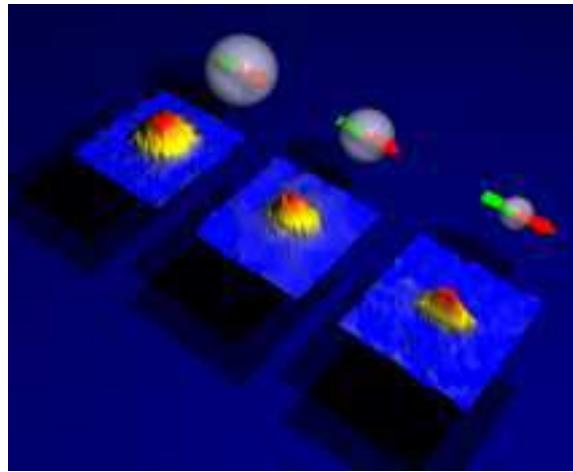


# Bogoliubov Theory of Dipolar Bose Gases at Finite Temperature



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# Overview

1. Bogoliubov theory
2. Landau theory
3. Contact and dipole-dipole interaction
4. Low-temperature series of state variables
5. Sound velocities

# Bosons in Fock space

- Annihilator  $\hat{\psi}(\vec{x})$
- Creator  $\hat{\psi}^\dagger(\vec{x})$
- Canonical commutator relation  $[\hat{\psi}(\vec{x}), \hat{\psi}(\vec{x}')] = [\hat{\psi}^\dagger(\vec{x}), \hat{\psi}^\dagger(\vec{x}')] = 0$   
 $[\hat{\psi}(\vec{x}), \hat{\psi}^\dagger(\vec{x}')] = \delta(\vec{x} - \vec{x}')$
- Particle-density operator  $\hat{n}(\vec{x}) = \hat{\psi}^\dagger(\vec{x})\hat{\psi}(\vec{x})$

# Choice of Hamiltonian

- Kinetic energy

$$\hat{H}_{\text{kin}} = - \int d^3x \hat{\psi}^\dagger(\vec{x}) \frac{\hbar^2}{2m} \Delta \hat{\psi}(\vec{x})$$

- Chemical potential

$$\hat{H}_\mu = - \int d^3x \hat{\psi}^\dagger(\vec{x}) \mu \hat{\psi}(\vec{x})$$

- Boost

$$\hat{H}_{\text{boost}} = \int d^3x \hat{\psi}^\dagger(\vec{x}) \frac{\hbar}{i} \vec{w} \vec{\nabla} \hat{\psi}(\vec{x})$$

- Interaction energy

$$\hat{H}_{\text{int}} = \frac{1}{2} \int d^3x \int d^3x' \hat{\psi}^\dagger(\vec{x}) \hat{\psi}^\dagger(\vec{x}') V(\vec{x} - \vec{x}') \hat{\psi}(\vec{x}') \hat{\psi}(\vec{x})$$

→ Homogeneous system with general two-particle interaction

# Fock space in Fourier representation

- New annihilators
- New creators

$$\hat{\psi}(\vec{x}) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} e^{i\vec{k}\vec{x}} \hat{b}_{\vec{k}}$$

$$\hat{\psi}^\dagger(\vec{x}) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} e^{-i\vec{k}\vec{x}} \hat{b}_{\vec{k}}^\dagger$$

- Canonical commutator relation

$$[\hat{b}_{\vec{k}}, \hat{b}_{\vec{k}'}] = [\hat{b}_{\vec{k}}^\dagger, \hat{b}_{\vec{k}'}^\dagger] = 0$$

$$[\hat{b}_{\vec{k}}, \hat{b}_{\vec{k}'}^\dagger] = \delta_{\vec{k}, \vec{k}'}$$

- Particle-number operator

$$\hat{N}_{\vec{k}} = \hat{b}_{\vec{k}}^\dagger \hat{b}_{\vec{k}}$$

# Hamiltonian in Fourier representation

$$\hat{H} = \sum_{\vec{k}} \varepsilon_{\vec{k}} \hat{b}_{\vec{k}}^\dagger \hat{b}_{\vec{k}} + \frac{1}{2V} \sum_{\vec{k}, \vec{k}', \vec{q}} V_{\vec{q}} \hat{b}_{\vec{k}-\vec{q}}^\dagger \hat{b}_{\vec{k}'+\vec{q}}^\dagger \hat{b}_{\vec{k}'} \hat{b}_{\vec{k}}$$

→ Free dispersion

$$\varepsilon_{\vec{k}} = \frac{\hbar^2 \vec{k}^2}{2m} + \hbar \vec{w} \vec{k} - \mu$$

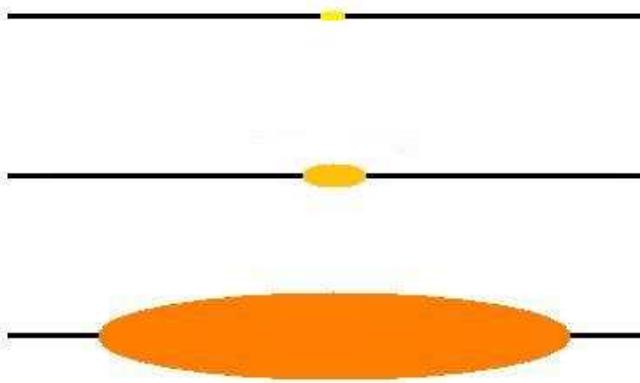
→ Interaction replaced by Fourier transform

$$V_{\vec{q}} = \frac{1}{V} \int d^3x e^{i\vec{q}\vec{x}} V(\vec{x})$$

→ Momentum automatically conserved

# Approximation of interaction terms

- Interaction of particles in ground state most probable



- Interaction term becomes:

$$\begin{aligned} \frac{1}{2V} \sum_{\vec{k}, \vec{k}', \vec{q}} V_{\vec{q}} \hat{b}_{\vec{k}-\vec{q}}^\dagger \hat{b}_{\vec{k}'+\vec{q}}^\dagger \hat{b}_{\vec{k}'} \hat{b}_{\vec{k}} &= \frac{1}{2V} V_{\vec{0}} \hat{b}_{\vec{0}}^\dagger \hat{b}_{\vec{0}}^\dagger \hat{b}_{\vec{0}} \hat{b}_{\vec{0}} + \frac{1}{V} \sum_{\vec{k} \neq \vec{0}} (V_{\vec{0}} + V_{\vec{k}}) \hat{b}_{\vec{0}}^\dagger \hat{b}_{\vec{0}} \hat{b}_{\vec{k}}^\dagger \hat{b}_{\vec{k}} \\ &\quad + \frac{1}{2V} \sum_{\vec{k} \neq \vec{0}} V_{\vec{k}} (\hat{b}_{\vec{0}}^\dagger \hat{b}_{\vec{0}}^\dagger \hat{b}_{\vec{k}} \hat{b}_{-\vec{k}} + \hat{b}_{\vec{0}} \hat{b}_{\vec{0}} \hat{b}_{\vec{k}}^\dagger \hat{b}_{-\vec{k}}^\dagger) + \dots \end{aligned}$$

# C-number approximation

- Macroscopic number of particles in ground state:  
 $N_{\vec{0}} \gg 1$   
 $N_{\vec{0}} \approx N_{\vec{0}} \pm 1$
- Mean-field ansatz:  
 $\hat{b}_{\vec{0}} = \hat{b}_{\vec{0}}^\dagger = \sqrt{N_{\vec{0}}}$

→ Annihilators and creators only in second order

$$\begin{aligned}\hat{H} = & V n_{\vec{0}} \left( \frac{n_{\vec{0}}}{2} V_{\vec{0}} - \mu \right) + \sum_{\vec{k} \neq \vec{0}} \left[ \frac{\hbar^2 \vec{k}^2}{2m} - \mu + \hbar \vec{w} \vec{k} + n_{\vec{0}} (V_{\vec{0}} + V_{\vec{k}}) \right] \hat{b}_{\vec{k}}^\dagger \hat{b}_{\vec{k}} \\ & + \frac{n_{\vec{0}}}{2} \sum_{\vec{k} \neq \vec{0}} V_{\vec{k}} (\hat{b}_{\vec{k}} \hat{b}_{-\vec{k}} + \hat{b}_{\vec{k}}^\dagger \hat{b}_{-\vec{k}}^\dagger) + \dots\end{aligned}$$

# Diagonal Hamiltonian

- Linear transformation of annihilators and creators:

$$\begin{aligned}\hat{B}_{\vec{k}} &= u_{\vec{k}} \hat{b}_{\vec{k}} + v_{\vec{k}} \hat{b}_{-\vec{k}}^{\dagger} \\ \hat{B}_{\vec{k}}^{\dagger} &= u_{\vec{k}}^* \hat{b}_{\vec{k}}^{\dagger} + v_{\vec{k}}^* \hat{b}_{-\vec{k}}\end{aligned}$$

1. Bosonic commutator relation

$$[\hat{B}_{\vec{k}}, \hat{B}_{\vec{k}'}] = [\hat{B}_{\vec{k}}^{\dagger}, \hat{B}_{\vec{k}'}^{\dagger}] = 0$$

$$[\hat{B}_{\vec{k}}, \hat{B}_{\vec{k}'}^{\dagger}] = \delta_{\vec{k}, \vec{k}'}$$

Choice of parameters:

$$u_{\vec{k}} = \frac{E_{\vec{k}} + \epsilon_{\vec{k}}}{2\sqrt{E_{\vec{k}}\epsilon_{\vec{k}}}}, \quad v_{\vec{k}} = \frac{E_{\vec{k}} - \epsilon_{\vec{k}}}{2\sqrt{E_{\vec{k}}\epsilon_{\vec{k}}}}$$

2. Diagonal form of Hamiltonian



Inverse transformation:

$$\begin{aligned}\hat{b}_{\vec{k}} &= u_{-\vec{k}}^* \hat{B}_{\vec{k}} + v_{\vec{k}} \hat{B}_{-\vec{k}}^{\dagger} \\ \hat{b}_{\vec{k}}^{\dagger} &= u_{-\vec{k}} \hat{B}_{\vec{k}}^{\dagger} + v_{\vec{k}}^* \hat{B}_{-\vec{k}}\end{aligned}$$

# Effective potential

$$\hat{H} = V n_{\vec{0}} \left( \frac{n_{\vec{0}}}{2} V_{\vec{0}} - \mu \right) + \frac{1}{2} \sum_{\vec{k} \neq \vec{0}} \left( E_{\vec{k}} - \varepsilon_{\vec{k}} - n_{\vec{0}} V_{\vec{k}} \right) + \sum_{\vec{k} \neq \vec{0}} \left( E_{\vec{k}} + \hbar \vec{w} \vec{k} \right) \hat{B}_{\vec{k}}^\dagger \hat{B}_{\vec{k}}$$

→ Bogoliubov dispersion

$$E_{\vec{k}} = \sqrt{\varepsilon_{\vec{k}}^2 + 2n_{\vec{0}} V_{\vec{k}} \varepsilon_{\vec{k}}}$$

→ Free dispersion

$$\varepsilon_{\vec{k}} = \frac{\hbar^2 \vec{k}^2}{2m} - \mu + n_{\vec{0}} V_{\vec{0}}$$

→ Partition function  
in grand-canonical ensamble

$$Z = \text{tr} \left( e^{-\beta \hat{H}} \right)$$

→ Effective potential

$$V_{\text{eff}} = -\frac{1}{\beta} \ln Z$$

$$V_{\text{eff}} = V n_{\vec{0}} \left( \frac{n_{\vec{0}}}{2} V_{\vec{0}} - \mu \right) + \frac{1}{2} \sum_{\vec{k} \neq \vec{0}} \left( E_{\vec{k}} - \varepsilon_{\vec{k}} - n_{\vec{0}} V_{\vec{k}} \right) + \frac{1}{\beta} \sum_{\vec{k} \neq \vec{0}} \ln \left( 1 - e^{-\beta(E_{\vec{k}} + \hbar \vec{w} \vec{k})} \right)$$

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# Landau theory

- Order parameter

$$n_{\vec{0}}$$

- Condition for equilibrium

$$\frac{\partial V_{\text{eff}}}{\partial n_{\vec{0}}} = 0$$

- Treating fluctuations as perturbation → artificial parameter

$$\eta$$

$$V_{\text{eff}} = V n_{\vec{0}} \left( \frac{n_{\vec{0}}}{2} V_{\vec{0}} - \mu \right) + \frac{1}{2} \eta \sum_{\vec{k} \neq \vec{0}} (E_{\vec{k}} - \varepsilon_{\vec{k}} - n_{\vec{0}} V_{\vec{k}}) + \frac{1}{\beta} \eta \sum_{\vec{k} \neq \vec{0}} \ln \left( 1 - e^{-\beta(E_{\vec{k}} + \hbar \vec{w} \cdot \vec{k})} \right)$$

$$n_{\vec{0}} = \frac{\mu}{V_{\vec{0}}} - \eta \frac{1}{2V V_{\vec{0}}} \sum_{\vec{k} \neq \vec{0}} \left[ \left( 1 + \frac{2}{e^{\beta E_{\vec{k}}} - 1} \right) \frac{\partial E_{\vec{k}}}{\partial n_{\vec{0}}} - 2(V_{\vec{0}} + V_{\vec{k}}) \right] \rightarrow \text{Selfconsistent equation for condensate density}$$

$$n_{\vec{0}} = n_{\vec{0}}^{(0)} + \eta n_{\vec{0}}^{(1)} + \dots$$

→ Iterative solution

# Free Energy

$$F(T, V, \mu) = V_{\text{eff}} \left( n_{\vec{0}}^{(0)} + \eta n_{\vec{0}}^{(1)} + \dots \right)$$

$$\begin{aligned} F(T, V, \mu) &= -\frac{V}{2} \frac{\mu^2}{V_{\vec{0}}} + \frac{\eta}{2} \sum_{\vec{k} \neq \vec{0}} \left[ \sqrt{\left( \frac{\hbar^2 \vec{k}^2}{2m} \right)^2 + 2\mu \frac{V_{\vec{k}}}{V_{\vec{0}}} \frac{\hbar^2 \vec{k}^2}{2m}} - \frac{\hbar^2 \vec{k}^2}{2m} - \mu \frac{V_{\vec{k}}}{V_{\vec{0}}} \right] \\ &\quad + \frac{\eta}{\beta} \sum_{\vec{k} \neq \vec{0}} \ln \left( 1 - e^{-\beta \sqrt{\left( \frac{\hbar^2 \vec{k}^2}{2m} \right)^2 + 2\mu \frac{V_{\vec{k}}}{V_{\vec{0}}} \frac{\hbar^2 \vec{k}^2}{2m}}} \right) + \dots \end{aligned}$$

- Total differential

$$dF = -SdT - pdV - Nd\mu$$

→ State variables

$$n(T, V, \mu) = -\frac{1}{V} \left( \frac{\partial F}{\partial \mu} \right)_{T, V}$$

$$p(T, V, \mu) = -\left( \frac{\partial F}{\partial V} \right)_{T, \mu}$$

$$S(T, V, \mu) = -\left( \frac{\partial F}{\partial T} \right)_{V, \mu}$$

# Superfluid density

- Boost changes free energy in thermal fluctuations

$$\Delta F = \frac{\eta}{\beta} \sum_{\vec{k} \neq 0} \ln \left( 1 - e^{-\beta E_{\vec{k}}} e^{-\beta \hbar \vec{w} \cdot \vec{k}} \right)$$

- Taylor series for small velocities

$$\Delta F = \frac{\eta}{\beta} \sum_{\vec{k} \neq 0} \ln \left( 1 - e^{-\beta E_{\vec{k}}} \right) - \eta \frac{\beta \hbar^2}{2} \sum_{\vec{k} \neq 0} (\vec{w} \cdot \vec{k})^2 \frac{e^{\beta E_{\vec{k}}}}{(e^{\beta E_{\vec{k}}} - 1)^2} + \dots$$

- Momentum

$$p_\alpha = - \frac{\partial \Delta F}{\partial w_\alpha}$$

$$\eta \beta \hbar^2 \sum_{\vec{k} \neq 0} \frac{e^{\beta E_{\vec{k}}}}{(e^{\beta E_{\vec{k}}} - 1)^2} \sum_{j=1}^3 k_\alpha k_j w_j = m V \sum_{j=1}^3 (n_n)_{\alpha j} w_j$$

$$\vec{p} = m V n_n \vec{w}$$

- Tensor of superfluid density

$$(n_s)_{\alpha j} = n \delta_{\alpha j} - (n_n)_{\alpha j}$$

# Characteristic quantities

## Condensate depletion

$$\Delta n_{\vec{0}} = n - n_{\vec{0}}$$

- Condensate density from variation of  $V_{\text{eff}}$  with respect to order parameter

$$n_{\vec{0}}$$

## Superfluid depletion

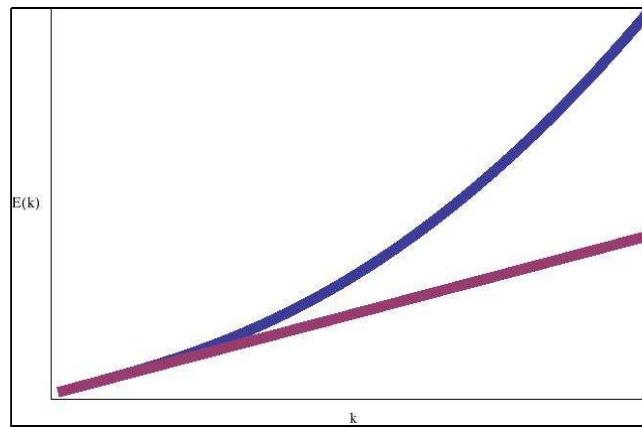
$$n_n = n - n_s$$

- Superfluid density from variation of  $F$  with respect to boost velocity  $\vec{w}$

- Particle density from derivative of free energy

## Bogoliubov dispersion

$$E_{\vec{k}} = \sqrt{\left(\frac{\hbar^2 \vec{k}^2}{2m}\right)^2 + 2nV_{\vec{k}} \frac{\hbar^2 \vec{k}^2}{2m}}$$



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# Contact and dipole-dipole interaction

## 1. Contact interaction

$$V_c(\vec{x}) = g\delta(\vec{x})$$

- Strength of delta function

$$g = \frac{4\pi\hbar^2 a}{m}$$

- S-wave scattering length  $a$

## 2. Dipole-dipole interaction

$$V_{dd}(\vec{x}) = \frac{C_{dd}}{4\pi|\vec{x}|^3} (1 - 3\cos^2\theta)$$

- magnetic  $C_{dd} = \mu_0 m^2$
- electric  $C_{dd} = 4\pi d^2$

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$$V(\vec{x}) = V_c(\vec{x}) + V_{dd}(\vec{x}) \longrightarrow V_{\vec{k}} = g \left[ 1 + \varepsilon_{dd} (3\cos^2\theta - 1) \right]$$

