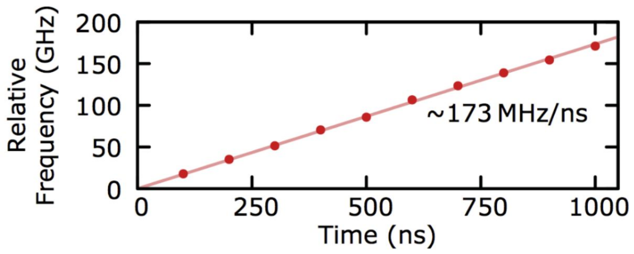
Quantum Mechanical Description of Thermo-Optic Interaction

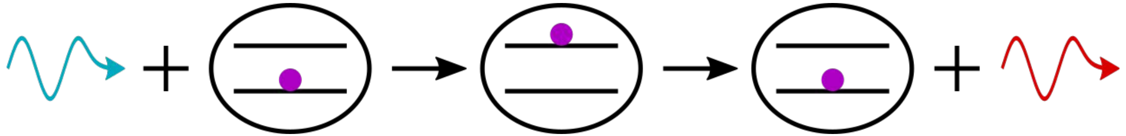
Enrico Stein, Axel Pelster

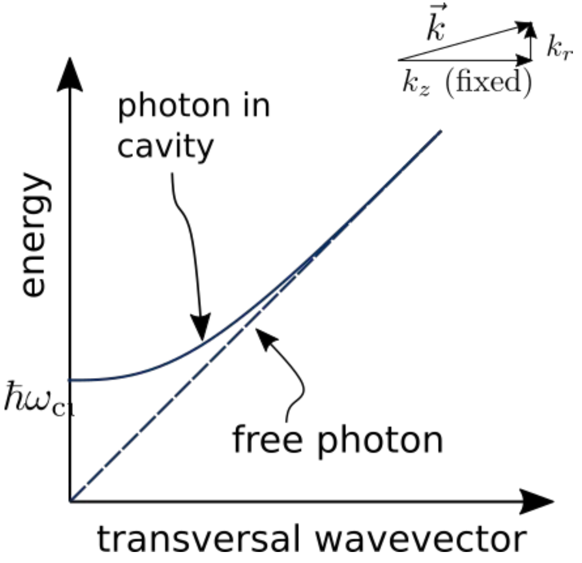


Dung, et alii, Nature Photonics **11**, 565 (2017)

Photons in Dye-Filled Cavity

- **Effective photon mass**
- **Effective harmonic trap**
- **Thermalisation through interaction with dye molecules:**





⇒ **Thermal gas of massive bosons in 2D harmonic potential**

Photon-Photon Interaction

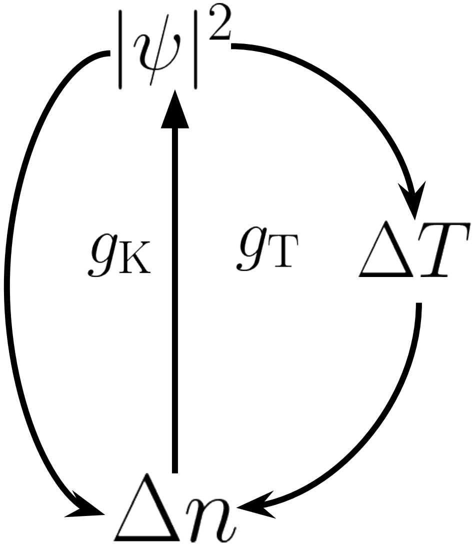
Photon Energy: $E(t) = E_{\mathrm{HO}} + \hbar\omega_{\mathrm{c}} \int d^2x \; [\Delta n_{\mathrm{K}} + \Delta n_{\mathrm{T}}(t)] \, |\psi|^2$

Kerr Effect

- Change of refractive index due to intensity:

$$\Delta n_{\mathrm{K}} \sim |\psi|^2$$

⇒ **Contact interaction**



Thermo-Optic Effect

- Heating of dye medium due to intensity
- 3D heat diffusion:

