

# Engineering Feshbach Resonances by Time-Periodic Driving

Christoph Dauer, Axel Pelster and Sebastian Eggert

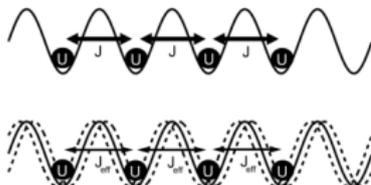
*Department of Physics and Research Center OPTIMAS,  
Technische Universität Kaiserslautern, Germany*

DPG Spring Meeting,  
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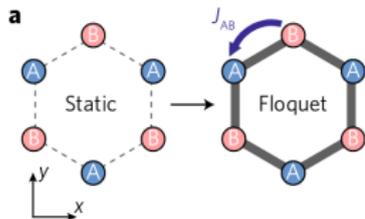


## Tuning Ultracold Gases

### Floquet Engineering



H. Lignier et al. PRL 99, 220403 (2007)



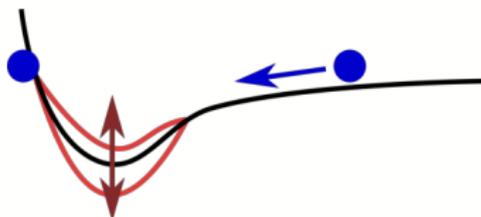
N. Fläschner et al. Nat 14, 265 (2018)  
 Review: A. Eckardt RMP 89, 011004 (2017)

### Ultracold Quantum Gases

$$i\hbar \frac{\partial}{\partial t} \phi = \left( -\frac{\hbar^2}{2m} \Delta + V_{\text{ext}}(\mathbf{r}, t) + \frac{4\pi\hbar^2 a}{m} |\phi|^2 \right) \phi$$

### Tunable parameters

- Kinetic energy
- Trap, optical lattice
- Interaction via Feshbach resonance
  - Magnetic Feshbach resonance
  - Optical Feshbach resonance
  - Driving induced scattering resonance [1,2]

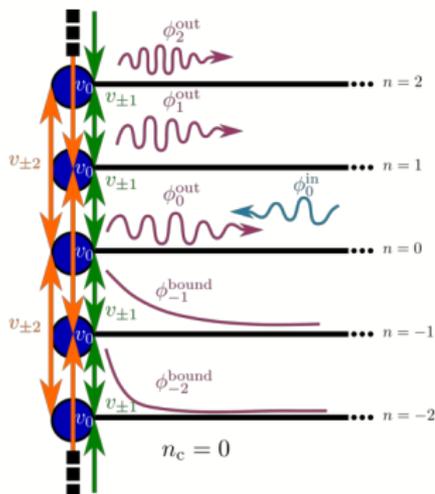


[1] Smith PRL 115, 193002 (2015)  
 [2] Sykes et al. PRA 95, 062705 (2018)

## Floquet Scattering

- Time-periodic scattering potential:  $v(r, t) = \sum_{n=-\infty}^{\infty} e^{-in\omega t} v_n(r)$
- Floquet approach:  $\psi(\mathbf{r}, t) = e^{-i\epsilon t/\hbar} \sum_{n=-\infty}^{\infty} e^{-in\omega t} \phi_n(\mathbf{r})$
- Floquet-partial wave expansion:  $\phi_n(\mathbf{r}) = \sum_{l=0}^{\infty} R_{n,l}(r) P_l(\cos \theta)$
- Radial-Floquet equation:
 
$$\left[ \Delta_r + k_n^2 - \frac{l(l+1)}{r^2} - v_0(r) \right] R_{l,n}(r) = \sum_{m \neq 0} v_m(r) R_{l,n-m}(r)$$

- Dispersion  $\frac{\hbar^2}{2m} k_n^2 = \epsilon + n\hbar\omega$
- Asymptotic wave function:
 
$$\phi_n(\mathbf{r}) = \delta_{n,0} e^{ikr} + f_n \frac{e^{ik_n r}}{r}$$
- Critical index:  $n_c = \lceil -\frac{\epsilon}{\hbar\omega} \rceil$ 
  - Free states, if  $n \geq n_c : k_n \in \mathbb{R}$
  - Bound states, if  $n < n_c : k_n \in \mathbb{C}$
- Time averaged scattering length [1]
 
$$a_{\text{scatt}} = -\lim_{\epsilon \rightarrow 0} f_0$$



- Contact potential, s-wave scattering length  $a(t) = \bar{a} + a_1 \cos(\omega t)$   
 $v(r, t) = \frac{2a(t)}{r^2} \delta(r) \frac{\partial}{\partial r} r \Rightarrow v_0(r) = \frac{2\bar{a}}{r^2} \delta(r) \frac{\partial}{\partial r} r, \quad v_{\pm 1}(r) = \frac{a_1}{r^2} \delta(r) \frac{\partial}{\partial r} r$
- Radial-Floquet equation,  $l = 0$  (low-energy limit)  
 $[\Delta_r + k_n^2 - v_0(r)] R_{l=0, n}(r) = \sum_{m=\pm 1} v_m(r) R_{l=0, n-m}(r)$

- Floquet modes [3]:

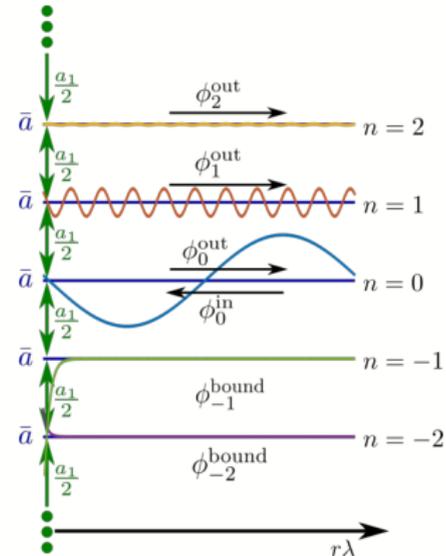
$$R_{l=0, n}(r) = \frac{\delta_{n,0}}{2} \frac{ie^{-ik_n r}}{k_n r} - D_n \frac{ie^{ik_n r}}{k_n r}$$

- Evaluation of  $\delta$ -function and continuity condition:

$$\left(\frac{i}{k_n \bar{a}} - 1\right) D_n - \frac{a_1}{2\bar{a}} (D_{n+1} + D_{n-1}) = \gamma_n$$

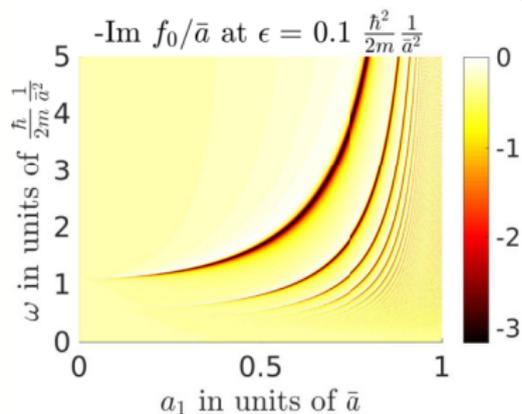
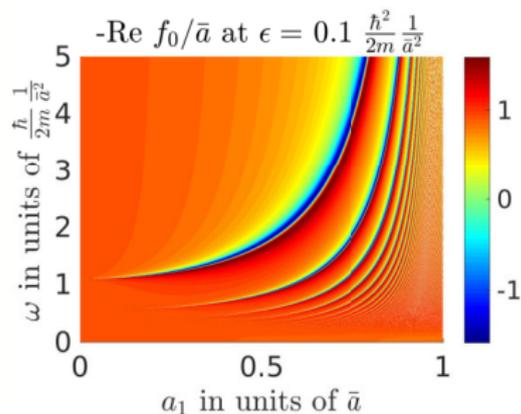
- Solution via continued fraction
- Calculate scattering properties

$$f_n = \frac{-i}{k_n} \left(D_n - \delta_{n,0} \frac{1}{2}\right)$$



## Driving Induced Scattering Resonances

- Lines of resonances in  $\omega$ - $a_1$  plane for positive  $\bar{a}$
- Incoming particle has energy  $\epsilon$
- Length scale:  $\bar{a}$
- Energy scale:  $E_D = \frac{\hbar^2}{2m} \frac{1}{\bar{a}^2}$
- Amplitude determines elastic scattering:  $\langle\langle \sigma \rangle\rangle_{\text{el}} = 4\pi |f_0|^2$
- Total cross section via Floquet-optical theorem [2]:  $\langle\langle \sigma \rangle\rangle = 4\pi \frac{\text{Im } f_0}{k_0}$



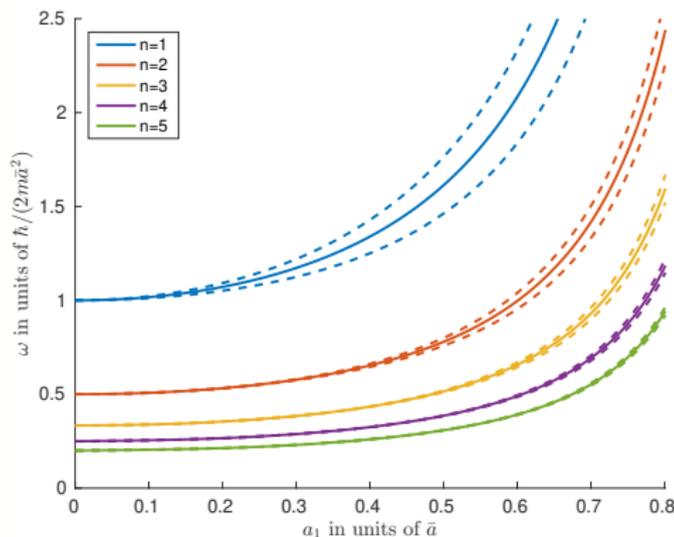
[2] Sykes et al. PRA 95, 062705 (2018)

## Resonances in Scattering Length

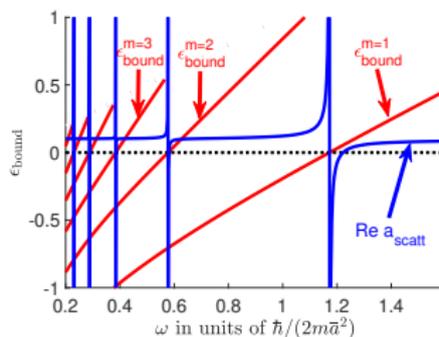
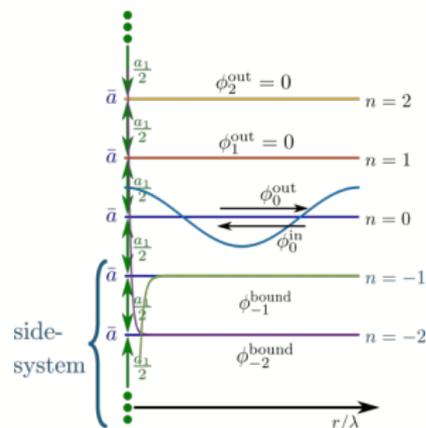
Low-energy physics: Scattering length around resonance:

$$a_{\text{scatt}}(\omega) \approx a_{\text{BG}} \left( 1 - \frac{\delta}{\omega - \omega_0} \right)$$

- Energy scale is determined by choosing  $\bar{a}$
- Resonance width  $\delta^{(n)}$  is set by  $a_1$
- Enhancement of scattering length by choosing  $\omega$  relative to  $\omega_0^{(n)}$



