Dipolar Quantum Gases

Aristeu Lima

- Bosons: Gross-Pitaevskii Theory
- Fermions: Collective Motion in the Normal Phase

Physical Motivation

- Dipole-Dipole Interaction (DDI) potential

$$V_{\mathrm{dd}}(\mathbf{x}) = \frac{C_{\mathrm{dd}}}{4\pi|\mathbf{x}|^3}\left[1 - 3\frac{z^2}{|\mathbf{x}|^2}\right]$$

- Magnetic systems: $C_{\mathrm{dd}} = \mu_0 m^2$, with $m \sim 10\ \mu_B$

Boson: ^{52}Cr A. Griesmaier et al., PRL **94**, 160401 (2005)
Fermion: ^{53}Cr R. Chicireanu et al., PRA **73**, 053406 (2006)
Both: Dy M. Lu et al., PRL **104**, 063001 (2010)

- Electric systems: $C_{\mathrm{dd}} = 4\pi d^2$, with $d \sim 1$ Debye

Fermion: ^{40}K^{87}Rb S. Ospelkaus et al., Science **32**, 231 (2008)
Boson: ^{41}K^{87}Rb K. Aikawa et al., NJP **11**, 055035 (2009)

Part I: Dipolar Bose Gases

- Mean-field: Gross-Pitaevskii Theory
- Time-of-flight Expansion: Experimental Confirmation in ^{52}Cr
- Finite Temperatures: Anisotropic Shift of Critical Temperature

Field Equations

- Hamilton Operator

$$\hat{H} = \int \mathrm{d}^3x\, \hat{\Psi}^\dagger(\mathbf{x},t) \left[-\frac{\hbar^2\nabla^2}{2M} + U_{\mathrm{trap}}(\mathbf{x}) + \frac{1}{2}\int \mathrm{d}^3x'\, \hat{\Psi}^\dagger(\mathbf{x}',t) V_{\mathrm{int}}\left(\mathbf{x}-\mathbf{x}'\right) \hat{\Psi}(\mathbf{x}',t) \right] \hat{\Psi}(\mathbf{x},t).$$

- Equal Time Commutation Relations

$$\left[\hat{\Psi}(\mathbf{x},t), \hat{\Psi}^\dagger(\mathbf{x}',t)\right] = \delta(\mathbf{x}-\mathbf{x}'), \quad \left[\hat{\Psi}^\dagger(\mathbf{x},t), \hat{\Psi}^\dagger(\mathbf{x}',t)\right] = 0, \quad \left[\hat{\Psi}(\mathbf{x},t), \hat{\Psi}(\mathbf{x}',t)\right] = 0.$$

- Heisenberg Equation

$$i\hbar\frac{\partial}{\partial t}\hat{\Psi}(\mathbf{x},t) = \left[-\frac{\hbar^2\nabla^2}{2M} + U_{\mathrm{trap}}(\mathbf{x}) + \int \mathrm{d}^3x'\, \hat{\Psi}^\dagger(\mathbf{x}',t) V_{\mathrm{int}}\left(\mathbf{x}-\mathbf{x}'\right) \hat{\Psi}(\mathbf{x}',t) \right] \hat{\Psi}(\mathbf{x},t).$$

- Role of 1-p Ground State $\hat{\Psi}(\mathbf{x},t) = \hat{a}_0(t)\phi_0(\mathbf{x}) + \sum_\nu{}' \hat{a}_\nu(t)\phi_\nu(\mathbf{x})$

- Bogoliubov Prescription $\hat{\Psi}(\mathbf{x},t) = \Psi(\mathbf{x},t) + \delta\hat{\psi}(\mathbf{x},t)$

- Gross-Pitaevskii Equation ($\delta\hat{\psi}(\mathbf{x},t) = 0$)

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{x},t) = \left[-\frac{\hbar^2\nabla^2}{2M} + U_{\mathrm{trap}}(\mathbf{x}) + \int \mathrm{d}^3x'\, \Psi^*(\mathbf{x}',t) V_{\mathrm{int}}\left(\mathbf{x}-\mathbf{x}'\right) \Psi(\mathbf{x}',t) \right] \Psi(\mathbf{x},t)$$

Classical Field Theory

- Action Principle $\delta\mathcal{A}[\Psi,\Psi^*]=0$

with
$$\mathcal{A}[\Psi,\Psi^*]=\int \mathrm{d}^3x\int_{t_1}^{t_2}\mathrm{d}t\,\Psi^*(\mathbf{x},t)\left[i\hbar\frac{\partial}{\partial t}-H(\mathbf{x},t)\right]\Psi(\mathbf{x},t)$$

- Hamiltonian
$$H(\mathbf{x},t)=-\frac{\hbar^2\nabla^2}{2M}+U_{\mathrm{trap}}(\mathbf{x})+\frac{1}{2}\int \mathrm{d}^3x'\,V_{\mathrm{int}}(\mathbf{x}-\mathbf{x}')|\Psi(\mathbf{x}',t)|^2$$

- Phase Factorization $\Psi(\mathbf{x},t)=e^{iM\chi(\mathbf{x},t)/\hbar}\sqrt{n(\mathbf{x},t)}$
- New Action

$$\mathcal{A}[n,\chi]=-M\int_{t_1}^{t_2}\mathrm{d}t\int \mathrm{d}^3x\,\sqrt{n(\mathbf{x},t)}\left[\dot{\chi}(\mathbf{x},t)+\frac{1}{2}\nabla\chi(\mathbf{x},t)\cdot\nabla\chi(\mathbf{x},t)+H_0(\mathbf{x},t)\right]\sqrt{n(\mathbf{x},t)}$$

- Thomas-Fermi Hamiltonian
$$H_0(\mathbf{x},t)=U_{\mathrm{trap}}(\mathbf{x})+\frac{1}{2}\int \mathrm{d}^3x'\,V_{\mathrm{int}}(\mathbf{x}-\mathbf{x}')n(\mathbf{x}',t)$$

- Interaction (Dipoles along the z axis)
$$V_{\mathrm{int}}(\mathbf{x})=g\delta(\mathbf{x})+\frac{C_{\mathrm{dd}}}{4\pi|\mathbf{x}|^3}\left[1-3\frac{z^2}{|\mathbf{x}|^2}\right];\quad g=\frac{4\pi\hbar^2a_s}{M}$$