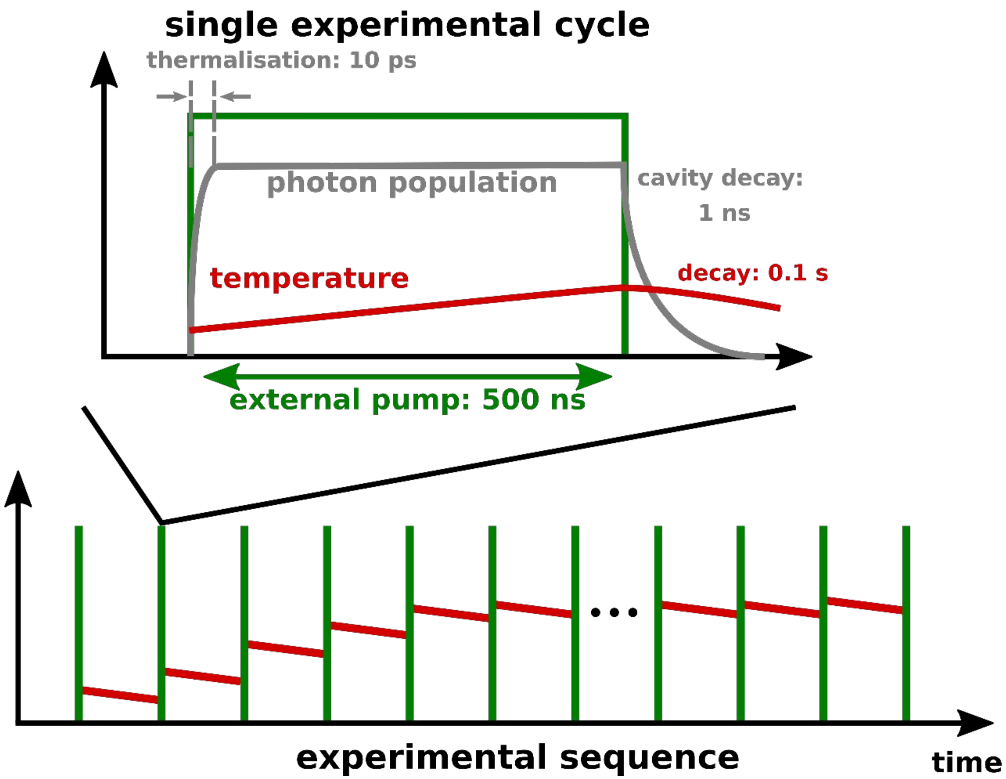
$$\Delta n_{\mathrm{T}}(t) \sim \int_0^t dt' \int d^2x' \, G(\mathbf{x}-\mathbf{x}', t-t') |\psi(\mathbf{x}', t')|^2$$

⇒ **Non-local, retarded interaction**

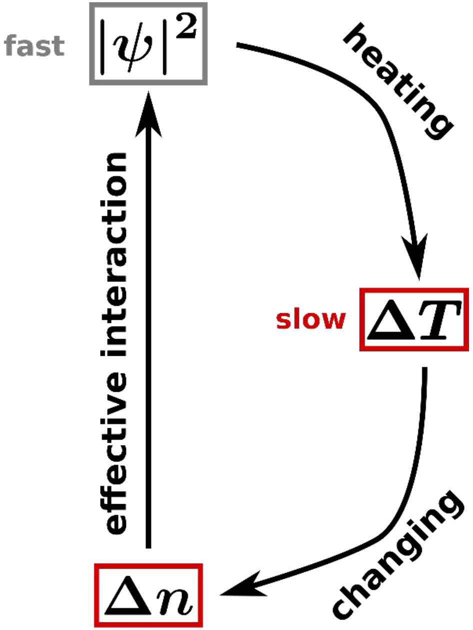
Leading mechanism

Stein, Vewinger and Pelster, NJP **21**, 103044 (2019)
Stein and Pelster, NJP **24**, 023032 (2022)

Time Scales



single experimental cycle
thermalisation: 10 ps
photon population
cavity decay:
1 ns
temperature
decay: 0.1 s
external pump: 500 ns
...
experimental sequence
time



fast
$|\psi|^2$
heating
slow
ΔT
changing
Δn
effective interaction

Hamiltonian Description

- Thermo-optic Hamiltonian:

$$\hat{H}(t) = \int d^2x \; \hat{\Psi}^\dagger \Big\{ h_0(\mathbf{x}) + \gamma T(\mathbf{x},t) \Big\} \hat{\Psi}$$

energy shift from heating

- Temperature diffusion:

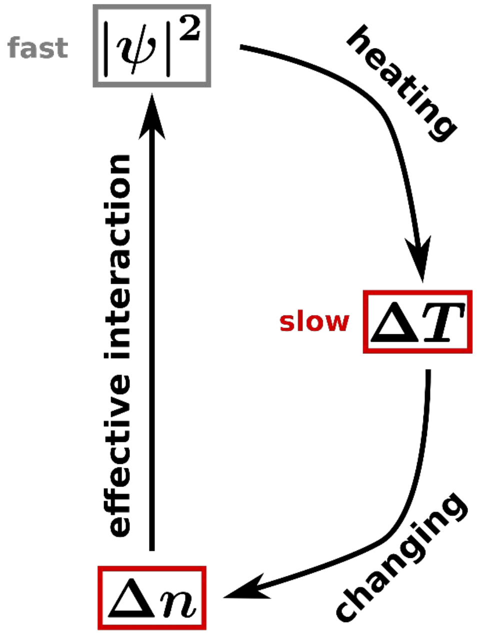
$$\partial_t T = \left\{ \mathcal{D}\nabla^2 - \frac{1}{\tau} \right\} T + B n(\mathbf{x},t)$$

photons as heat source

- Photon density:

$$n(\mathbf{x},t) = \left\langle \hat{\Psi}^\dagger(\mathbf{x},t)\hat{\Psi}(\mathbf{x},t) \right\rangle$$

Stein and Pelster, arXiv:2203.16955 (2022)



Short-Time Approximation:

Interaction

- No temperature diffusion in a single cycle:

$$\hat{H}(t) = \int d^2x \, \hat{\Psi}^\dagger \left\{ h_0(\mathbf{x}) + g(t) n(\mathbf{x}, 0) \right\} \hat{\Psi}$$

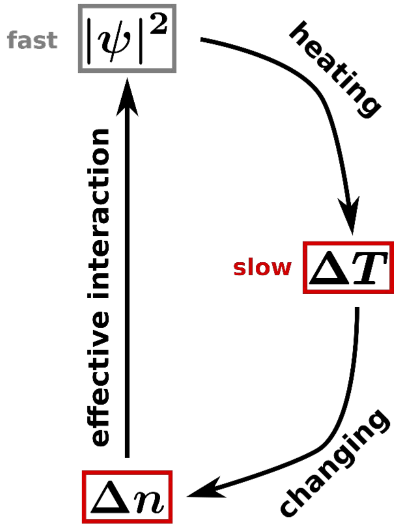
- Adiabatic time dependence:

$$g(t) = t\gamma B$$

- Expansion in eigenmodes of $h_0(\mathbf{x})$:

$$\hat{H}(t) = \sum_{mn} \left[E_m(0)\delta_{mn} + g(t) F_{mn} \right] \hat{a}_m^\dagger \hat{a}_n$$

