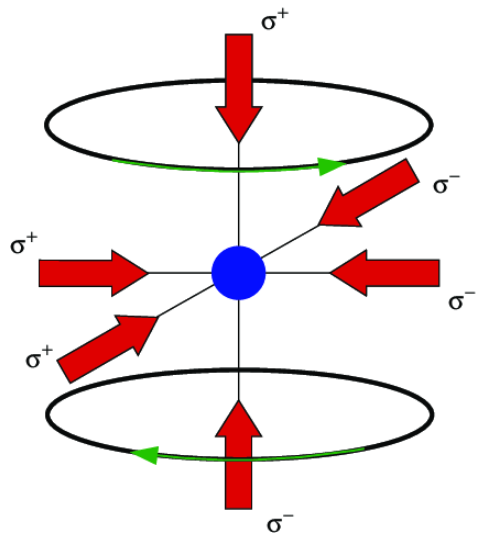
T=0: Pure Bose condensate

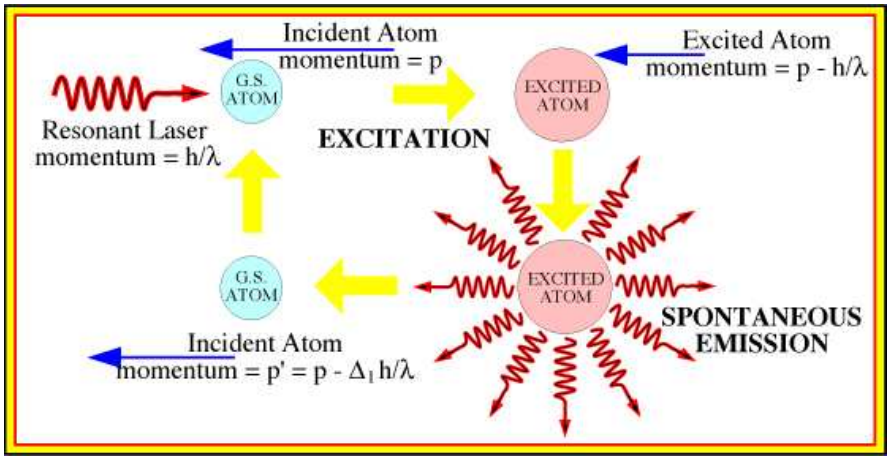
"Giant matter wave"

- $\lambda_{\mathrm{dB}} = \dfrac{\hbar}{\sqrt{2Mk_BT}}$
- $n = \dfrac{1}{d^3}$
- $\dfrac{\lambda_{\mathrm{dB}}}{d} \approx 1$
- $T_c \approx \dfrac{\hbar^2 n^{3/2}}{2Mk_B}$