Variational Approach

- Ansatz

$$n(\mathbf{x},t) = n_0(t)\left(1-\frac{x^2}{R_x^2(t)}-\frac{y^2}{R_y^2(t)}-\frac{z^2}{R_z^2(t)}\right); \quad n_0(t)=\frac{15N}{8\pi R_x(t)R_y(t)R_z(t)}$$

$$\chi(\mathbf{x},t) = \frac{1}{2}\alpha_x(t)x^2+\frac{1}{2}\alpha_y(t)y^2+\frac{1}{2}\alpha_z(t)z^2 \star$$

- Flow Energy

$$E_{\text{flow}}(t)=\frac{M}{2}\int \mathrm{d}^3x\,\nabla\chi(\mathbf{x},t)\cdot\nabla\chi(\mathbf{x},t)=\frac{MN}{14}\left[\alpha_x^2(t)R_x^2(t)+\alpha_y^2(t)R_y^2(t)+\alpha_z^2(t)R_z^2(t)\right]$$

- Trapping Energy

$$E_{\text{trap}}(t)=\int \mathrm{d}^3x\,n(\mathbf{x},t)U_{\text{trap}}(\mathbf{x})=\frac{MN}{14}\left(\omega_x^2R_x^2(t)+\omega_y^2R_y^2(t)+\omega_z^2R_z^2(t)\right)$$

- Contact Interaction

$$E_\delta(t)=\frac{1}{2}\int \mathrm{d}^3x\int \mathrm{d}^3x'\,n(\mathbf{x},t)g\delta(\mathbf{x}-\mathbf{x}')n(\mathbf{x}',t)=\frac{15gN^2}{28\pi R_x(t)R_y(t)R_z(t)}$$

$\star$ V. M- Perez-Garcia et al PRL **77**, 5320 (1996)

Variational Approach

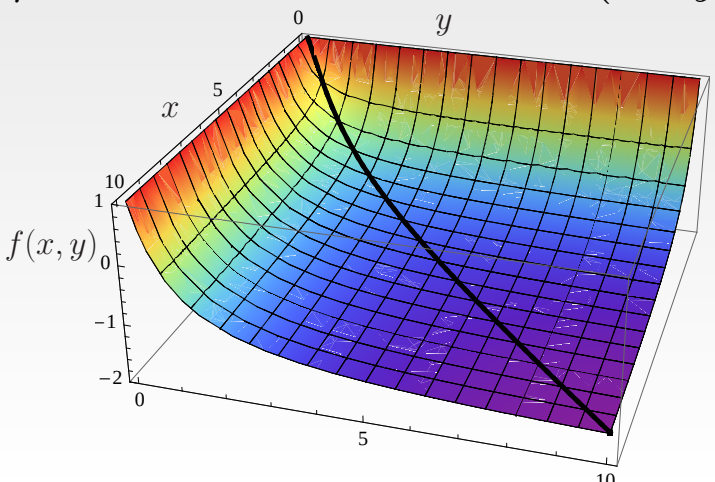
- Dipole-Dipole Interaction

$$
\begin{aligned}
E_{\mathrm{dd}}(t) &= \frac{1}{2}\int \mathrm{d}^3x \int \mathrm{d}^3x'\, n(\mathbf{x},t) V_{\mathrm{dd}}(\mathbf{x}-\mathbf{x}') n(\mathbf{x}',t) \\
&= \frac{1}{2}\int \frac{\mathrm{d}^3k}{(2\pi)^3} \tilde{n}(\mathbf{k},t)\tilde{V}_{\mathrm{dd}}(\mathbf{k})\tilde{n}(-\mathbf{k},t) \\
&= -\frac{15 g \epsilon_{\mathrm{dd}} N^2}{28\pi R_x(t) R_y(t) R_z(t)} f\left(\frac{R_x(t)}{R_z(t)}, \frac{R_y(t)}{R_z(t)}\right)
\end{aligned}
$$

with

$$
f(x,y) = 1 + 3xy \frac{E(\varphi,k) - F(\varphi,k)}{(1-y^2)\sqrt{1-x^2}}
$$

$F(\varphi,k)$ and $E(\varphi,k)$ elliptic integrals; $\epsilon_{\mathrm{dd}} = C_{\mathrm{dd}}/3g$
$\varphi = \arcsin\sqrt{1-x^2}$ and $k^2 = (1-y^2)/(1-x^2)$



Equations of Motion

- Phase Parameters $\alpha_i(t) = \frac{\dot{R}_i(t)}{R_i(t)}$

- Thomas-Fermi Radii

$$\frac{NM}{7}\ddot{R}_i(t) = -\frac{\partial}{\partial R_i}E_{\text{total}}\left(R_x, R_y, R_z\right) \quad \star$$

with $E_{\text{total}} = E_{\text{trap}} + E_\delta + E_{\text{dd}}$

- Cylinder Symmetric Case
  $(\omega_x = \omega_y = \omega_\rho, R_x = R_y = R_\rho, f_s(x) = f(x,x))$ $*$

$$\ddot{R}_\rho(t) = -\omega_\rho^2 R_\rho(t) + \frac{15gN}{4\pi M R_\rho(t)^3 R_z(t)}\left\{1 - \epsilon_{\text{dd}}\left[1 + \frac{3}{2}\frac{R_\rho^2(t) f_s(R_\rho(t)/R_z(t))}{R_\rho^2(t) - R_z^2(t)}\right]\right\},$$

$$\ddot{R}_z(t) = -\omega_z^2 R_z(t) + \frac{15gN}{4\pi M R_\rho^2(t) R_z^2(t)}\left\{1 + 2\epsilon_{\text{dd}}\left[1 + \frac{3}{2}\frac{R_z^2(t) f_s(R_\rho(t)/R_z(t))}{R_\rho^2(t) - R_z^2(t)}\right]\right\}.$$

$\star$ S. Giovanazzi et al. PRA **74**, 013621 (2006)

$*$ Duncan H. J. O'Dell et al. PRL **92**, 250401(2004)

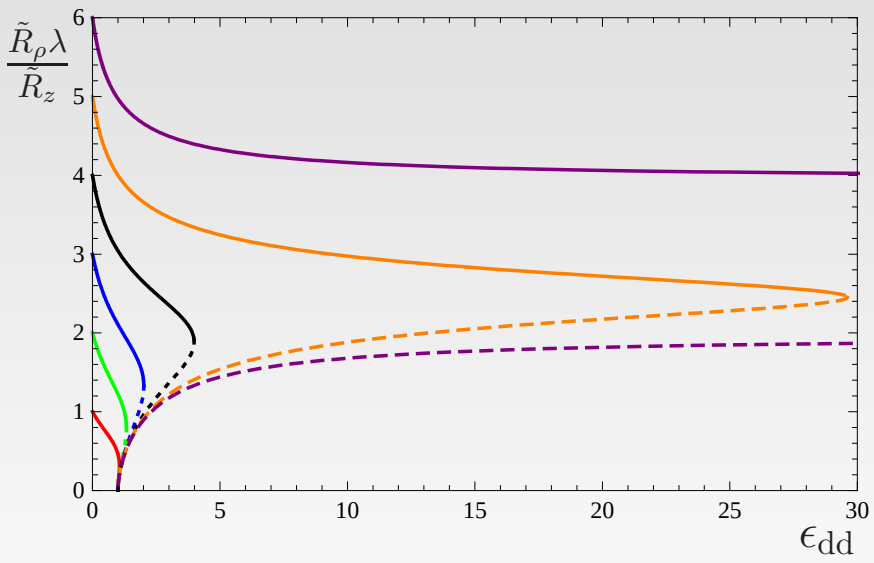
Static Properties

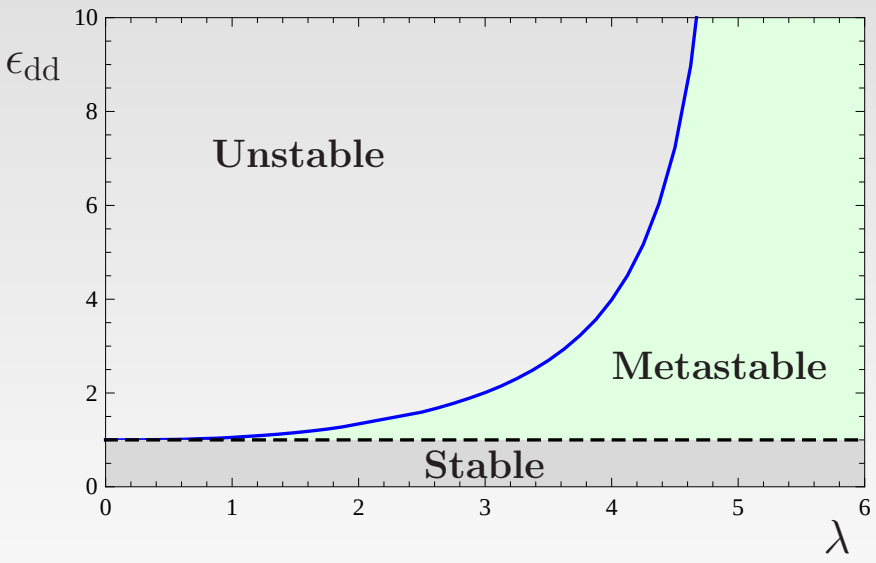
- Dimensionless Units $\tilde{R}_i = \dfrac{R_i}{R_i^{(0)}}.$

with

$$R_i^{(0)} = \left(\frac{2\mu^{(0)}}{M\omega_i^2}\right)^{1/2}, \qquad \mu^{(0)} = gn_0$$

- Aspect Ratio and Stability Diagram $\ddot{R}_\rho(t) = \ddot{R}_z(t) = 0$ $(\lambda = \omega_\rho/\omega_z)$





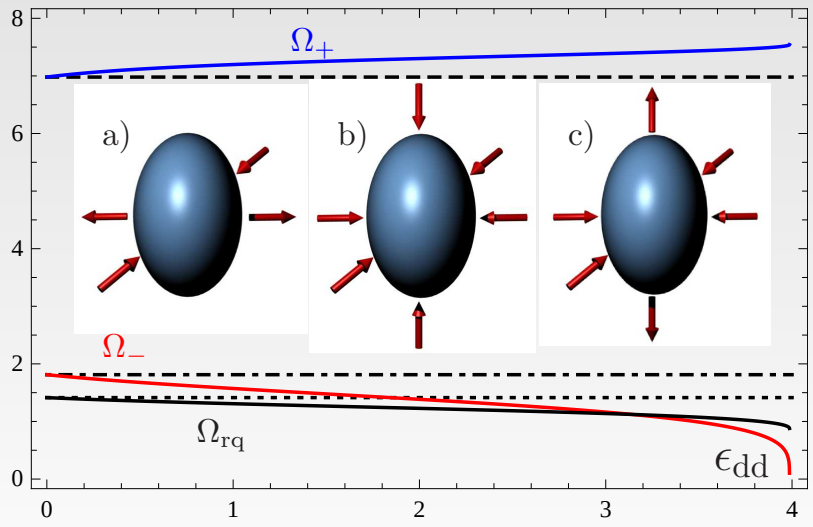
Hydrodynamic Excitations

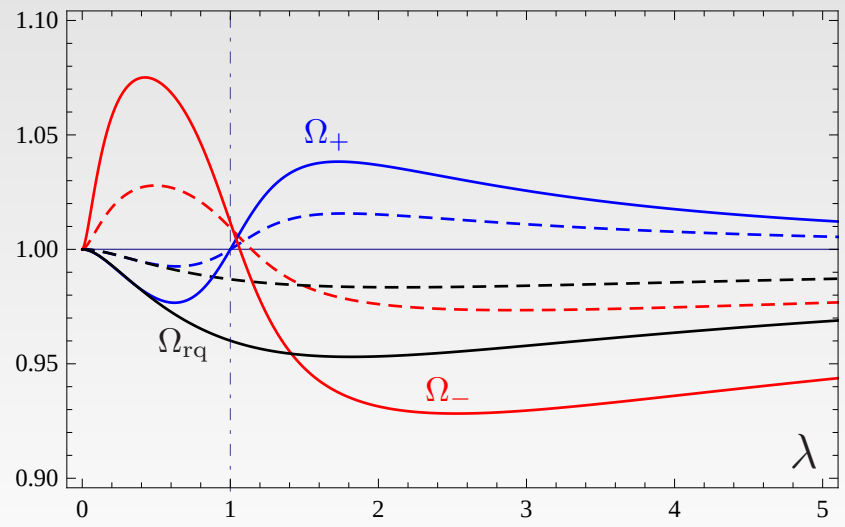
- Oscillations around Equilibrium $R_i(t) = R_i^{\mathrm{eq}}(0) + \eta_i e^{i\Omega t}$
- Eigenvalue Problem

$$O_{ij}\eta_j = \Omega^2 \eta_j$$

with

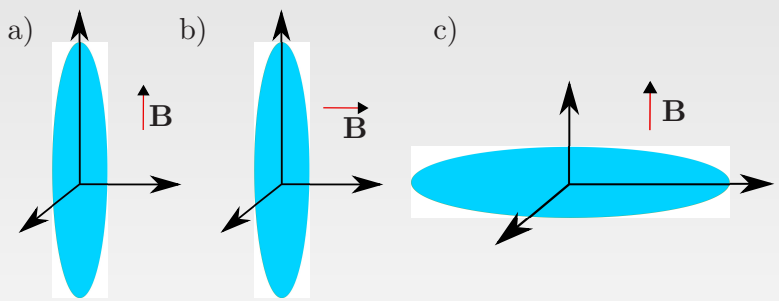
$$O_{ij} = \frac{7}{NM}\frac{\partial^2}{\partial R_i \partial R_j} E_{\mathrm{tot}}\left(R_x, R_y, R_z\right)\Big|_{\sum_k R_k = R_k^{\mathrm{eq}}(0)}$$

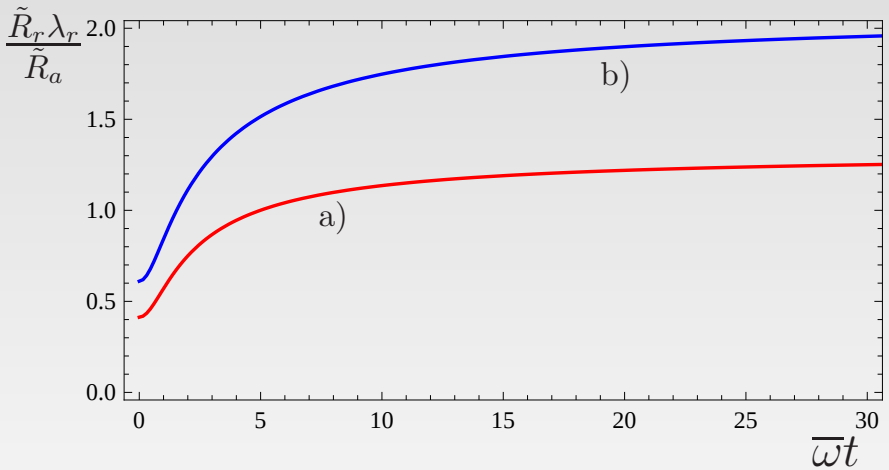




Time-of-Flight for Dy $\epsilon_{\rm dd} \approx 0.42$

- Set ω_i to zero in the Eqs. of motion
- Take equilibrium values as initial condition
- a)$\lambda_x = \lambda_y = 0.5$ and $\mathbf{B} = B\hat{z}$
- b)$\lambda_x = \lambda_y = 0.5$, but with $\mathbf{B} = B\hat{y}$ is equivalent to c)
- c)$\lambda_x = 1$, $\lambda_y = 2$, with $\mathbf{B} = B\hat{z}$





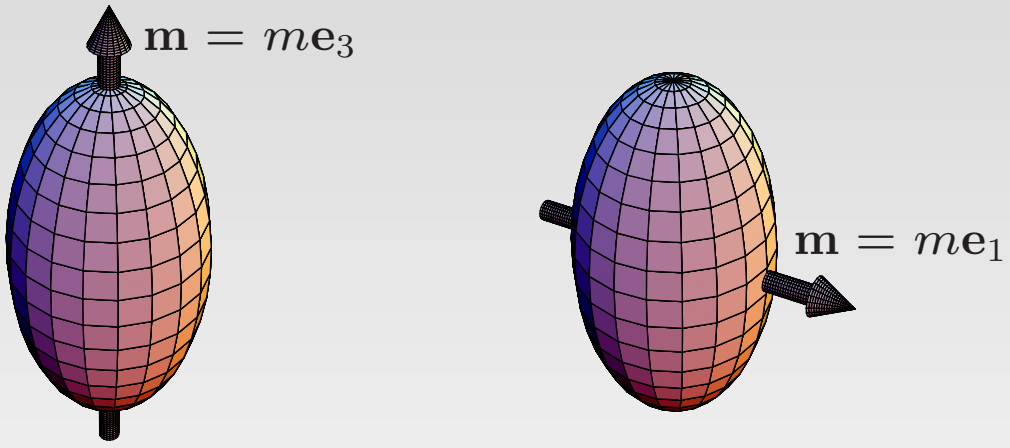
Theory: S. Giovanazzi et al. PRA **74**, 013621 (2006)

Experiment: T. Lahaye et al. Nature **448**, 672 (2007)

Finite Temperature

T_c-Shift in ^{52}Cr

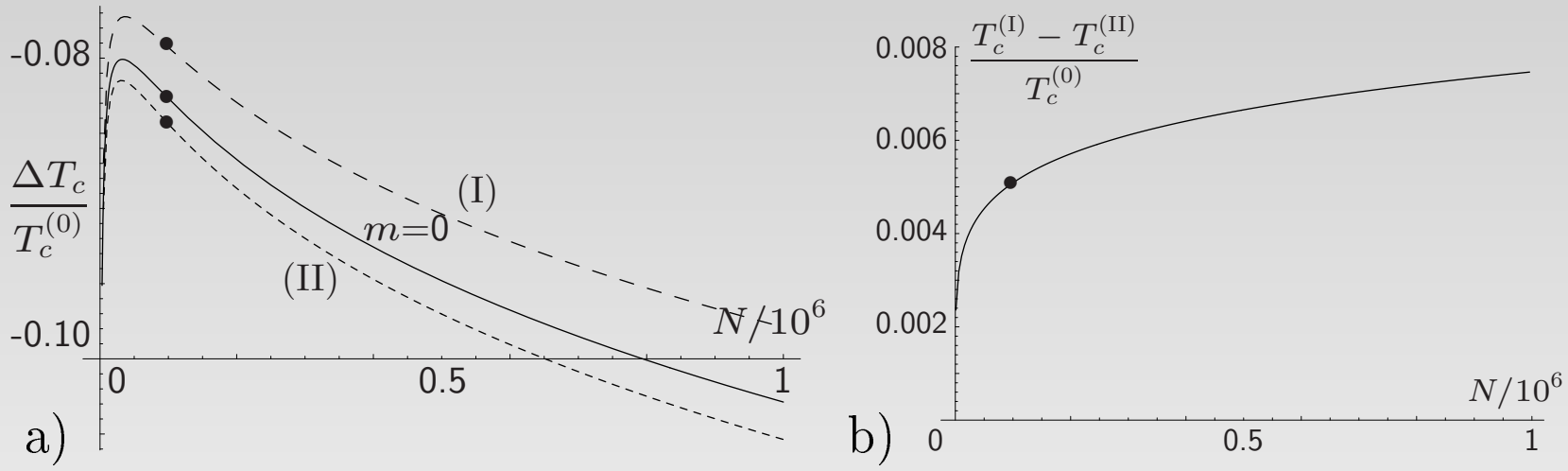
- Trap frequencies obey $\omega_x = \omega_y \neq \omega_z$
- Configuration of the magnetization geometry



case I case II

Finite Temperature

T_c-Shift in ^{52}Cr



K. Glaum et al. PRL **98**, 080407 (2007)

Part II: Dipolar Fermi Gases

- Semiclassical Hartree-Fock theory
- Equilibrium configuration
- Low-lying excitations
- Time-of-flight expansion
- Outlook

Hydrodynamic x collisionless

- Phase-space distribution $f(\mathbf{x}, \mathbf{k}, t)$ of particles in presence of mean-field potential $U(\mathbf{x}, \mathbf{k}, t)$ obeys Boltzmann-Vlasov eq.

$$\frac{\partial f}{\partial t} + \left(\frac{\hbar \mathbf{k}}{M} + \frac{1}{\hbar}\frac{\partial U}{\partial \mathbf{k}}\right) \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{1}{\hbar}\frac{\partial U}{\partial \mathbf{x}} \cdot \frac{\partial f}{\partial \mathbf{k}} = I_{\mathrm{Coll}}[f]$$

- Relaxation time approxation:
  $I_{\mathrm{Coll}}[f] = -\frac{f - f_{\mathrm{le}}}{\tau_R}$ with f rescaled
  le stands for *local equilibrium* and τ_R is the relaxation time

- $I_{\mathrm{Coll}}[f]$ vanishes in
  - Collisionless regime because $\overline{\omega}\tau_R \to \infty$
  - Hydrodynamic regime because $f = f_{\mathrm{le}}$, which means $\overline{\omega}\tau_R \to 0$

- Thus, the problem simplifies for $\overline{\omega}\tau_R \gg 1$ (collisionless regime) and $\overline{\omega}\tau_R \ll 1$ (hydrodynamic regime)

Estimative of τ_R

- Boltzmann-Vlasov eq. for DDI remains very hard to solve, due to anisotropy
- Approximation of the DDI through an effective contact interaction with scattering length $a_{\rm dd} = MC_{\rm dd}/(4\pi\hbar^2)$ gives $\frac{1}{\overline{\omega}\tau_R} = (N^{1/3} a_{\rm dd}\sqrt{M\overline{\omega}/\hbar})^2 F(T/T_{\rm F})$, with $F(T/T_{\rm F}) \sim 0.1$ in quantum regime (L.Vichi and S.Stringari PRA, **60**, 4734 (1999))
- Thus, for $N = 4 \times 10^4$ dipoles, one obtains
  - $\overline{\omega}\tau_R \approx 0.01$ for KRb ($d = .57\ D$) and $\overline{\omega}\tau_R \approx 2 \times 10^{-7}$ for LiCs ($d = 5.53\ D$) polar molecules
  - $\overline{\omega}\tau_R \approx 75$ for ^{53}Cr ($m = 6\ \mu_B$) and $\overline{\omega}\tau_R \approx 4 \times 10^3$ for ^{163}Dy ($m = 10\ \mu_B$) atoms

Variational Hartree-Fock

- Action $\mathcal{A} = \int_{t_1}^{t_2} \mathrm{d}t \langle \Psi | i\hbar \frac{\partial}{\partial t} - H | \Psi \rangle$ Slater determinant
- Common-phase factorization $\psi_i(x,t) = e^{iM\chi(x,t)/\hbar} |\psi_i(x,t)|$
- Velocity field $\mathbf{v} = \nabla \chi(x,t)$
- Time-even Slater determinant
  $\Psi_0(x_1, \cdots, x_N, t) = \mathrm{SD}\left[|\psi(x,t)|\right]$
- Time-even one-body density matrix
  $\rho_0(x, x'; t) = \prod_{i=2}^{N} \int \mathrm{d}^3 x_i \Psi_0^*(x', \cdots, x_N, t) \Psi_0(x, \cdots, x_N, t)$
- Particle density $\rho_0(x;t) = \rho_0(x,x;t)$

Variational Hartree-Fock

- New action Momentum density Flow energy

$$\mathcal{A} = -M\int \mathrm{d}t\int \mathrm{d}^3x\left\{\dot{\chi}(x,t)\rho_0(x;t) + \frac{\rho_0(x;t)}{2}\left[\nabla\chi(x,t)\right]^2\right\}$$
$$-\int \mathrm{d}t\langle\Psi_0|H|\Psi_0\rangle$$

- Energy contributions

$$\langle\Psi_0|H|\Psi_0\rangle = \langle\Psi_0|H_{\mathrm{kin}}|\Psi_0\rangle + \langle\Psi_0|H_{\mathrm{tr}}|\Psi_0\rangle + \langle\Psi_0|H_{\mathrm{int}}|\Psi_0\rangle$$

- Decomposition of $\langle\Psi_0|H_{\mathrm{int}}|\Psi_0\rangle$ into direct and exchange

$$E^{\mathrm{Dir}} = \frac{1}{2}\int \mathrm{d}^3x\mathrm{d}^3x' V_{\mathrm{int}}(x,x')\rho_0(x,x;t)\rho_0(x',x';t)$$
$$E^{\mathrm{Ex}} = -\frac{1}{2}\int \mathrm{d}^3x\mathrm{d}^3x' V_{\mathrm{int}}(x,x')\rho_0(x,x';t)\rho_0(x',x;t)$$

Variational Hartree-Fock

- Equations of motion

$$\frac{\delta \mathcal{A}}{\delta \chi(x,t)} = 0, \qquad \frac{\delta \mathcal{A}}{\delta \rho_0(x,x';t)} = 0$$

- If $\langle \Psi_0 | H | \Psi_0 \rangle$ is a functional of the density $\rho_0(x;t)$ alone, one obtains the continuity and Euler equations
- If not, other approaches are needed, like in DFT
- In this work, we switch to Wigner space

$$\begin{aligned} \nu_0\left(\mathbf{x},\mathbf{k};t\right) &= \int \mathrm{d}^3 s\, \rho_0\left(\mathbf{x}+\frac{\mathbf{s}}{2},\mathbf{x}-\frac{\mathbf{s}}{2};t\right)\, e^{-i\mathbf{k}\cdot\mathbf{s}} \\ \rho_0(\mathbf{x},\mathbf{x}';t) &= \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\, \nu_0\left(\frac{\mathbf{x}+\mathbf{x}'}{2},\mathbf{k};t\right) e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \end{aligned}$$

Variational Hartree-Fock

- Particle density $\rho_0(\mathbf{x};t) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\, \nu_0\left(\mathbf{x},\mathbf{k};t\right)$
- Momentum distribution $\rho_0(\mathbf{k};t) = \int \frac{\mathrm{d}^3x}{(2\pi)^3}\, \nu_0\left(\mathbf{x},\mathbf{k};t\right)$
- Trapping and kinetic energies $U_{\mathrm{tr}}(\mathbf{x}) = \frac{M}{2}\left(x^2\omega_x^2 + y^2\omega_y^2 + z^2\omega_z^2\right)$

$$
\begin{aligned}
E_{\mathrm{tr}} &= \int \frac{\mathrm{d}^3x\mathrm{d}^3k}{(2\pi)^3}\, \nu_0\left(\mathbf{x},\mathbf{k};t\right) U_{\mathrm{tr}}(\mathbf{x}) \\
E_{\mathrm{kin}} &= \int \frac{\mathrm{d}^3x\mathrm{d}^3k}{(2\pi)^3}\, \nu_0\left(\mathbf{x},\mathbf{k};t\right) \frac{\hbar^2\mathbf{k}^2}{2M}
\end{aligned}
$$

- Hartree

$$
E_{\mathrm{dd}}^{\mathrm{Dir}} = \int \frac{\mathrm{d}^3x\mathrm{d}^3k\mathrm{d}^3x'\mathrm{d}^3k'}{2(2\pi)^6} \nu_0(\mathbf{x},\mathbf{k};t) V_{\mathrm{dd}}(\mathbf{x}-\mathbf{x}') \nu_0(\mathbf{x}',\mathbf{k}';t)
$$

- Fock

$$
E_{\mathrm{dd}}^{\mathrm{Ex}} = -\int \frac{\mathrm{d}^3X\mathrm{d}^3k\mathrm{d}^3s\mathrm{d}^3k'}{2(2\pi)^6} \nu_0(\mathbf{X},\mathbf{k};t) V_{\mathrm{dd}}(\mathbf{s}) \nu_0(\mathbf{X},\mathbf{k}';t)\, e^{i\mathbf{s}\cdot(\mathbf{k}-\mathbf{k}')}
$$

Equations of motion

- Ansatz

$$\chi(x,t) = \frac{1}{2}\left[\alpha_x(t)x^2 + \alpha_y(t)y^2 + \alpha_z(t)z^2\right]$$

$$\nu_0(\mathbf{x},\mathbf{k};t) = \Theta\left(1 - \sum_i \frac{x_i^2}{R_i(t)^2} - \sum_i \frac{k_i^2}{K_i(t)^2}\right)$$

- Action (Energy)

$$A = -\int_{t_1}^{t_2} \mathrm{d}t \frac{\overline{R}^3\overline{K}^3}{3\cdot 2^7}\left\{\frac{M}{2}\sum_i\left[\dot{\alpha}_i R_i^2 + \alpha_i^2 R_i^2 + \omega_i^2 R_i^2\right] + \sum_i \frac{\hbar^2 K_i^2}{2M}\right.$$

$$\left. -c_0\overline{K}^3\left[f\left(\frac{R_x}{R_z},\frac{R_y}{R_z}\right) - f\left(\frac{K_z}{K_x},\frac{K_z}{K_y}\right)\right]\right\} - \int_{t_1}^{t_2}\mathrm{d}t\,\mu(t)\left(\frac{\overline{R}^3\overline{K}^3}{48} - N\right)$$

with $c_0 = \frac{2^{10}C_{\mathrm{dd}}}{3^4\cdot 5\cdot 7\cdot \pi^3} \approx 0.0116\, C_{\mathrm{dd}}$ and $\overline{\bullet} = (\bullet_x\, \bullet_y\, \bullet_z)^{\frac{1}{3}}$

Equations of motion (unitless)

- Auxiliary equations of motion $\alpha_i = \dot{R}_i / R_i$
- Particle conservation $\overline{R}^3 \overline{K}^3 = 1$
- Momentum deformation* $K_z^2 - K_x^2 = \frac{3c_3\epsilon_{\rm dd}}{\overline{R}^3}\left[-1 + \frac{(2K_x^2+K_z^2)}{2(K_x^2-K_z^2)} f_s\left(\frac{K_z}{K_x}\right)\right]$

  with $c_3 = \frac{2^{\frac{38}{3}}}{3^{\frac{23}{6}}\cdot 5\cdot 7\cdot \pi^2} \approx 0.2791$ and $\epsilon_{\rm dd} = \frac{C_{\rm dd}}{4\pi}\left(\frac{M^3\overline{\omega}}{\hbar^5}\right)^{\frac{1}{2}} N^{\frac{1}{6}}$

- Finally $\frac{1}{\omega_i^2}\frac{d^2 R_i}{dt^2} = -R_i + \sum_j \frac{K_j^2}{3R_i} - \epsilon_{\rm dd} Q_i(R, K)$

$$Q_x(\mathbf{r},\mathbf{k}) = \frac{c_3}{x^2 y z}\left[f\left(\frac{x}{z},\frac{y}{z}\right) - \frac{x}{z} f_1\left(\frac{x}{z},\frac{y}{z}\right) - f_s\left(\frac{k_z}{k_x}\right)\right]$$

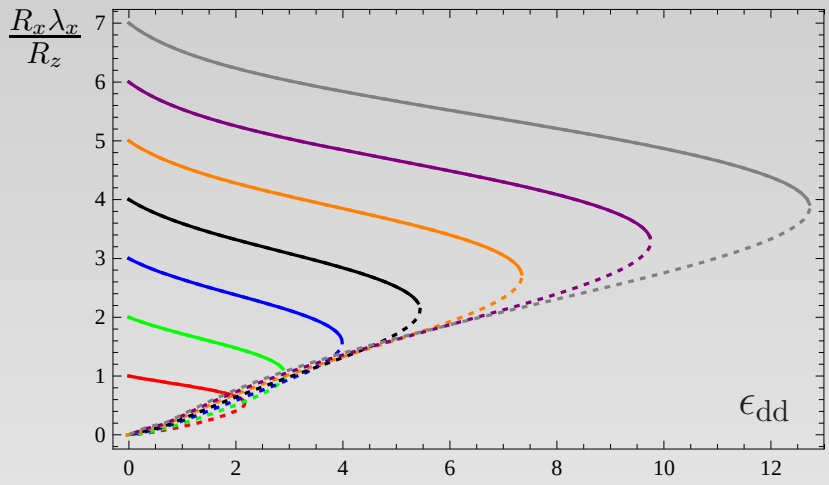
$$Q_y(\mathbf{r},\mathbf{k}) = \frac{c_3}{x y^2 z}\left[f\left(\frac{x}{z},\frac{y}{z}\right) - \frac{y}{z} f_2\left(\frac{x}{z},\frac{y}{z}\right) - f_s\left(\frac{k_z}{k_x}\right)\right]$$

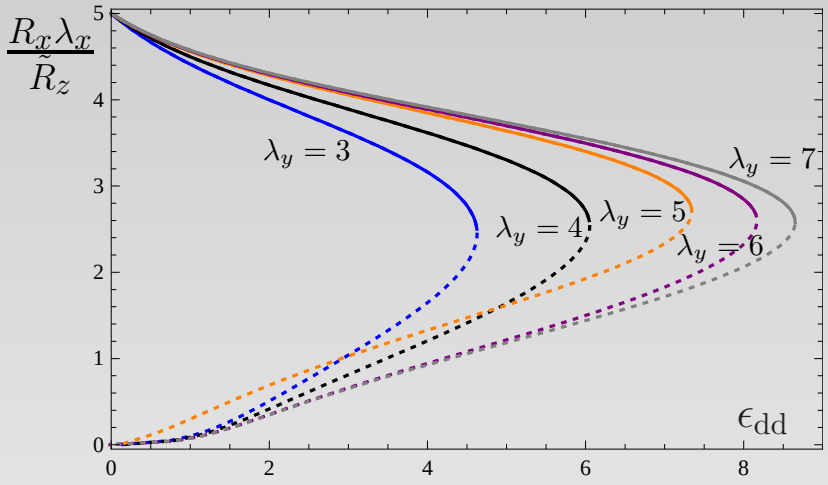
$$Q_z(\mathbf{r},\mathbf{k}) = \frac{c_3}{x y z^2}\left[f\left(\frac{x}{z},\frac{y}{z}\right) + \frac{x}{z} f_1\left(\frac{x}{z},\frac{y}{z}\right) + \frac{y}{z} f_2\left(\frac{x}{z},\frac{y}{z}\right) - f_s\left(\frac{k_z}{k_x}\right)\right]$$

*T. Miyakawa et al, PRA **77**, 061603(R) (2008)

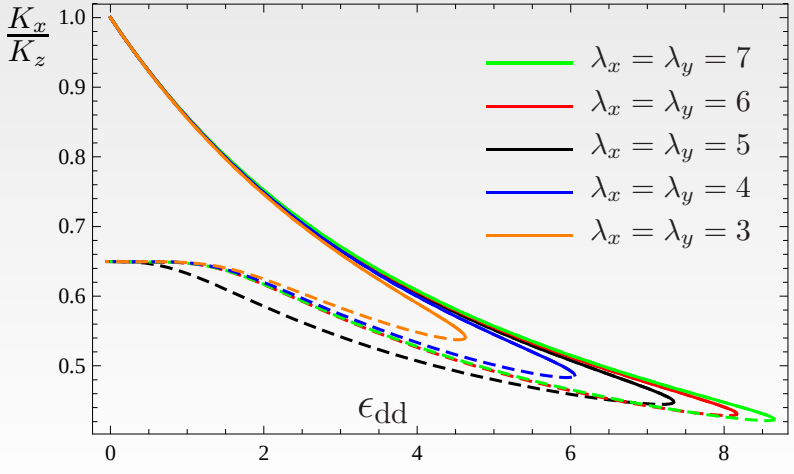
Static properties

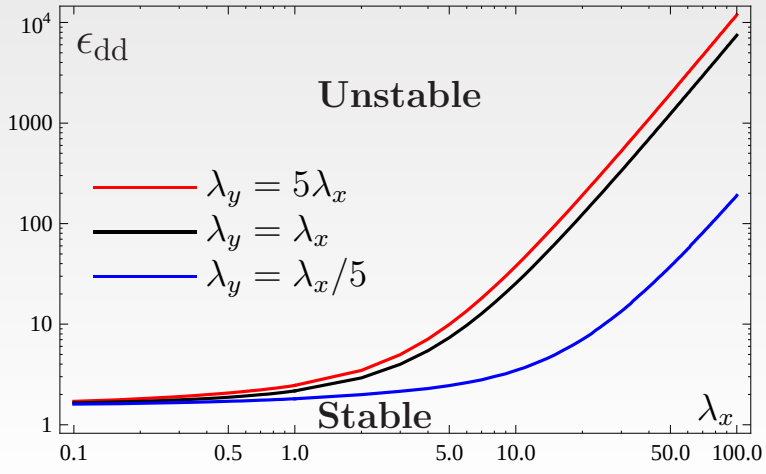
- Aspect ratio in real space





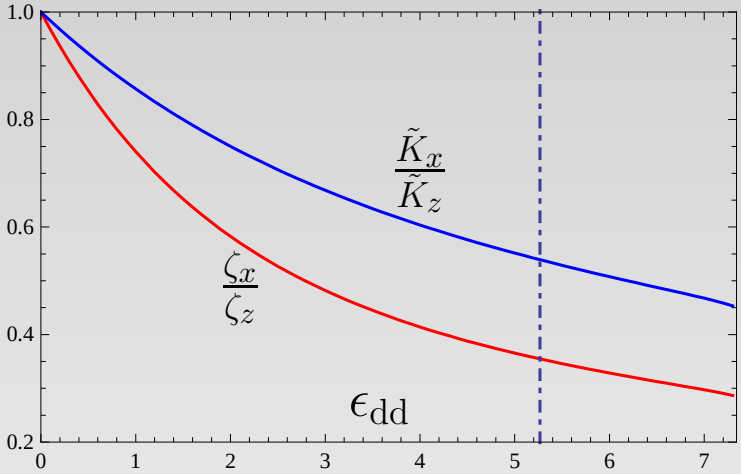
- Aspect ratio in momentum space and stability diagram



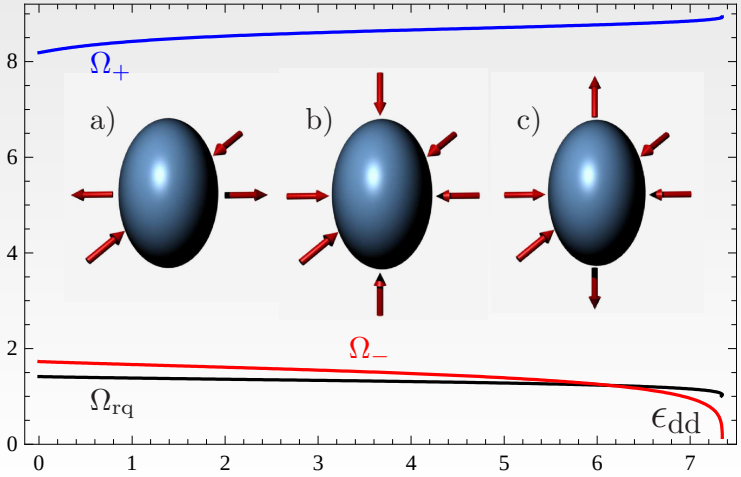


Low-lying excitations

- Linearization $R_i(t) = R_i(0) + \eta_i e^{i\Omega t}$; $K_i(t) = K_i(0) + \zeta_i e^{i\Omega t}$
- Momentum anisotropic breathing oscillations $\lambda_x = \lambda_y = 5$

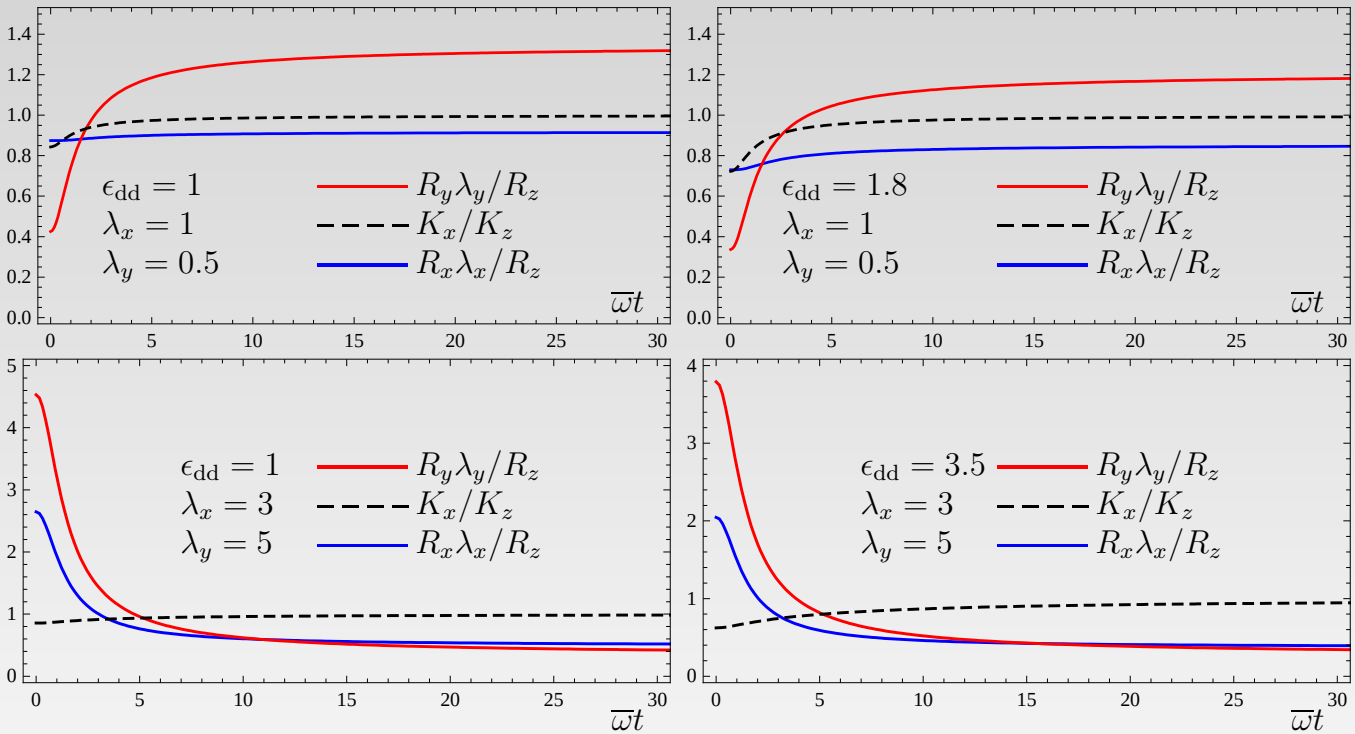


- Oscillation modes in real space $\lambda_x = \lambda_y = 5$



Time-of-flight expansion

- Numerically solve $\frac{1}{\omega_i^2}\frac{d^2R_i}{dt^2} = \sum_j \frac{K_j^2}{3R_i} - \epsilon_{dd} Q_i(R, K)$
- Expansion dynamics



Summary and outlook

We have studied a dipolar Fermi gas and considered

- Static properties (aspect ratios, symmetry of the momentum space, stability diagram, etc.)
- Low-lying excitations (Mono-, quadru- and radial quadrupole hydrodynamic modes)
- Time-of-flight dynamics
  A. Lima and A. Pelster PRA(R) **81**, 021606 (2010), PRA **81**, 063629 (2010)

Summary and outlook

We have studied a dipolar Fermi gas and considered

- Static properties (aspect ratios, symmetry of the momentum space, stability diagram, etc.)
- Low-lying excitations (Mono-, quadru- and radial quadrupole hydrodynamic modes)
- Time-of-flight dynamics
  A. Lima and A. Pelster PRA(R) **81**, 021606 (2010), PRA **81**, 063629 (2010)

We want to look at

- Detecting superfluidity (scissor mode?)
- Spin degrees of freedom (Ferromagnetism, Einstein-de-Haas)

Summary and outlook

We have studied a dipolar Fermi gas and considered

- Static properties (aspect ratios, symmetry of the momentum space, stability diagram, etc.)
- Low-lying excitations (Mono-, quadru- and radial quadrupole hydrodynamic modes)
- Time-of-flight dynamics
  A. Lima and A. Pelster PRA(R) **81**, 021606 (2010), PRA **81**, 063629 (2010)

We want to look at

- Detecting superfluidity (scissor mode?)
- Spin degrees of freedom (Ferromagnetism, Einstein-de-Haas)

THANK YOU