⇒ Diagonalisation

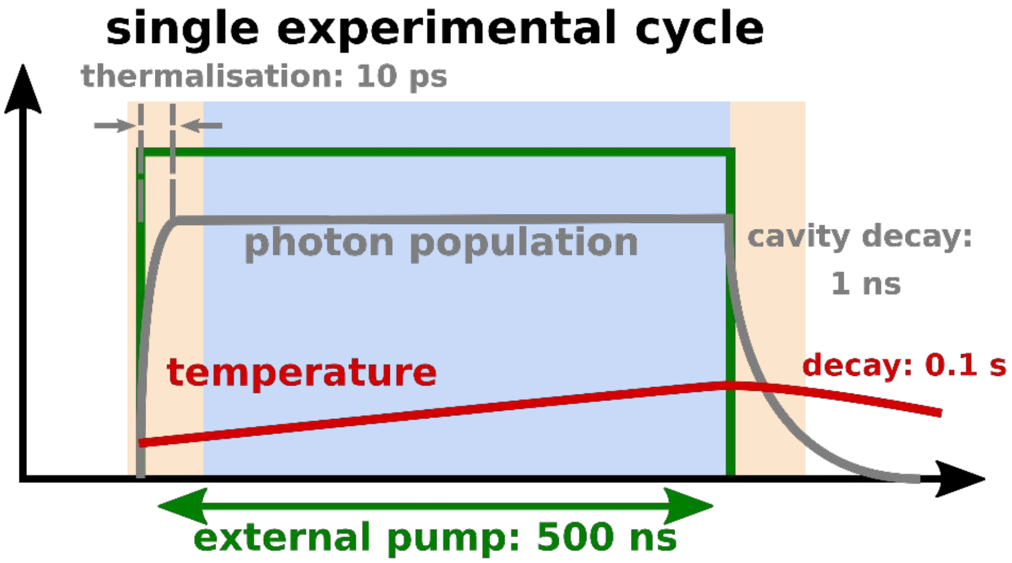


$$F_{mn} = \int d^2x \; \psi_m^* n(\mathbf{x}, 0) \psi_n$$

Stein and Pelster, arXiv:2203.16955 (2022)

Short-Time Approximation:

Thermalisation



$$\hat{H} = \sum_{mn} \left[E_m(0)\delta_{mn} + g(t)F_{mn}\right] \hat{a}_m^\dagger \hat{a}_n$$

- Neglect dissipative dynamics
- Only thermal steady state:

$$F_{mn} = \int d^2x \; \psi_m^* \underbrace{\sum_l N_l(0)|\psi_l|^2}_{\text{thermal density}} \psi_n$$

$$N_l(0) = \frac{1}{e^{\beta[E_l(0)-\mu]}-1}$$

⇒ Hartree-Fock analogue

Stein and Pelster, arXiv:2203.16955 (2022)

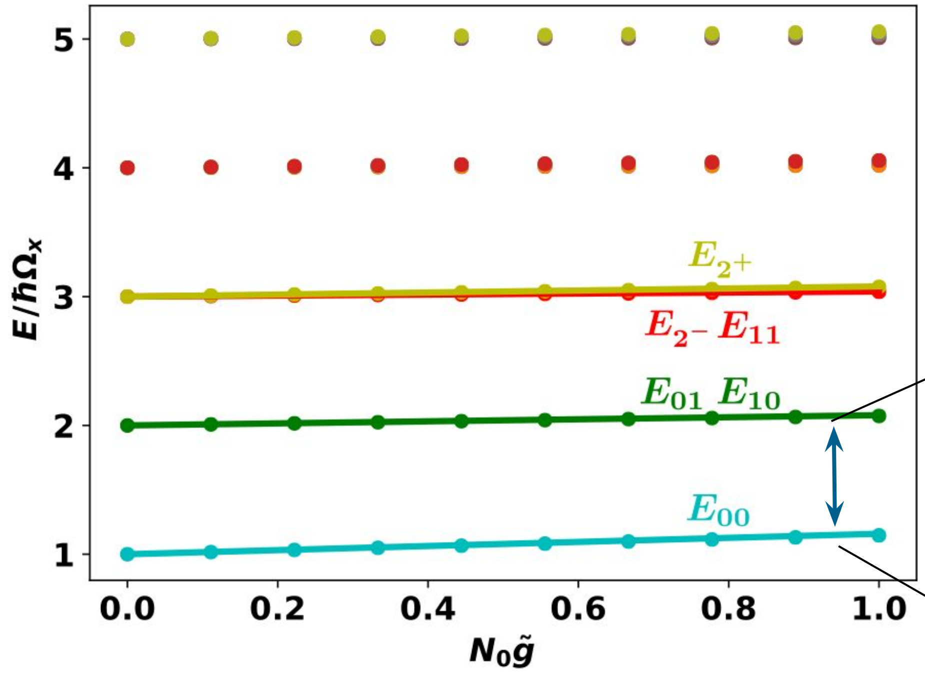
Harmonic Potential:

Spectrum

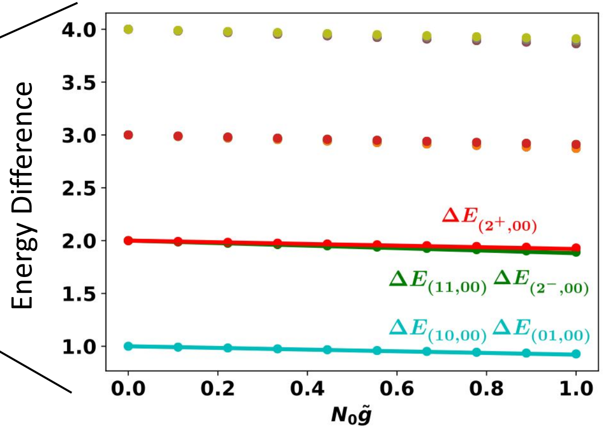
$$V = \frac{m\Omega^2}{2}\left(x^2 + y^2\right)$$

$$@N = 10.000,\ \ N_0/N \approx 1$$

- Dots: Exact diagonalisation
- Lines: Perturbation theory



Stein and Pelster, in preparation



Harmonic Potential:

Variational Ansatz

$$V = \frac{m\Omega^2}{2}\left(x^2 + y^2\right)$$

$$@N = 10.000, \;\; N_0/N \approx 1$$

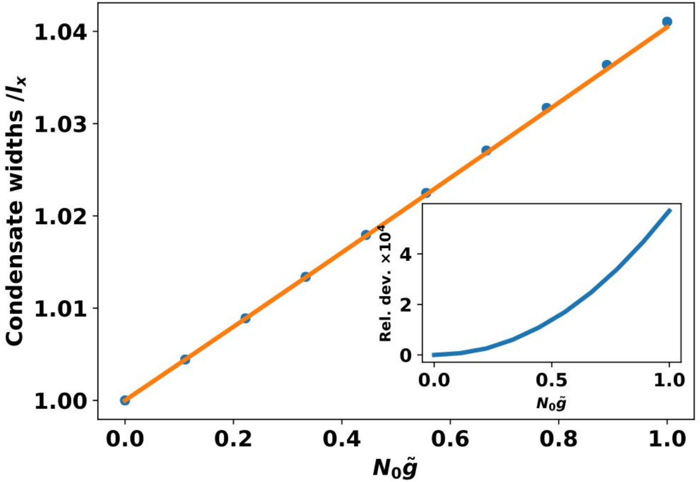
- Ground state **only**
- Energy functional:

$$E = \int d^2x \; \psi^* \left\{ \hat{h}_0 + g(t)|\psi(\mathbf{x},0)|^2 \right\} \psi$$

- Ansatz:

$$\psi = \sqrt{\frac{N}{\pi l_x l_y \alpha_x \alpha_y}} e^{-\frac{x^2}{2l_x^2\alpha_x^2} - \frac{y^2}{2l_y^2\alpha_y^2}}$$

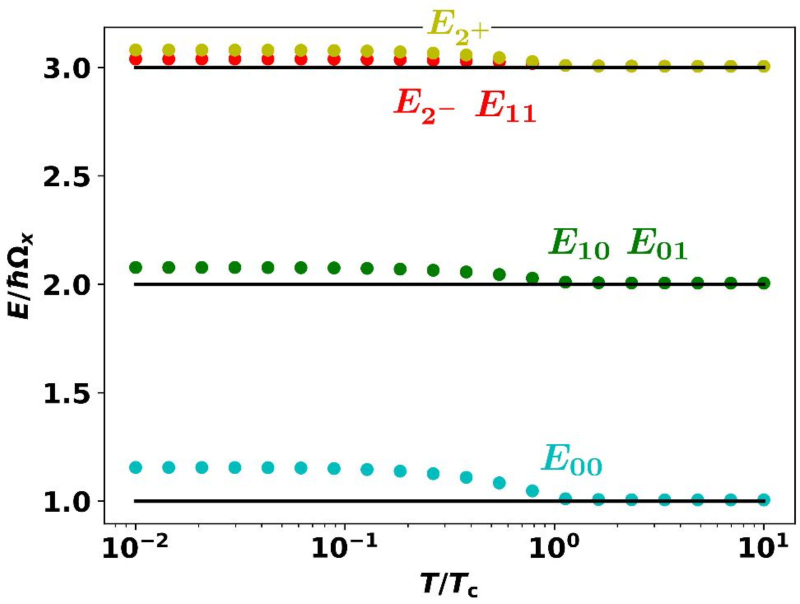
- Condition: $\alpha_x(0) = 1 = \alpha_y(0)$