Relative-interaction strength:

$$\varepsilon_{dd} = \frac{C_{dd}}{3g}$$

# Dipole-enhancement functions

Integration over cosine gives:

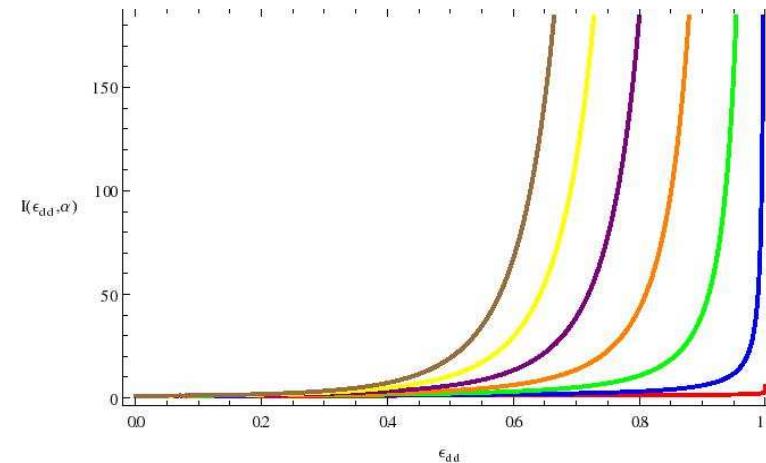
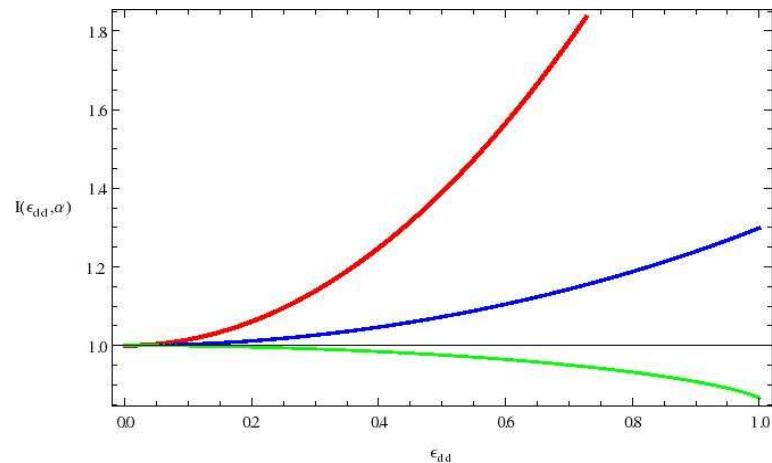
$$I(\epsilon_{dd}, \alpha) = \int_0^1 dc \left[ 1 + \epsilon_{dd} (3c^2 - 1) \right]^\alpha$$

1. Convergent  $I$ -functions for:

$$\alpha = \frac{5}{2}, \frac{3}{2}, \frac{1}{2}$$

2. Divergent  $I$ -functions for:

$$\alpha = -\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2}, -\frac{7}{2}, -\frac{9}{2}, -\frac{11}{2}, -\frac{13}{2}$$



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# Tools

- Thermodynamic limit

$$\sum_{\vec{k} \neq 0} = \frac{V}{(2\pi)^3} \int d^3 \vec{k}$$

- Spherical coordinates

$$\frac{V}{(2\pi)^3} \int d^3 \vec{k} = \frac{V}{4\pi^2} \left( \frac{2m}{\hbar^2} \right)^{\frac{3}{2}} \int_0^1 d \cos \theta \int_0^\infty d\varepsilon \sqrt{\varepsilon}$$

Where:  $\varepsilon = \frac{\hbar^2 k^2}{2m}$

- Taylor series

$$e^x = 1 + x + \frac{x^2}{2!} + \dots$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \dots$$

# Condensate depletion

$$\left(\Delta n_{\vec{0}}\right)_{TF} = \eta \frac{1}{4\pi^2} \left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} \int_0^1 dc \frac{1}{\beta} \int_0^\infty d\varepsilon \sqrt{\varepsilon} \frac{d}{d\varepsilon} \ln \left(1 - e^{-\beta \sqrt{\varepsilon^2 + 2nV_{\vec{k}}\varepsilon}}\right)$$

- Logarithm can be presented as a series  $\ln(1 - e^x) = -\sum_{\nu=1}^{\infty} \frac{e^{-\nu x}}{\nu}$

- Riemann Zeta-function  $\sum_{\nu=1}^{\infty} \frac{1}{\nu^s} = \zeta(s)$

- Root can be expanded with:  $\varepsilon = \frac{\kappa^2}{\beta^2 n V_{\vec{0}}}$

- Gamma-function  $\int_0^\infty d\kappa \kappa^{\tau-1} e^{-\tau} = \Gamma(\tau)$

$$\begin{aligned} \Delta n_{\vec{0}} &= \eta \frac{8\pi^{-\frac{1}{2}}}{3} (na)^{\frac{3}{2}} I\left(\varepsilon_{dd}, \frac{3}{2}\right) + \eta \frac{1}{\lambda_{\text{dB}}^4} \frac{\pi^{\frac{3}{2}}}{6} (na)^{-\frac{1}{2}} I\left(\varepsilon_{dd}, -\frac{1}{2}\right) \\ &\quad + \eta \frac{1}{\lambda_{\text{dB}}^8} \frac{\pi^{\frac{7}{2}}}{480} (na)^{-\frac{5}{2}} I\left(\varepsilon_{dd}, -\frac{5}{2}\right) + \dots \end{aligned}$$

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$$\lambda_{\text{dB}} = \sqrt{\frac{2\pi\hbar^2}{mk_{\text{B}}T}}$$

# Free Energy

$$F(T, V, \mu) = -\frac{V}{2} \frac{\mu^2}{V_0} + \frac{\eta}{2} \sum_{\vec{k} \neq 0} \left[ \sqrt{\left( \frac{\hbar^2 \vec{k}^2}{2m} \right)^2 + 2\mu \frac{V_{\vec{k}}}{V_0} \frac{\hbar^2 \vec{k}^2}{2m}} - \frac{\hbar^2 \vec{k}^2}{2m} - \mu \frac{V_{\vec{k}}}{V_0} \right]$$

$$+ \frac{\eta}{\beta} \sum_{\vec{k} \neq 0} \ln \left( 1 - e^{-\beta \sqrt{\left( \frac{\hbar^2 \vec{k}^2}{2m} \right)^2 + 2\mu \frac{V_{\vec{k}}}{V_0} \frac{\hbar^2 \vec{k}^2}{2m}}} \right) + \dots$$

- Known methods lead to

$$F(T, V, \mu) = -\frac{V\mu^2}{2V_0} + \eta \frac{256\pi^2}{15} \frac{V\hbar^2}{m} \left( \frac{\mu a}{V_0} \right)^{\frac{5}{2}} I\left(\varepsilon_{dd}, \frac{5}{2}\right) - \eta \frac{1}{\lambda_{\text{dB}}^8} \frac{\pi^{\frac{9}{2}}}{45} \frac{V\hbar^2}{m} \left( \frac{\mu a}{V_0} \right)^{-\frac{3}{2}} I\left(\varepsilon_{dd}, -\frac{3}{2}\right)$$

$$- \eta \frac{1}{\lambda_{\text{dB}}^{12}} \frac{\pi^{\frac{13}{2}}}{252} \frac{V\hbar^2}{m} \left( \frac{\mu a}{V_0} \right)^{-\frac{7}{2}} I\left(\varepsilon_{dd}, -\frac{7}{2}\right) + \eta \frac{1}{\lambda_{\text{dB}}^{16}} \frac{3\pi^{\frac{15}{2}}}{1280} \frac{V\hbar^2}{m} \left( \frac{\mu a}{V_0} \right)^{-\frac{11}{2}} I\left(\varepsilon_{dd}, -\frac{11}{2}\right) + \dots$$

# Superfluid depletion

$$(n_n)_{\alpha j} = \eta \frac{\beta \hbar^2}{mV} \sum_{\vec{k} \neq 0} \frac{e^{\beta E_{\vec{k}}}}{(e^{\beta E_{\vec{k}}} - 1)^2} k_{\alpha} k_j$$

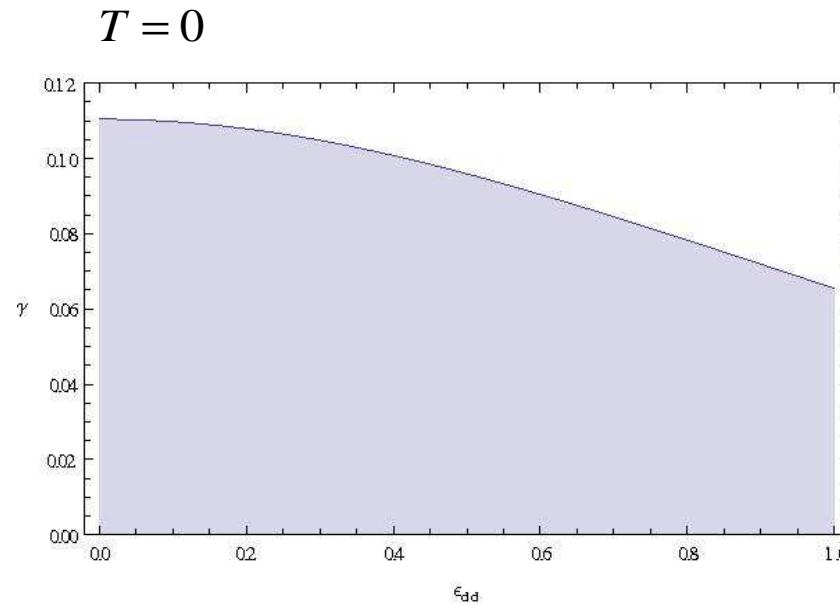
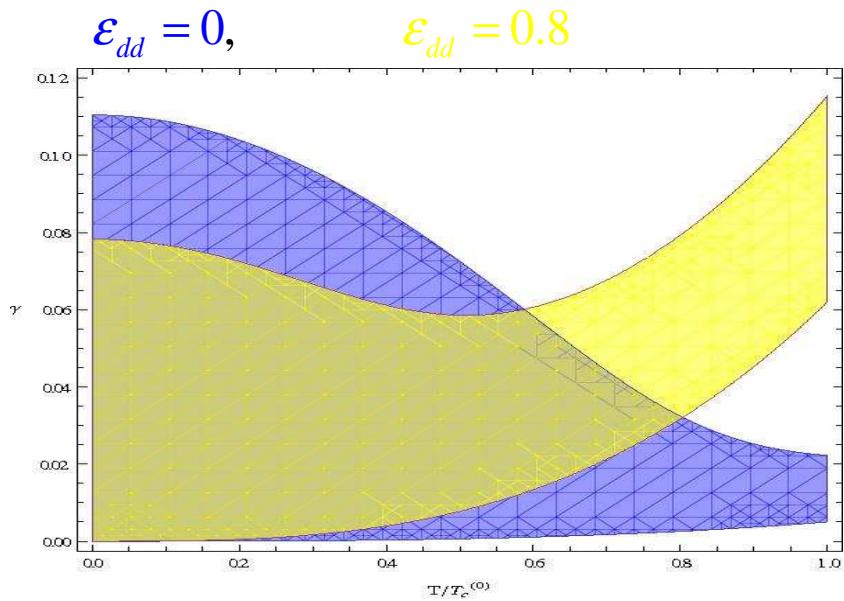
$$\left. \begin{array}{l} \bullet \text{ Spherical coordinates} \\ \bullet \text{ Cylindric symmetry} \end{array} \right\} \quad \begin{aligned} k_x^2 &= k_y^2 = \frac{1}{2} k^2 (1 - \cos^2 \theta) \\ k_z^2 &= k^2 \cos^2 \theta \end{aligned}$$

$$\begin{aligned} (n_n)_{perp/par} &= \eta \frac{1}{\lambda_{dB}^8} \frac{\pi^{\frac{7}{2}}}{30} (na)^{-\frac{5}{2}} J_{perp/par} \left( \epsilon_{dd}, -\frac{5}{2} \right) - \eta \frac{1}{\lambda_{dB}^{12}} \frac{\pi^{\frac{11}{2}}}{48} (na)^{-\frac{9}{2}} J_{perp/par} \left( \epsilon_{dd}, -\frac{9}{2} \right) \\ &\quad + \eta \frac{1}{\lambda_{dB}^{16}} \frac{11\pi^{\frac{15}{2}}}{256} (na)^{-\frac{13}{2}} J_{perp/par} \left( \epsilon_{dd}, -\frac{13}{2} \right) + \dots \end{aligned}$$

$$(n_n)_{par} \leq (n_n)_{perp}$$

# Area of validity

$$0 \leq \frac{\Delta n_0}{n} \leq \frac{1}{2} \quad \xrightarrow[T=0]{\gamma = na^3} \quad 0 \leq \gamma \leq \left[ \frac{3\sqrt{\pi}}{16} \frac{1}{I\left(\varepsilon_{dd}, \frac{3}{2}\right)} \right]^2$$



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# Two-Fluid Model

- Hydrodynamics with two fluids
- Superfluid  $\rightarrow$  no viscosity
- Landau-Khalatnikov equations for isotropic system
- Generalized L-K eqs. for anisotropic system  
by Carolin Wille (unpublished)
- Calculation of first two sound velocities

First sound: pressure wave

Second sound: entropy wave

$\rightarrow$  Dispersion

$$0 = \omega^4 - \omega^2 \left[ \left( \frac{\partial p}{\partial \rho} \right)_{\bar{s}} \vec{k}^2 + \frac{T\bar{s}^2}{c_v} \sum_{i,j,l} (\rho_n^{-1})_{il} (\rho_s)_{lj} k_i k_j \right] + \left( \frac{\partial p}{\partial \rho} \right)_T \vec{k}^2 \frac{T\bar{s}^2}{c_v} \sum_{i,j,l} (\rho_n^{-1})_{il} (\rho_s)_{lj} k_i k_j$$

# Sound velocities (T=0)

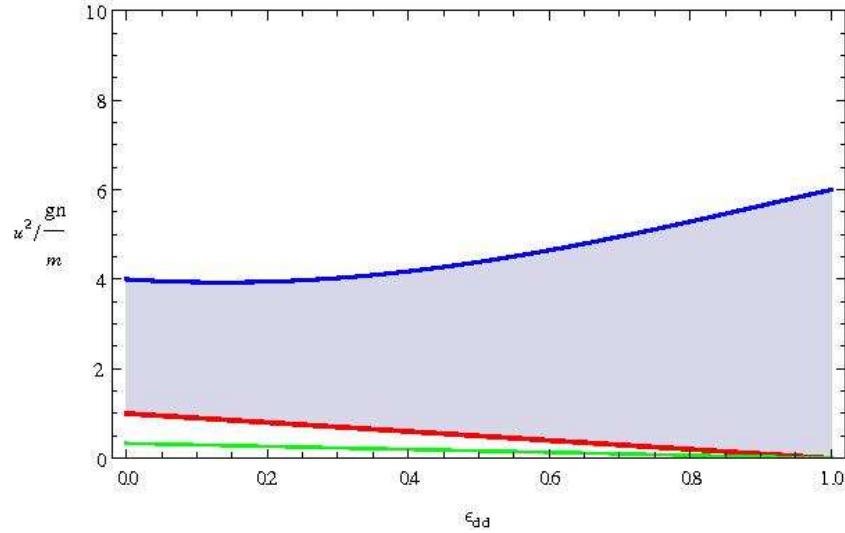
$$\omega_{1/2}^2(\vec{k}) = (k \sin \theta \quad k \cos \theta) \begin{pmatrix} (u_{perp}^2)_{1/2} & 0 \\ 0 & (u_{par}^2)_{1/2} \end{pmatrix} \begin{pmatrix} k \sin \theta \\ k \cos \theta \end{pmatrix}$$

$$(u_1^2)_{perp} = \frac{gn}{m} \left[ 1 - \varepsilon_{dd} + \frac{16}{\sqrt{\pi}} \sqrt{\gamma} I(\varepsilon_{dd}, \frac{5}{2}) \right]$$

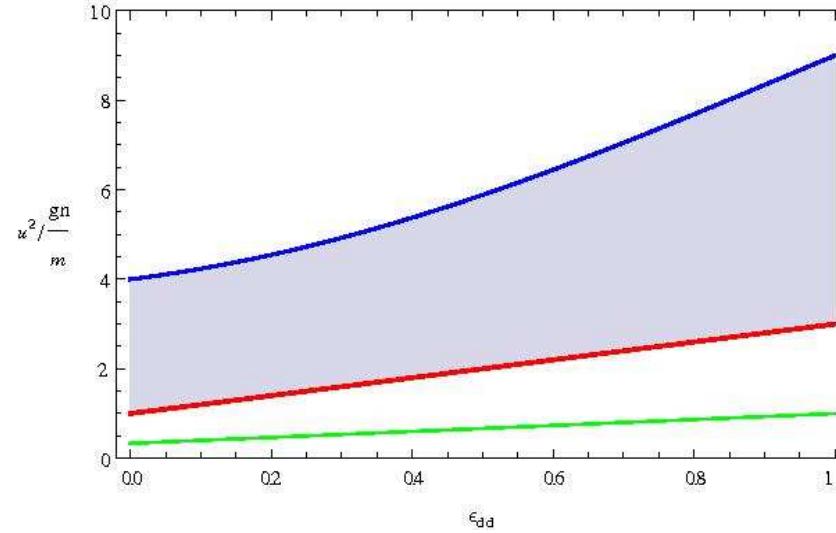
$$(u_2^2)_{perp} = \frac{gn}{m} \frac{1}{3} (1 - \varepsilon_{dd})$$

$$(u_1^2)_{par} = \frac{gn}{m} \left[ 1 + 2\varepsilon_{dd} + \frac{16}{\sqrt{\pi}} \sqrt{\gamma} I(\varepsilon_{dd}, \frac{5}{2}) \right]$$

$$(u_2^2)_{par} = \frac{gn}{m} \frac{1}{3} (1 + 2\varepsilon_{dd})$$



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# Conclusion

1. Bogoliubov theory

→ Diagonal Hamiltonian

→ Approach to thermodynamics

2. Landau theory

→ Variation of effective potential

→ State variables

3. Application for contact and dipole-dipole interaction

→ Low-temperature series of state variables

→ Hydrodynamic properties

4. Outlook

→ Temperature dependence of sound velocities

Thank You for Your attention!