- Explanation in coupled channel picture:  
Fano-Feshbach resonance involving dynamically bound states in side system
- Resonances if  $\epsilon = \epsilon_{\text{bound}}^{(m)}$
- For small  $a_1$ :  
$$\epsilon_{\text{bound}}^{(m)} \approx -E_D + |m|\hbar\omega + C^{(m)}a_1^2$$
- No losses through inelastic scattering at resonance position



## Influence of Higher Fourier Modes

- Drive with 2nd harmonic:

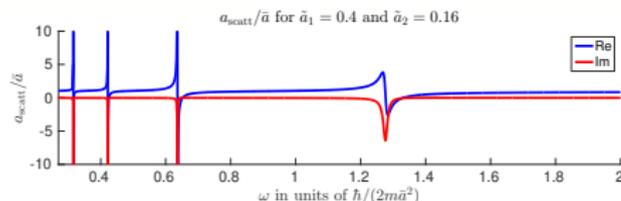
$$a(t) = \bar{a} + a_1 \cos(\omega t) + a_2 \cos(2\omega t)$$

- Recursion relation:

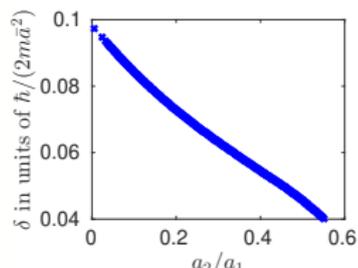
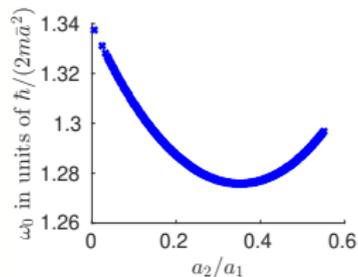
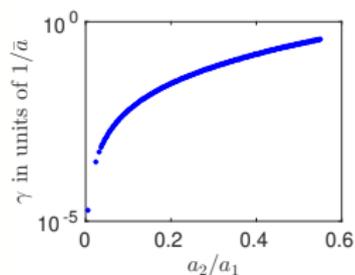
$$\left(\frac{i}{k_n \bar{a}} - 1\right) D_n - \sum_{m=\pm 1 \pm 2} \frac{a_m}{2\bar{a}} D_{n-m} = h_n$$

- Scattering length near resonance:

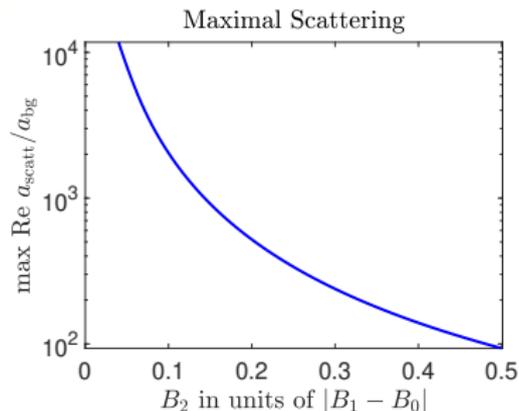
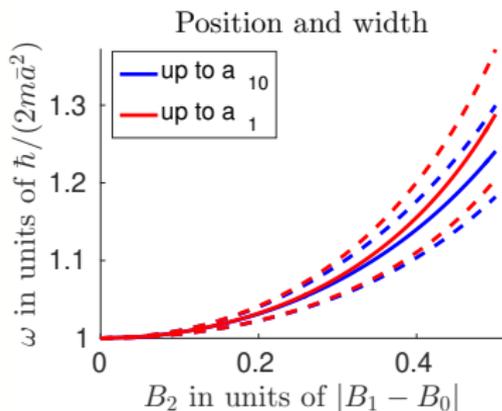
$$\frac{1}{a_{\text{scatt}}(\omega)} \approx \frac{1}{a_{\text{BG}}} \frac{\omega - \omega_0}{\omega - \omega_0 - \delta/\hbar} + i\gamma$$



- Finite maximal enhancement of scattering  
 $\max \text{Re } \tilde{a}_{\text{scatt}} = 1/(2\gamma)$
- Resonance position shifts
- Width of resonance decreases

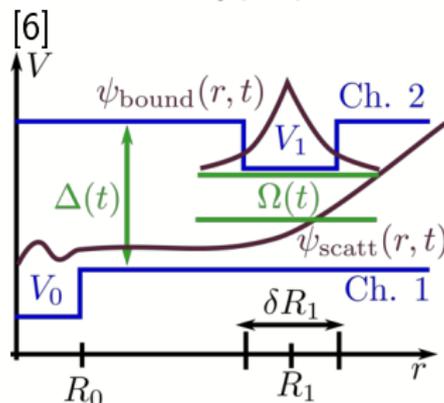
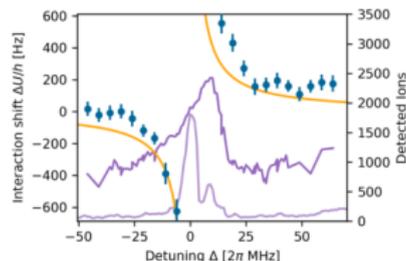


- Harmonically modulated  $B$ -field:  $a(t) = a_{\text{bg}} \left( 1 - \frac{\Delta}{B_2 \cos(\omega t) + B_1 - B_0} \right)$
- Drive with multiple Fourier coefficients  $a_n$
- $^{133}\text{Cs}$ ,  $B_0 = 547 \text{ G}$ ,  $\Delta = 7.5 \text{ G}$  [4]
- $\hbar/(2m\bar{a}^2) \approx 3 \text{ kHz}$



## Outlook: Two Channel Models

- Add internal degrees of freedom
 
$$\left[ \Delta_r + k_n^2 - \frac{l(l+1)}{r^2} - \underline{v}_0(r) \right] \mathbf{R}_{l,n}(r) = \sum_{m \neq 0} \underline{v}_m(r) \mathbf{R}_{l,n-m}(r)$$
- Model potential:  $\underline{v}(r, t) = \begin{pmatrix} -V_0 \theta(R_0 - r) & \Omega(t) \\ \Omega(t) & \Delta(t) - V_1 f_1(r) \end{pmatrix}$
- Magnetic Feshbach resonance [5]
  - $f_1(r) = \Theta(R_1 - r)$
  - $\Delta(t) = \Delta \mu B(t)$
  - $\Omega(t) = \text{const.}$  hyperfine-interaction
- Rydberg optical Feshbach resonance [6]
  - $f_1(r) = \begin{cases} 1, & -\delta R_1/2 < r + R_1 < +\delta R_1/2 \\ 0, & \text{otherwise} \end{cases}$
  - $\Delta(t)$  detuning
  - $\Omega(t)$  Rabi-frequency



[5] R. Duine et al. Phys. Rep. 396, 115 (2004)  
 [6] O. Thomas et al. Nat. Comm. 9, 2238 (2018)

# Bad Honnef Physics School on Methods of Path Integration in Modern Physics

organized by Stefan Kirchner and Axel Pelster

Bad Honnef (Germany); August 25 – 31, 2019

## Speakers and Topics:

Lawrence Schulman (Potsdam, USA): *Quantum Mechanics*

Andreas Wipf (Jena, Germany): *Statistical Field Theory*

Carlos Sa de Melo (Atlanta, USA): *Many-body Theory, BEC-BCS Crossover*

Jean Zinn-Justin (Paris, France): *Quantum Field Theory, Large-N Technique*

Victor Dotsenko (Paris, France): *Random Matrix Theory, Replica Trick*

Steve Simon (Oxford, UK): *Wilson Loops Spin, Topology, Holonomy Group*

Wolfhard Janke (Leipzig, Germany): *Quantum Monte Carlo*

Hagen Kleinert (Berlin, Germany): *Vortices and GIMPs*

<https://www.dpg-physik.de/veranstaltungen/2019>