- Dots: Exact diagonalisation
- Line: Variational approach

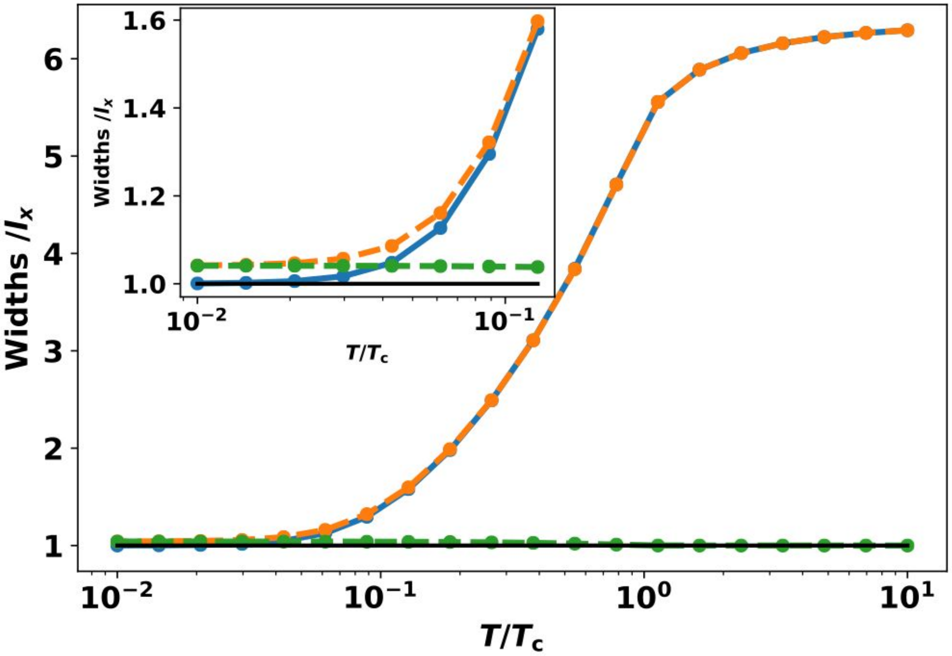
Dung et alii, Nature Photonics **11**, 565 (2017)