1.3 Cooling Techniques

Magneto-optical trap

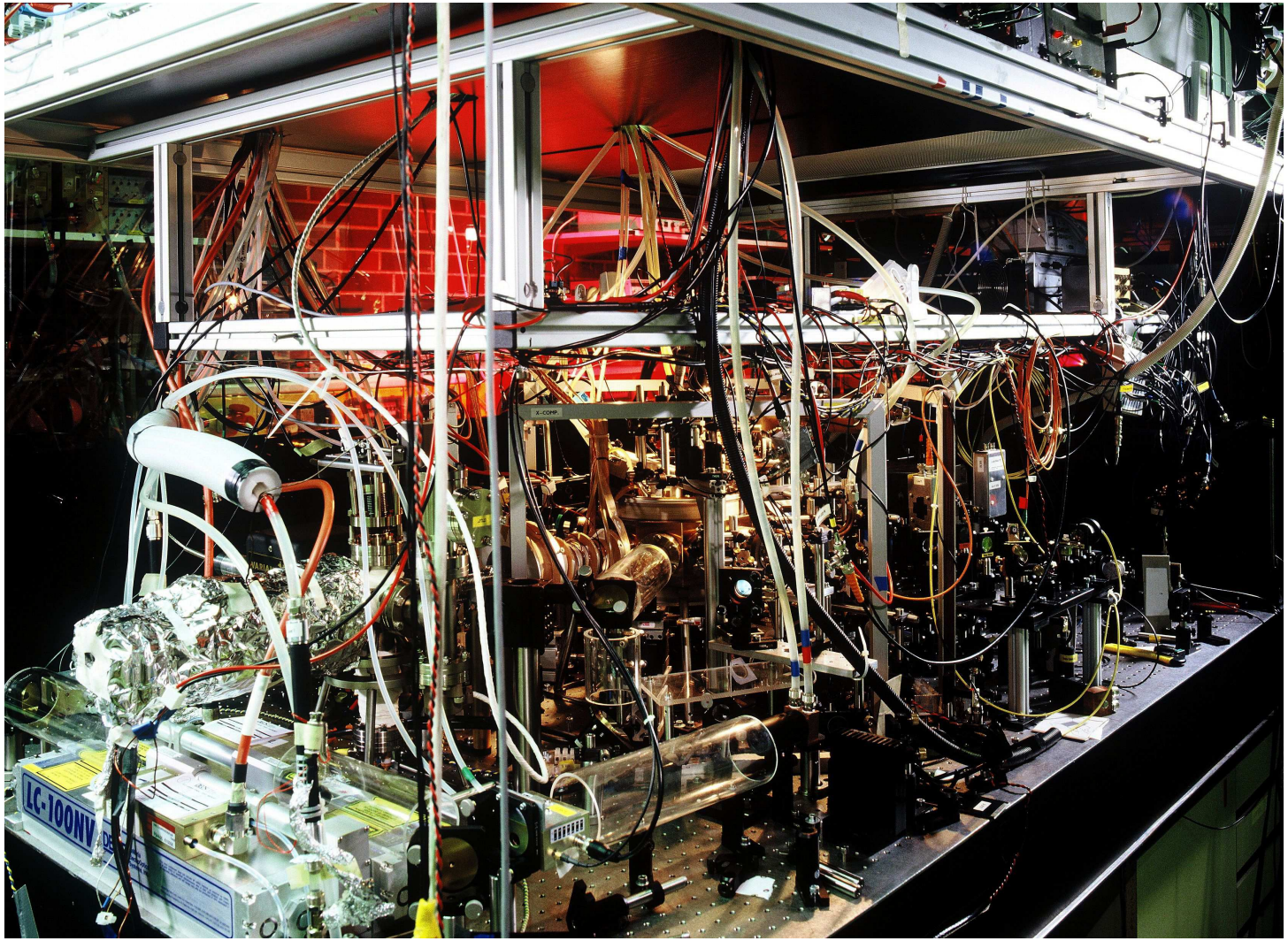




Laser cooling

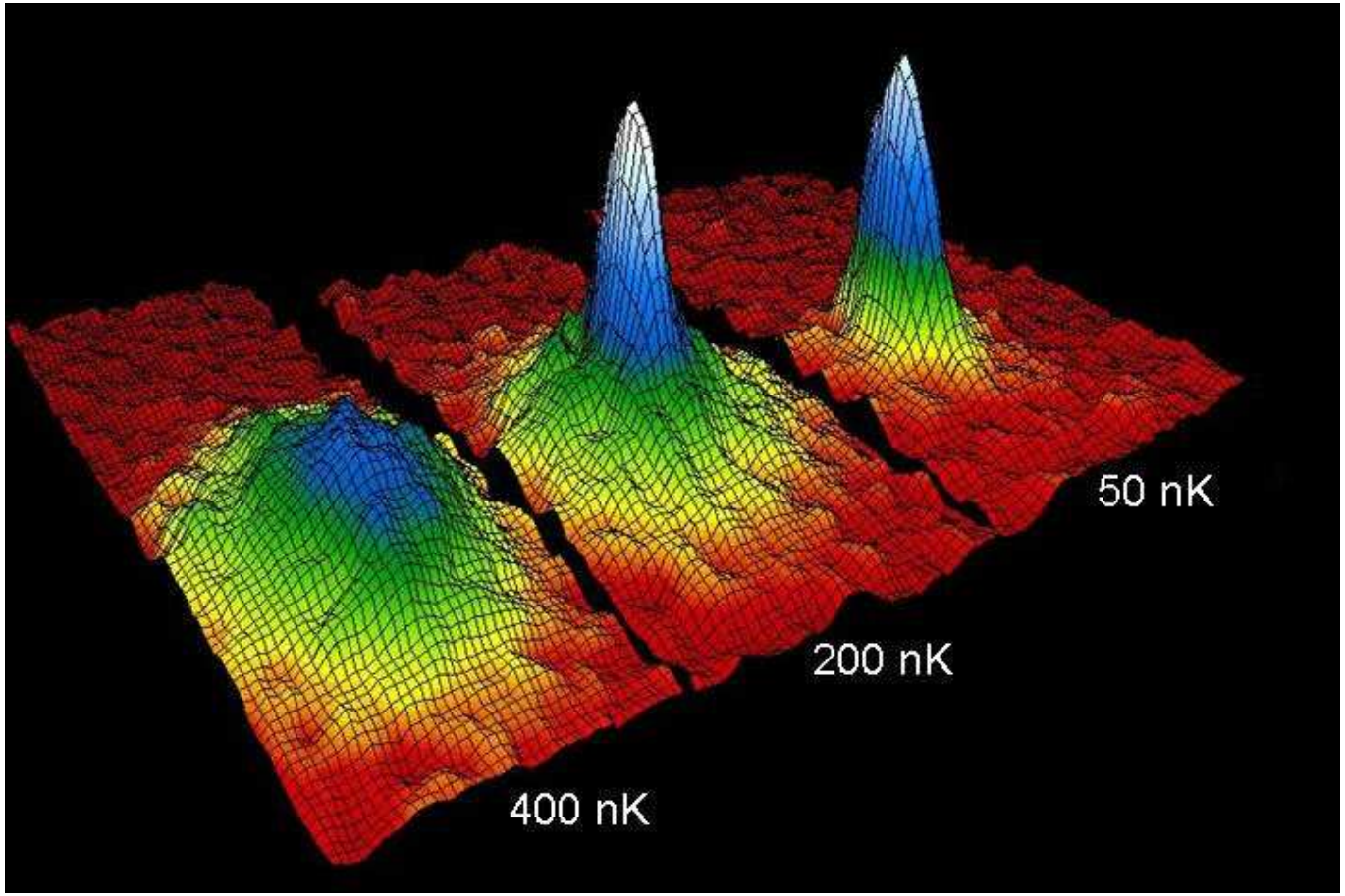
Evaporative cooling

1.4 Experimental Apparatus



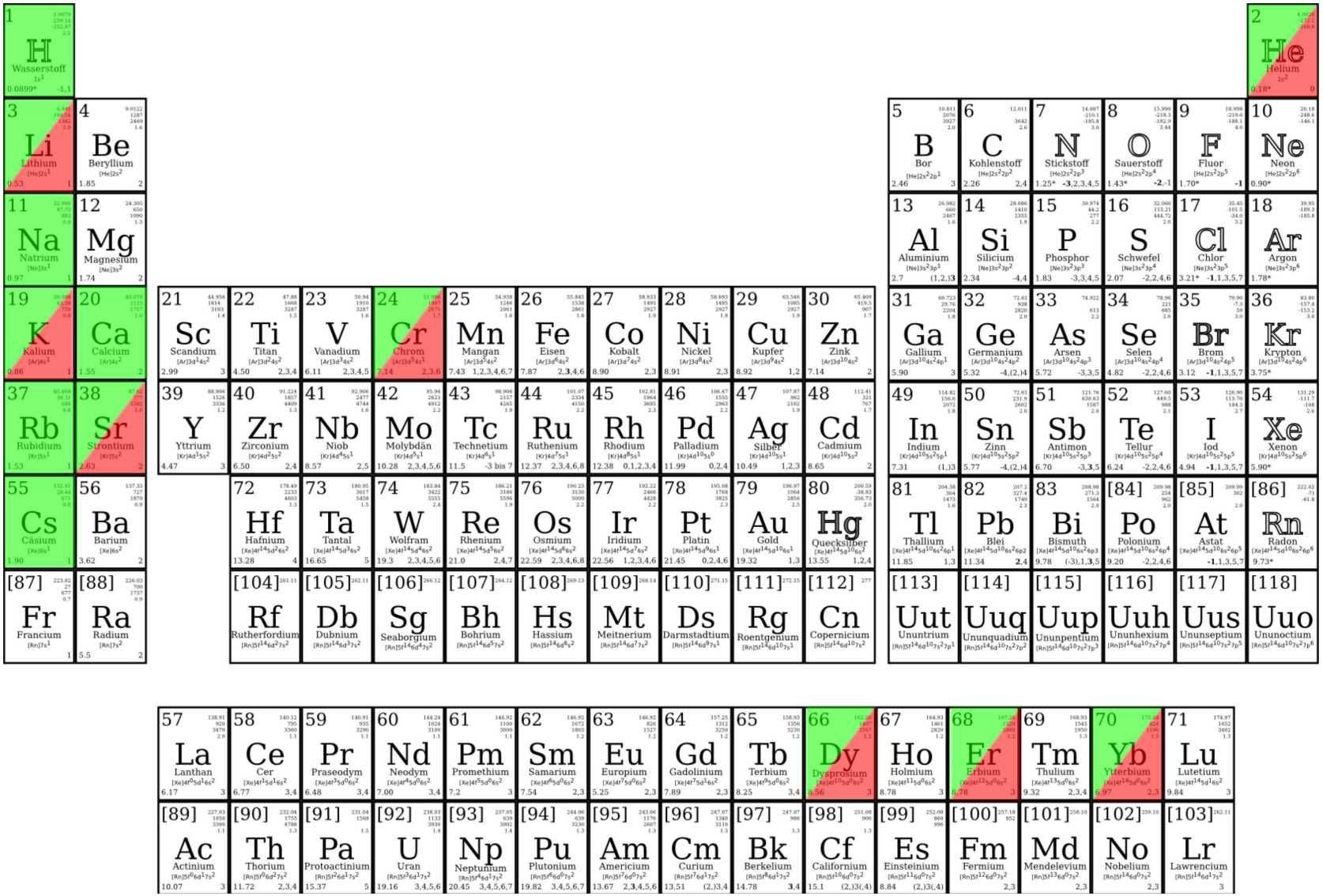
Costs about 1.000.000 EUR

1.5 Time-of-Flight Absorption Pictures



JILA (1995): $^{87}_{37}$Rb , N=20 000 , $\omega_1 = \omega_2 = \omega_3/\sqrt{8} = 2\pi\times$ 120 Hz

1.6 Periodic Table of Chemical Elements



Quantum degenerate bosons and fermions

1.7 Research Unit 2247 (2019-2022)

FOR 2247 From few to many-body physics with dipolar quantum gases

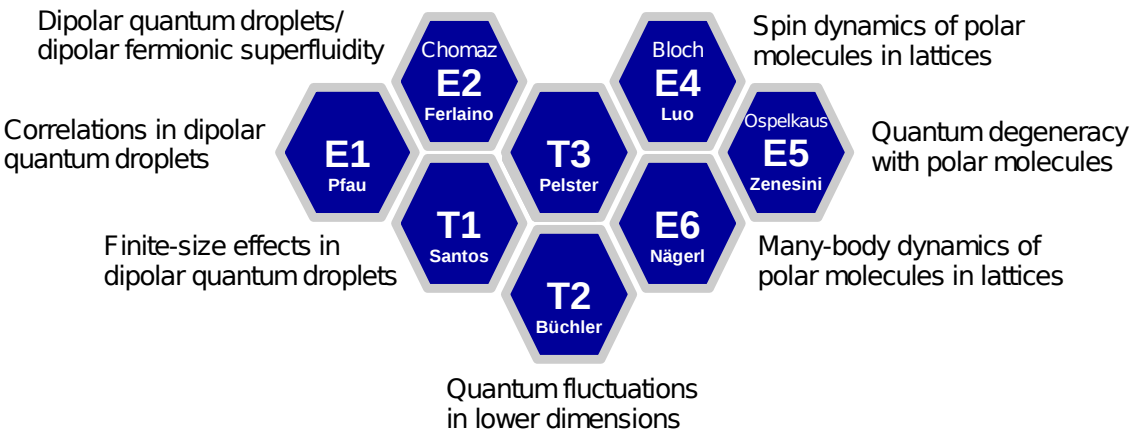
DFG FWF

Hannover, Innsbruck, Kaiserslautern, Munich, Stuttgart

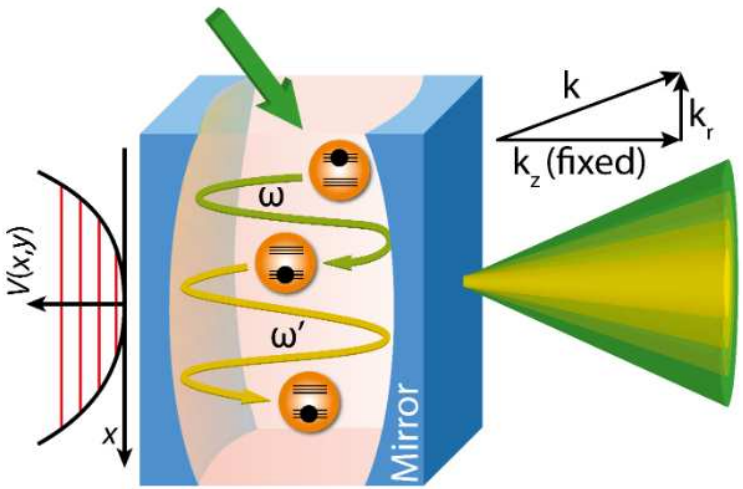
- **Project T3: Superfluidity in strong dipolar quantum gases**
  - **Dipolar Fermi gases: deformation of Fermi sphere**

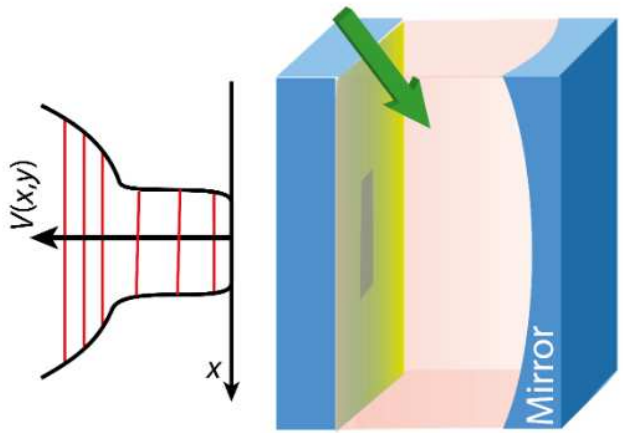
    Veljić, ... , Ferlaino, ..., Pelster, Balaž, NJP **20**, 093016 (2018)
  - **Dipolar Bose gases: quantum droplets**

    Pelster, Physik-Journal **18**, Nr. 6, 20 (2019)
- **Collaborations**



1.8 CRC/TR185 (2020-2024, decided end of May 2020)





Project B1
Schmitt, Kroha, Weitz
Dynamics in 2D

Project B6N
Pelster, Vewinger, von Freymann
Statics in 1D

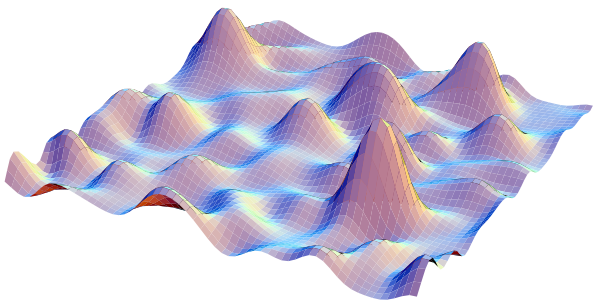
$\Longrightarrow$ **New center for photon BEC research in Kaiserslautern**

Klärs, Schmitt, Vewinger, and Weitz, Nature **468**, 545 (2010)

Overview: Pelster, Physik-Journal **10**, Nr. 1, 20 (2011)

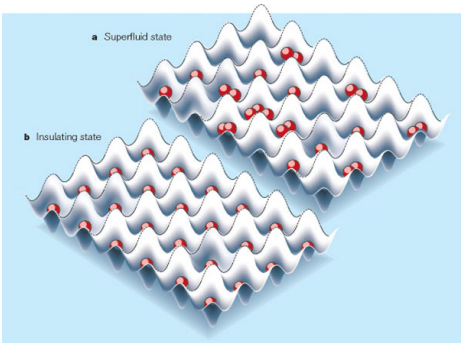
Pelster, Physik-Journal **13**, Nr. 3, 20 (2014)

1.9 Other Research Areas (Selection, Since 2015)



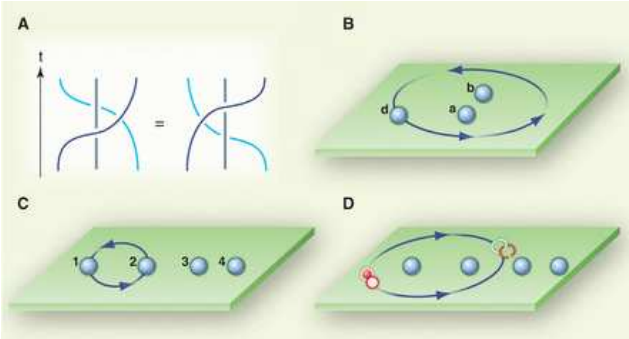
Dirty boson problem

NJP (2016), arXiv (2019)



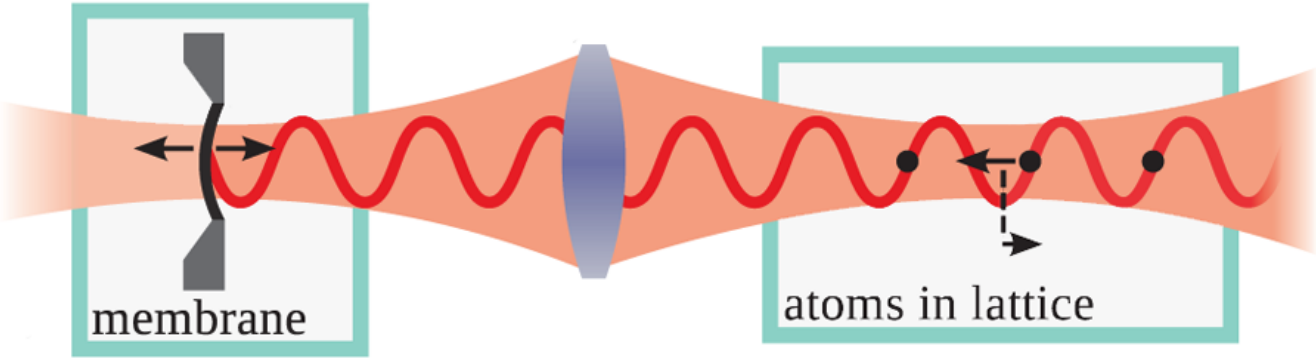
Optical lattices

PRB (2015), PRL (2016)



Anyonic statistics

NJP (2015)



Hybrid atom-optomechanical systems

PRL (2018), NJP (2019)

BECCAL

arXiv (2020)

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TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

OPTIMAS TU Kaiserslautern

SFB/TR 185

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DFG FWF

1. Ultracold Quantum Gases

2. BEC in Bubble Traps

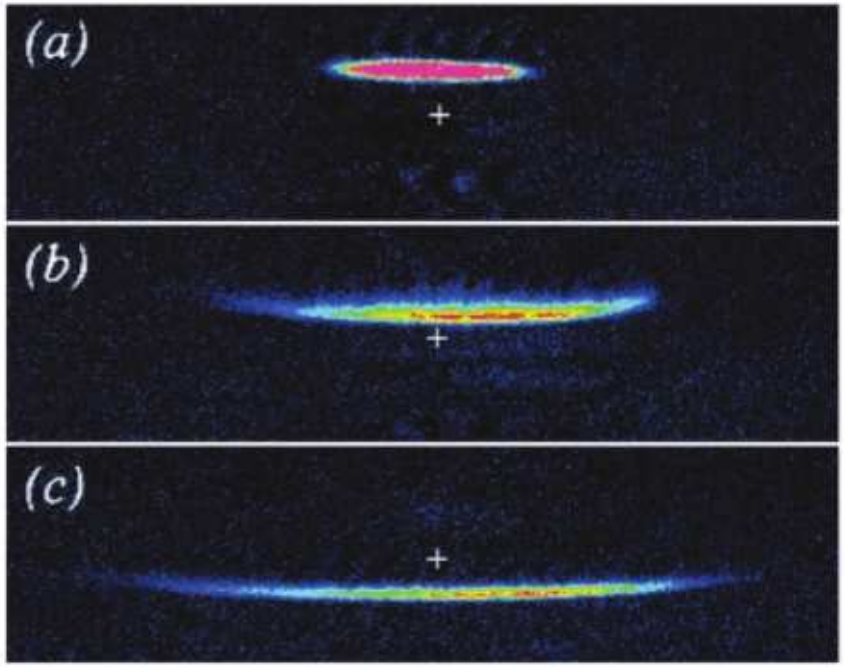
Móller, Santos, Bagnato, and Pelster, arXiv:2001.07443

3. Open Problems

BECCAL BOSE-EINSTEIN CONDENSATE AND COLD ATOM LABORATORY NASA DLR

2.1 Bubble Traps

On earth



Colombe et al., EPL **67**, 593 (2004)

Guo et al., PRL **124**, 025301 (2020)

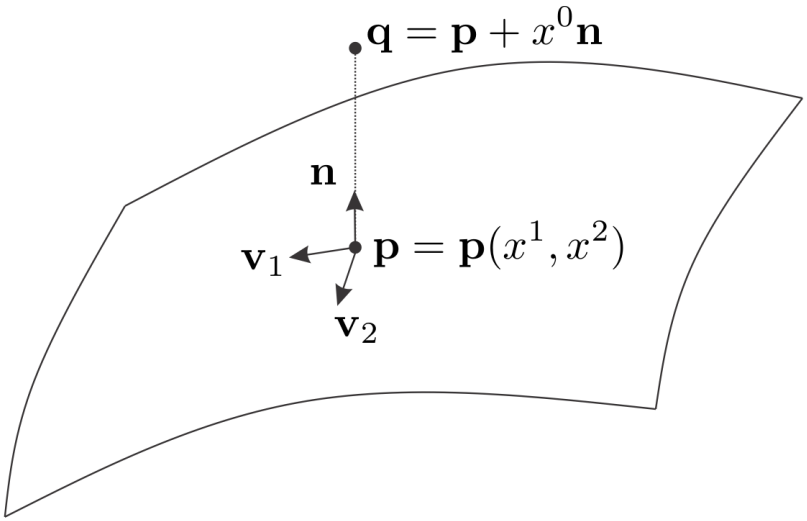
In microgravity

Gibney, Nature **557**, 151 (2018)

Freye et al., arXiv:1912.04849

2.2 Manifold

- **Gaussian normal coordinate system:**



- **Metric:**

$$G = \begin{pmatrix} 1 & 0 & 0 \\ 0 & & \\ 0 & & g \end{pmatrix}$$

- **Potential confinement:** $$V = \frac{M}{2}\,\omega^2\,(x^0)^2$$

2.3 Reducing Dimensionality

- **Particle number:** $N = \int dV \, |\Psi(x^0, x^1, x^2)|^2$

- **Energy:**

$$E = \int dV \, \Psi^* \left[-\frac{\hbar^2}{2M} \Delta + \frac{M}{2} \omega^2 (x^0)^2 + \frac{1}{2} g |\Psi|^2 - \mu \right] \Psi$$

- **Ansatz:**

$$\Psi(x^0, x^1, x^2) = \frac{e^{-(x^0)^2/2\sigma^2(x^1,x^2)}}{\sqrt[4]{\pi}\sqrt{\sigma(x^1, x^2)}} \psi(x^1, x^2)$$

- **Integrate variable x^0 perpendicular to manifold**

- **Extremize energy with respect to $\psi^*(x^1, x^2)$ and $\sigma(x^1, x^2)$**

Inspired by L. Salasnich et al., PRA **65**, 043614 (2002)

2.4 Quasi Two Dimensions

- **Particle number:** $$N = \int d^2x \, \sqrt{\det g} \, |\psi(x^1, x^2)|^2$$

- **Two-dimensional Gross-Pitaevskii equation:**

$$\left[-\frac{\hbar^2}{2M} \Delta_{\mathrm{LB}} + V_{\mathrm{eff}}(x^1, x^2) + \frac{\hbar^2}{4M\sigma^2} + \frac{M\omega^2\sigma^2}{4} + \frac{g|\psi|^2}{\sqrt{2\pi}\sigma} \right] \psi = \mu \, \psi$$

- **Laplace-Beltrami operator:**

$$\Delta_{\mathrm{LB}} = \frac{1}{\sqrt{\det g}} \frac{\partial}{\partial x^i} \left(\sqrt{\det g} \, g^{ij} \frac{\partial}{\partial x^j} \right)$$

- **Width:**

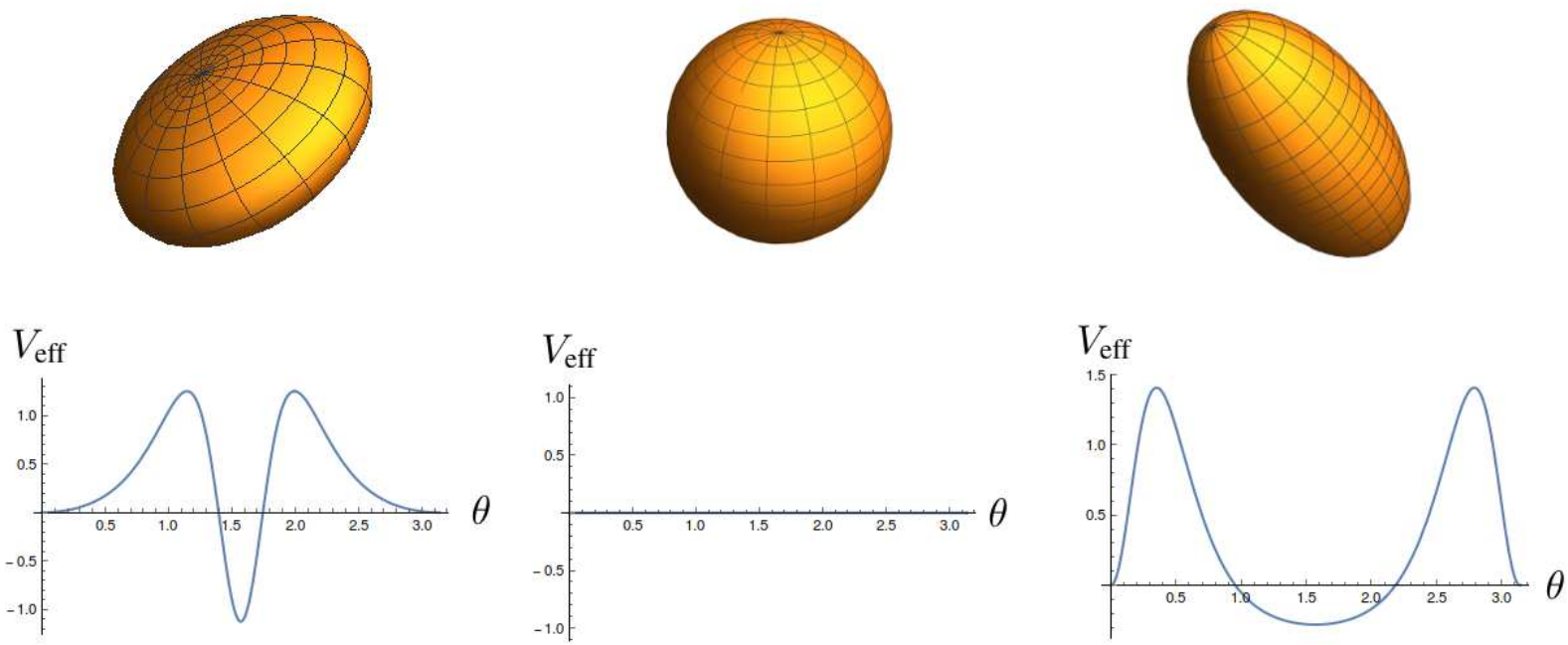
$$\frac{\sigma^4}{\sigma_{\mathrm{osc}}^4} = 1 + \frac{gM}{\sqrt{2\pi}\hbar^2} \sigma |\psi|^2 \,, \qquad \sigma_{\mathrm{osc}} = \sqrt{\frac{\hbar}{M\omega}}$$

2.5 Effective Potential

- **General expression:**

$$V_{\text{eff}}(x^1, x^2) = \frac{\hbar^2}{4M}\left[\frac{1}{2}\left(\frac{\partial \ln\sqrt{\det g}}{\partial x^0}\right)^2 + \frac{\partial^2 \ln\sqrt{\det g}}{\partial {x^0}^2}\right]\Bigg|_{x^0=0}$$

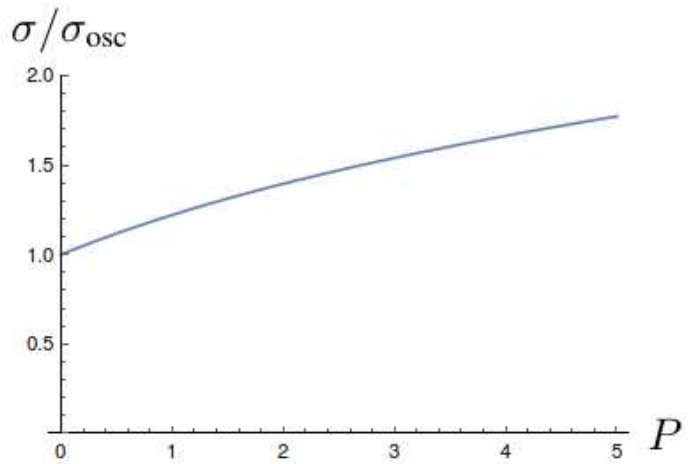
- **Examples:**

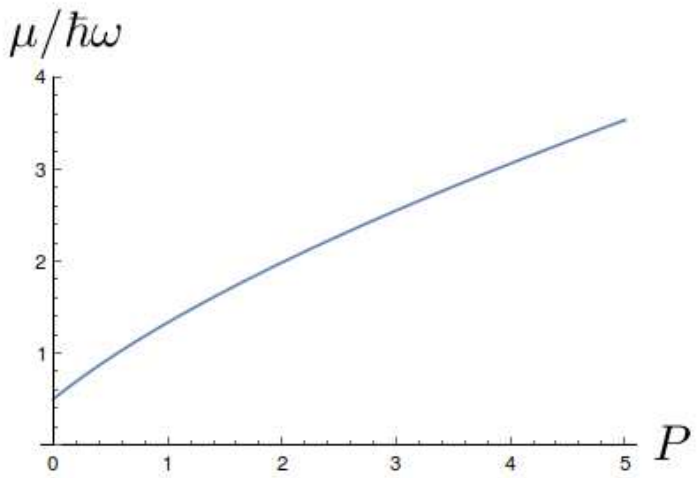


2.6 Equilibrium on Quasi Sphere

- **Wave function from normalization:** $\psi = \sqrt{\dfrac{N}{4\pi R^2}}$

- **Width:** **Equation of state:**





$$\left(\frac{\sigma}{\sigma_{\rm osc}}\right)^4 = 1 + P\,\frac{\sigma}{\sigma_{\rm osc}}$$

$$\frac{\mu}{\hbar\omega} = \frac{1}{4}\left(\frac{1}{\sigma^2/\sigma_{\rm osc}^2} + \frac{\sigma^2}{\sigma_{\rm osc}^2}\right) + \frac{P}{\sigma/\sigma_{\rm osc}}$$

- **Dimensionless interaction strength:** $P = \dfrac{a_{\rm s}\sigma_{\rm osc}N}{\sqrt{2\pi}R^2}$

$$a_{\rm s} = 100\ a_{\rm B}, \sigma_{\rm osc} = 1\ \mu{\rm m}, R = 10\ \mu{\rm m}, N = 10^5 \quad \Longrightarrow \quad P = 2.1$$

2.7 Low-Lying Excitations on Quasi Sphere

- **Action:**

$$S = \int dt \int dV\, \Psi^* \left[i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2M} \Delta - \frac{M}{2} \omega^2 (x^0)^2 - \frac{1}{2} g |\Psi|^2 \right] \Psi$$

- **Ansatz:**

$$\Psi(x^0, x^1, x^2, t) = \frac{\exp\left\{ -\frac{1}{2} \left[\frac{1}{\sigma^2(x^1, x^2, t)} + iB(x^1, x^2, t) \right] (x^0)^2 \right\}}{\sqrt[4]{\pi} \sqrt{\sigma(x^1, x^2, t)}} \psi(x^1, x^2, t)$$

- **Linear stability analysis**
- **Lower frequencies:**

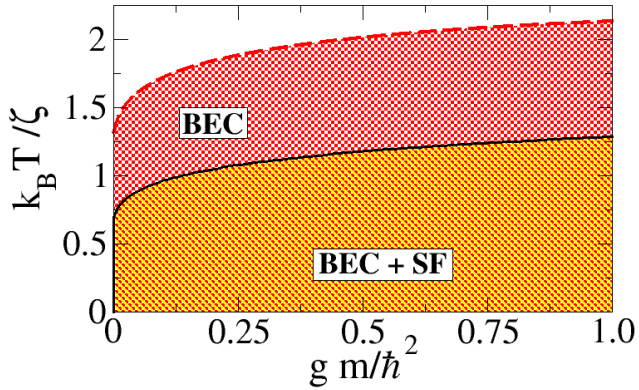
$$\Omega_l' = \frac{\hbar}{2MR^2} l(l+1) + \mathcal{O}(P)$$

Higher frequencies:

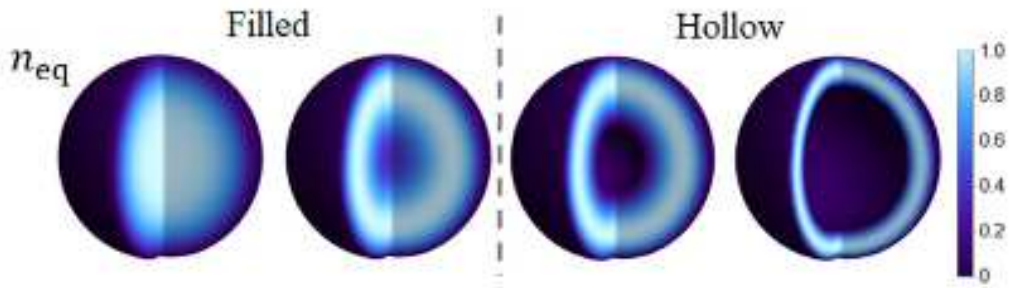
$$\Omega_l'' = 2\omega - \frac{\hbar}{2MR^2} l(l+1) + \mathcal{O}(P)$$

2.8 Outlook for Bubble Traps

- **Taking levitation into account**
  Breathing and quadrupole mode: Diniz et al., arXiv:1911.03513
- **Phase diagram for sphere:** Tononi and Salasnich, PRL **123**, 160403 (2019)
  - **Mermin-Wagner-Hohenberg theorem:** $\lim_{R\to\infty} T_c(R) = 0$
  - **Berezinski-Kosterlitz-Thouless physics:**



- **Dimensional crossover:** K. Sun et al., PRA **98**, 013609 (2018)



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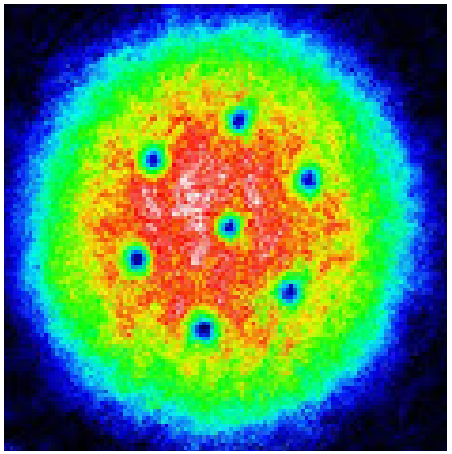
3. Open Problems

3.1 Vortex Dynamics in Bubble Traps

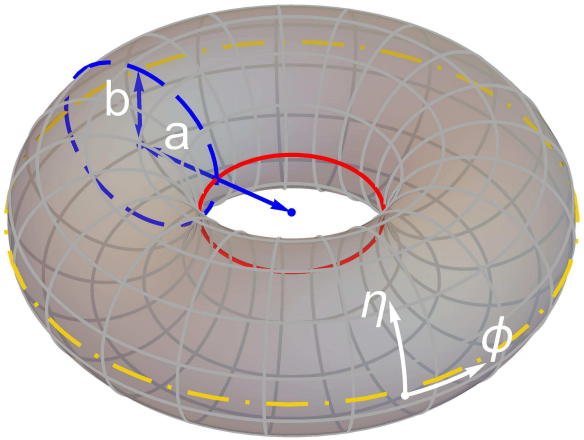
- **Far-field vortex dynamics in plane:**
  - **Hamiltonian equations**
  - **Vortex precession about trap center**
  - **Vortex-vortex interaction**

  Navarro et al., PRL **110**, 225301 (2013)



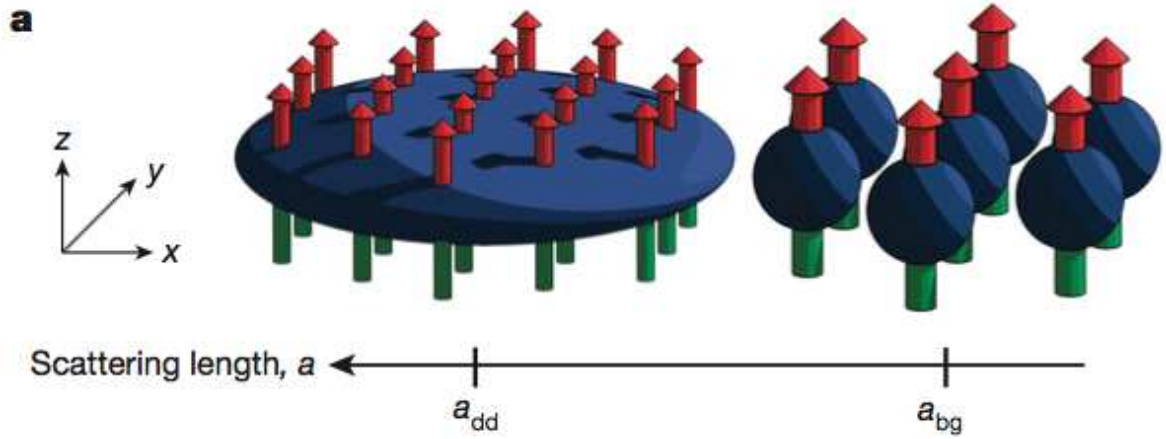
- **Vision for CAL at ISS:**
  Vortex precession in bubble trap as gyroscope

- **Preliminary studies**
  - **Vortex on cylinder:**
    Günther et al., PRA **96**, 063608 (2017)
  - **Dynamics of two vortices on cone:**
    Massignan and Fetter, PRA **99**, 063602 (2019)
  - **Vortex dynamics on torus:**
    Günther et al., arXiv:1911.11794



3.2 Quantum Droplets in Bubble Traps

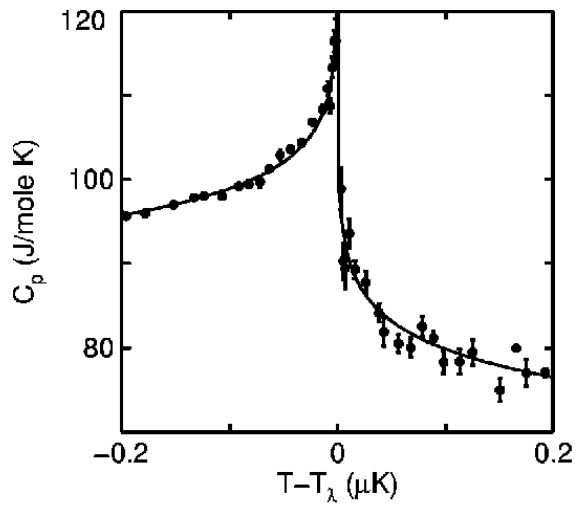
- **Physical notion:**
  - **Mean-field instability**
  - **Stabilization due to quantum fluctuations**
  - **Extended Gross-Pitaevskii equation**
- **Experiments:**
  - **Dipolar Bose gas (Dy, Er: Stuttgart, Innsbruck, Pisa)**



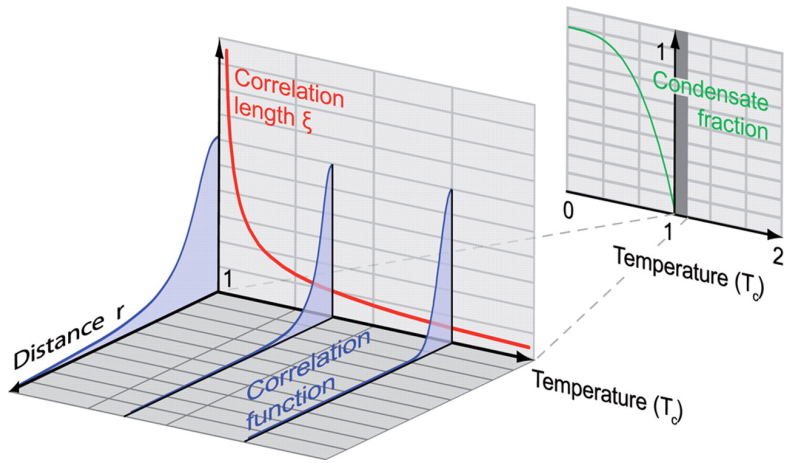
  - **Bose-Bose mixture (K: Barcelona, K-Rb: Florence)**
- **Vision for BECCAL:**
  Quantum droplets for K-Rb mixture

3.3 Critical Exponents

- **Superfluid helium (homogeneous case)**
  - **On earth:**
    $\alpha = -0.026 \pm 0.004$
    Ahlers, PRA **3**, 696 (1971)
  - **In microgravity:**
    $\alpha = -0.0127 \pm 0.0003$
    Lipa et al., PRB **68**, 174518 (2003)



- **Bose-Einstein condensate (harmonically trapped case)**
  - **On earth:**
    $\xi \sim |(T - T_c)/T_c|^{-\nu}$
    $\nu = 0.67 \pm 0.13$
    Donner et al., Science **315**, 1556 (2007)
  - **In microgravity:**
    BECCAL?



3.4 Optical Lattices

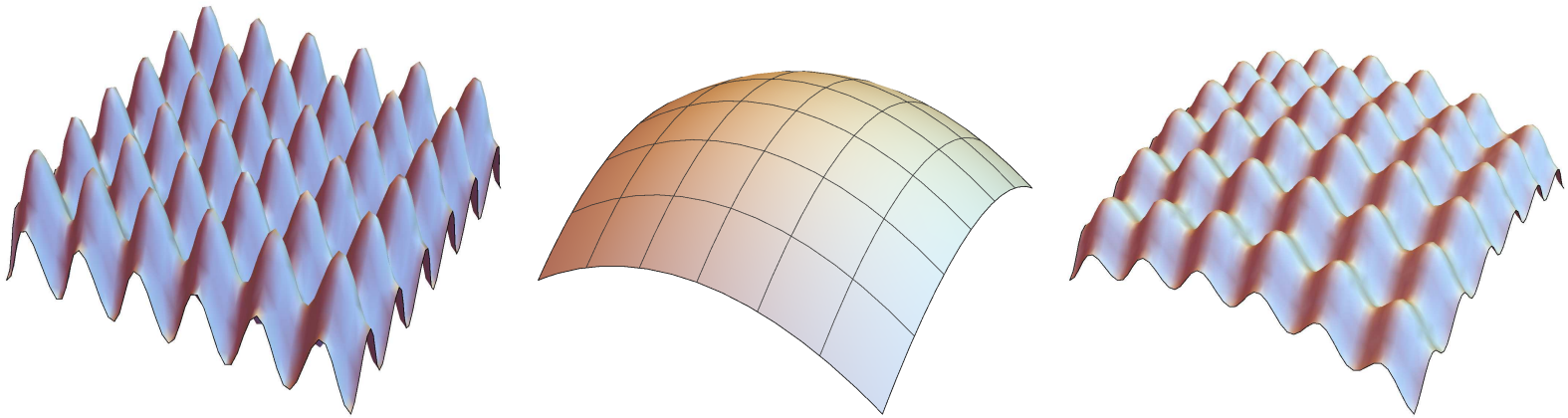
- **Flat optical lattice:**
  - **Continuum many-body Hamiltonian:**

$$\hat{H} = \int d^3x \left\{ \hat{\psi}^\dagger \left[-\frac{\hbar^2}{2M}\Delta + V_0 \sum_{k=1}^{3} \sin^2\left(\frac{\pi}{a}x_k\right) \right] \hat{\psi} + \frac{g}{2}\hat{\psi}^{\dagger 2}\hat{\psi}^2 \right\}$$

  - **Bose-Hubbard model:**

$$\hat{H} = -J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i \left(\hat{n}_i - 1\right) \; ; \qquad \hat{n}_i = \hat{b}_i^\dagger \hat{b}_i$$

- **Curved optical lattice: Inhomogeneous hopping**



Gödtel, diploma thesis, TU Kaiserslautern, 2017

3.5 Wilhelm and Else Heraeus Seminar

Exploring Quantum ManyBody Physics with Ultracold Atoms and Molecules

organized by Carlos Sá de Melo and Axel Pelster

Bad Honnef (Germany); December 14 – 18, 2020

Invited Speakers: Monika Aidelsburger (Germany), Alain Aspect (France), Waseem Bakr (USA), Nicholas Bigelow (USA), Doerte Blume (USA), Georg Bruun (Denmark), Andreas Buchleitner (Germany), Jean Dalibard (France), Francesca Ferlaino (Austria), Thomas Gasenzer (Germany), Tin-Lun Ho (USA), Michael Köhl (Germany), Giovanni Modugno (Italy), Cristiane Morais Smith (Netherlands), Jian-Wei Pan (China), Dmitry Petrov (France), Ernst Rasel (Germany), Monika Schleier-Smith (USA), Peter Schmelcher (Germany), Sandro Stringari (Italy), Päivi Törmä (Finland), Christof Weitenberg (Germany), Artur Widera (Germany), Jun Ye (USA), Wei Zhang (China)

WILHELM UND ELSE HERAEUS-STIFTUNG