Finite Temperatures



$$V = \frac{m\Omega^2}{2}\left(x^2 + y^2\right)$$

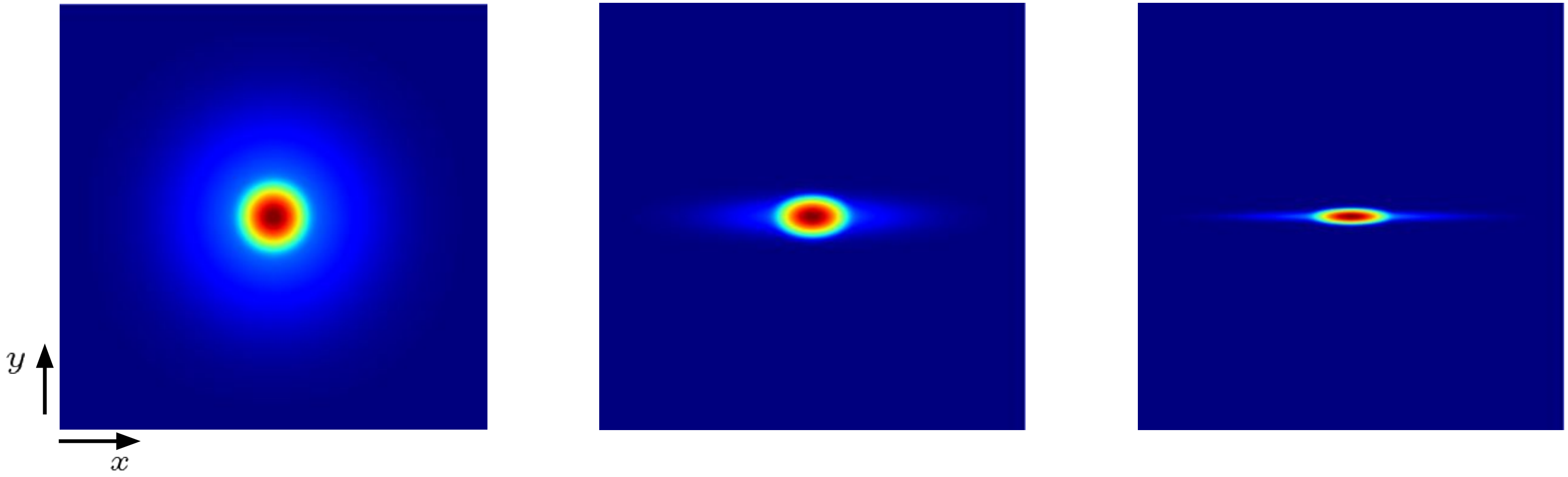
$$@N = 10.000$$



Stein and Pelster, in preparation

Crossover 2D ⇔ 1D

- **Trap aspect ratio:** $\lambda = \sqrt{\Omega_y/\Omega_x}$ $\qquad V = \frac{m\Omega^2}{2}\left(x^2 + \lambda^4 y^2\right)$



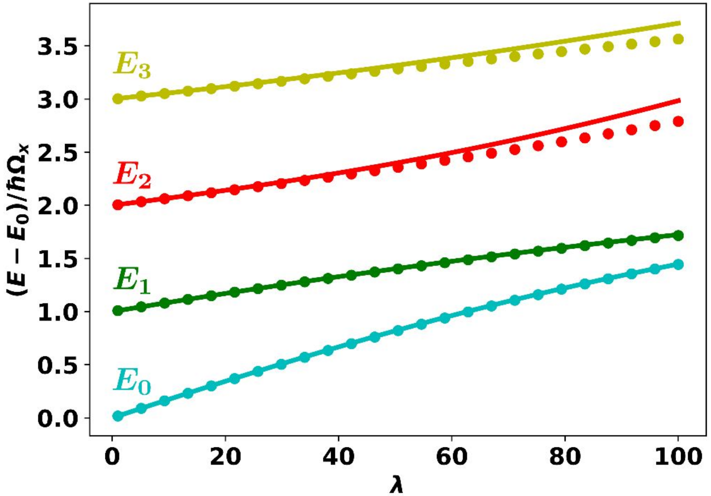
2D $\qquad \lambda = 1$

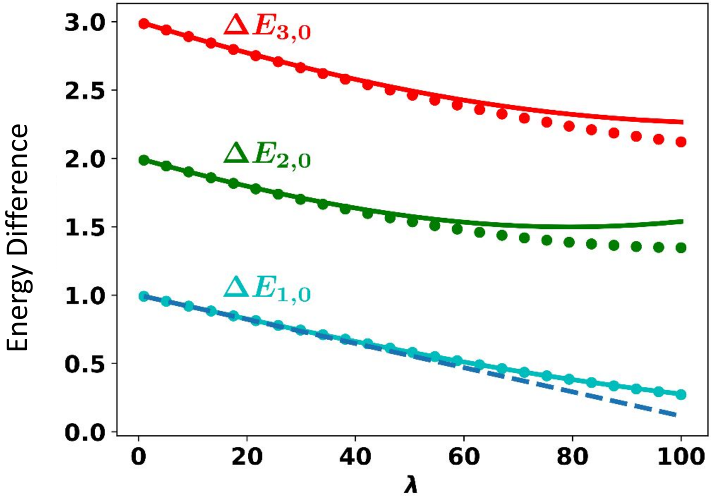
Details on poster!

1D $\qquad \lambda \to \infty$

Dimensional Crossover:

Spectrum





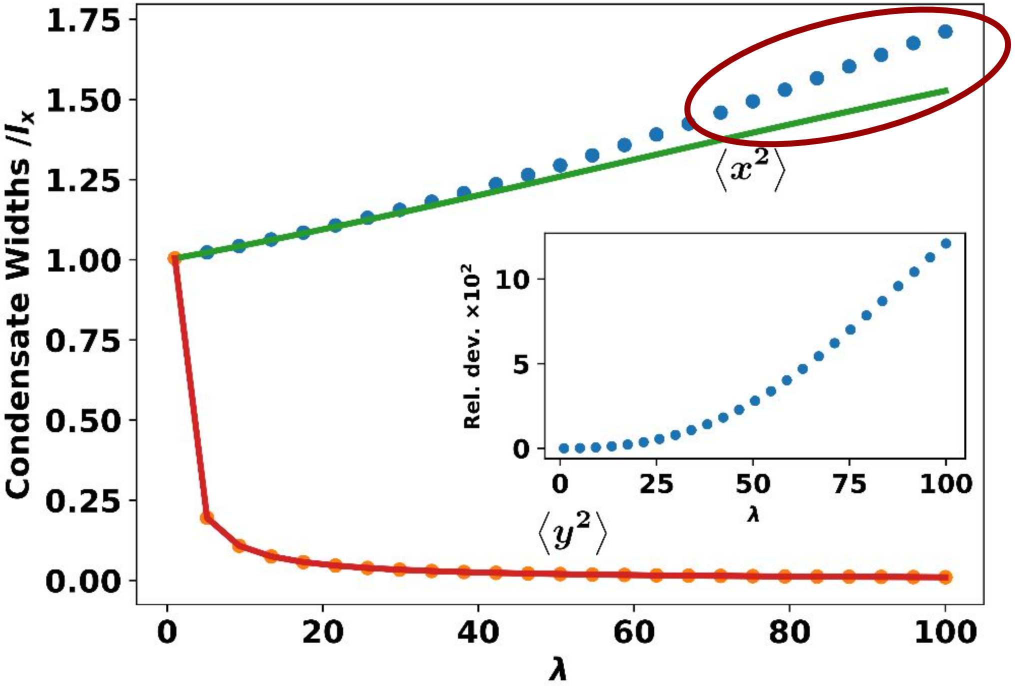
$$@N = 10.000, \quad N_0/N \approx 1, \quad \tilde{g} = 10^{-5}$$

- Effective 1D interaction strength:

$$\tilde{g}_{1\mathrm{D}} \propto \lambda$$

- Dots: Exact diagonalisation
- Line: Analytical approach

Dimensional Crossover:
Condensate Width



Deviation due to thermal cloud!

- Effective 1D interaction strength:

$$\tilde{g}_{1D} \propto \lambda$$

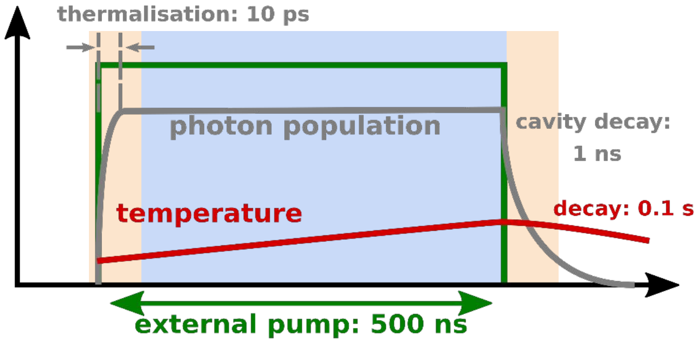
- Dots: Exact diagonalisation
- Line: Variational approach

$$V = \frac{m\Omega^2}{2}\left(x^2 + \lambda^4 y^2\right)$$

$$@N = 10.000,\ \ N_0/N \approx 1, \tilde{g} = 10^{-5}$$

Stein and Pelster, in preparation

Summary

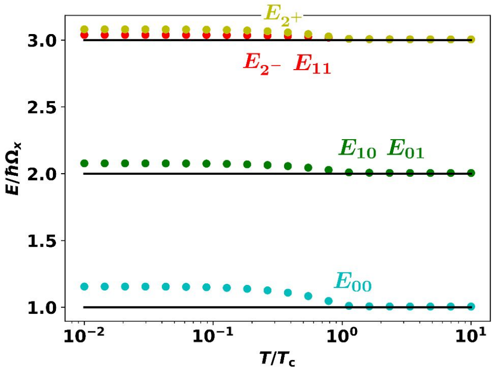


- Adiabatic description of thermo-optic interaction:

$$\hat{H}(t) = \sum_{mn} \left[E_m(0)\delta_{mn} + g(t)F_{mn} \right] \hat{a}_m^\dagger \hat{a}_n$$

- Access to new eigenstates and eigenenergies via diagonalisation $\Rightarrow$ Hartree-Fock analogue

- Describe finite temperatures:



- Relevant at dimensional crossover: