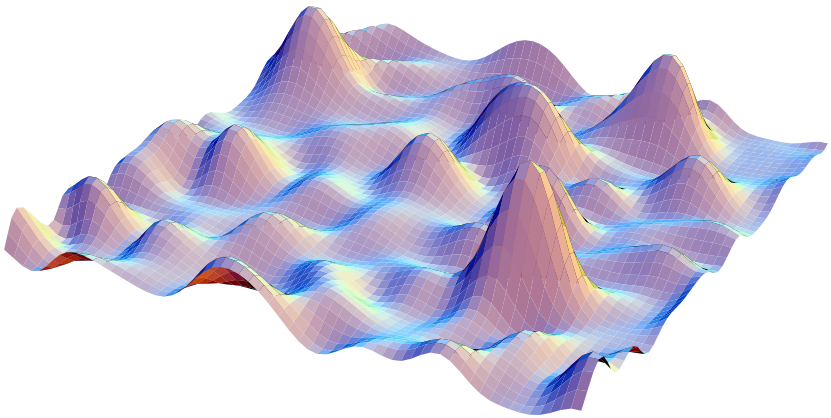
On the Dirty Boson Problem

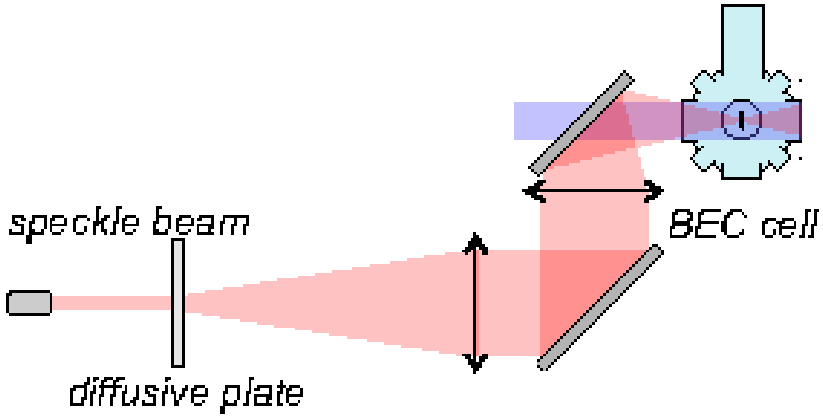
Axel Pelster

1. Experimental Realizations
2. Theoretical Description
3. Perturbative Results
4. Non-Perturbative Results
5. Outlook

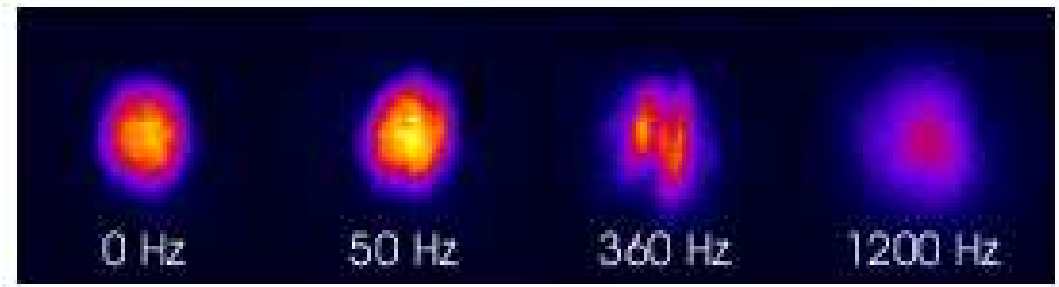


1.1 Laser Speckles: Controlled Randomness

Experimental Set-Up:

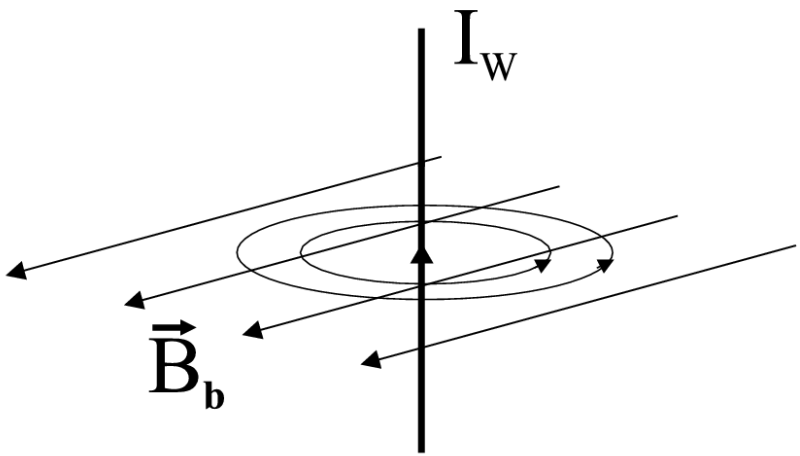


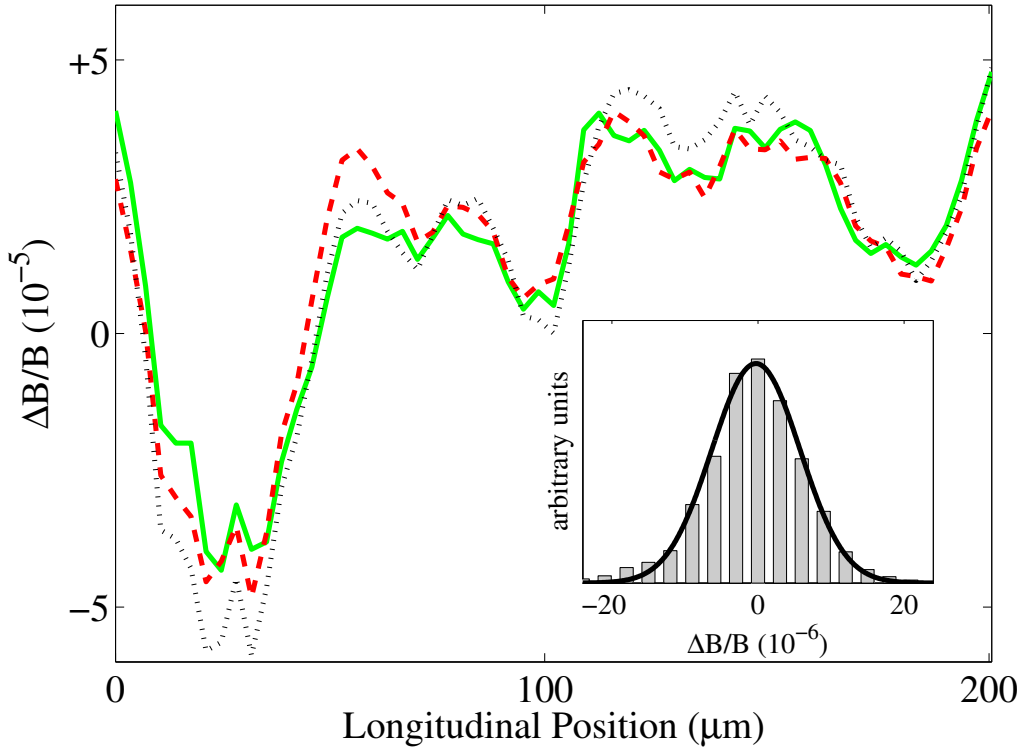
Fragmentation:



Lye et al., PRL **95**, 070401 (2005)

1.2 Wire Trap: Undesired Randomness





Distance: $d = 10\ \mu$m

Wire Width: $100\ \mu$m

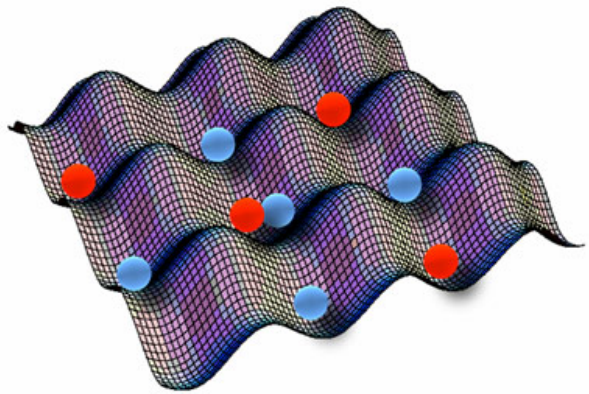
Magnetic Field: 10 G, 20 G, 30 G

Deviation: $\Delta B/B \approx 10^{-4}$

Krüger et al., PRA **76**, 063621 (2007)

1.3 Overview

- **Superfluid Helium in Porous Media**:
  Crooker et al., PRL **51**, 666 (1983)

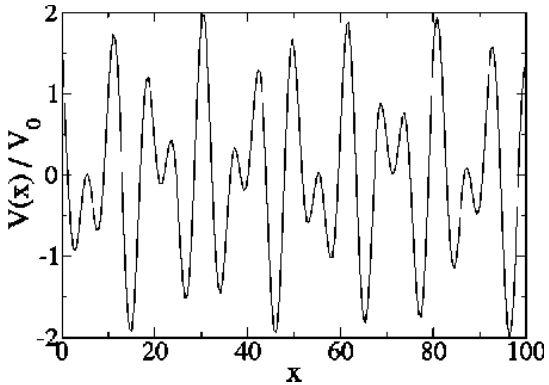


- **Laser Speckles**:
  Lye et al., PRL **95**, 070401 (2005)

  Clément et al., PRL **95**, 170409 (2005)

- **Wire Traps:**
  Krüger et al., PRA **76**, 063621 (2007)

  Fortàgh and Zimmermann, RMP **79**, 235 (2007)



- **Localized Atomic Species:**
  Gavish and Castin, PRL **95**, 020401 (2005)

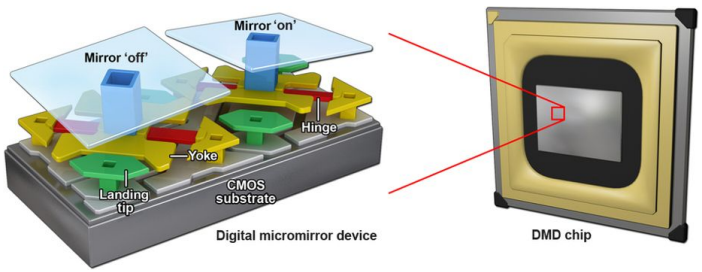
  Gadway et al., PRL **107**, 145306 (2011)

- **Incommensurate Lattices:**
  Damski et al., PRL **91**, 080403 (2003)

  Schulte et al., PRL **95**, 170411 (2005)

- **Digital Micromirror Devices:**
  Ren et al., Ann. Phys. (Berlin) **527**, 447 (2015)

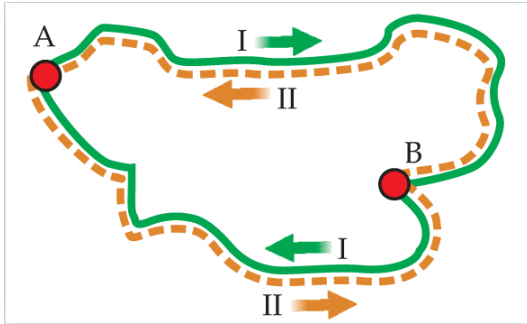
  Choi el al., Science **352**, 1547 (2016)



1.4 Anderson Localization

- **Wave effect (no interaction):**
  - **absence of diffusion**
  - **destructive interference**

  Anderson, PR **109**, 1492 (1958)



- **Initial realizations:**
  light, microwaves, sound waves, electrons in solids, . . .

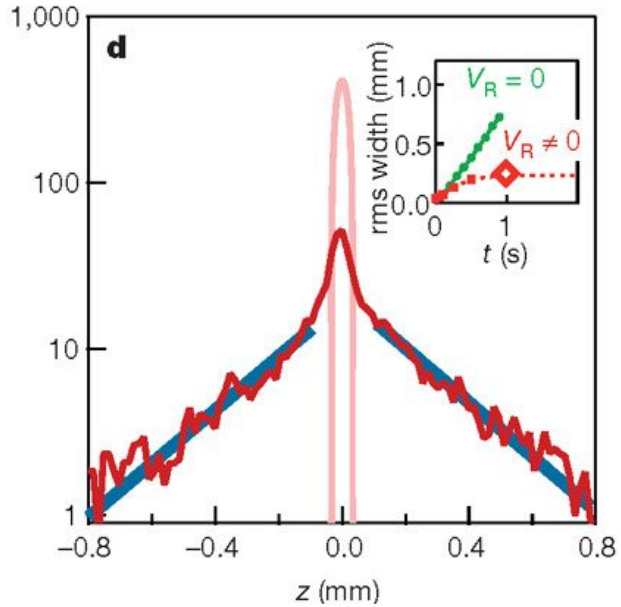
  Abrahams (Ed.), *50 Years of Anderson Localization* (World Scientific, 2010)

- **Quasi 1D BEC ^{87}Rb,**
  TOF with laser speckles:

  Billy et al., Nature **453**, 891 (2008)

- **Quasi 1D BEC ^{39}K,**
  TOF in incommensurate lattice:

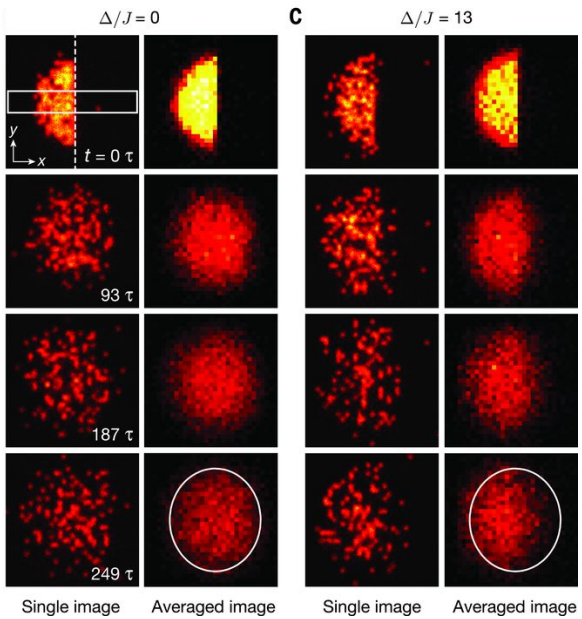
  Roati et al., Nature **453**, 895 (2008)



1.5 Many-Body Localization

- **Paradigm: isolated quantum many-body systems thermalize**
- **Exception:**
  - **disordered systems with interactions**
  - **particles localize, transport ceases, thermalization prevented**
- **Hallmark experiment:**

  Choi et al., Science **352**, 1547 (2016)
  - **2D optical lattice, ^{87}Rb, quenched DMD on-site disorder**
  - **quantum gas microscope: site-resolved measurement**
  - **diverging length scale at localization transition**



- **Logarithmic growth of entanglement:** Lukin et al., `arXiv:1805.09819`
- **More details upon AL and MBL:**
  Report about focus session upon disorder by Max Kiefer

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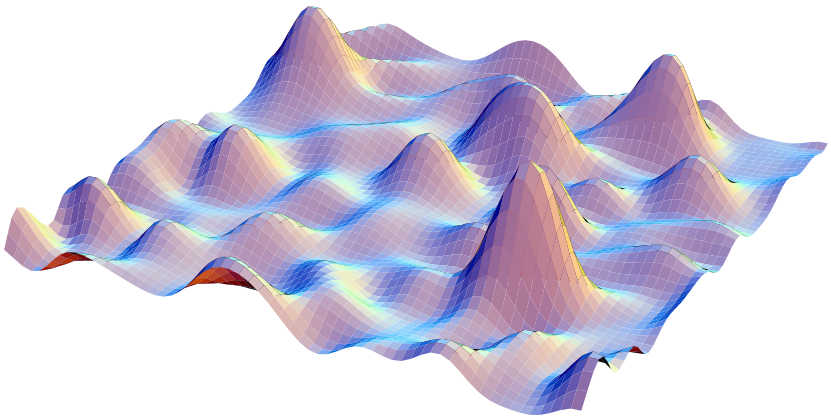

2.1 Model System

Action of a Bose gas:

$$\mathcal{A} = \int_0^{\hbar\beta} d\tau \int d^3x \left\{ \psi^*(\mathbf{x},\tau) \left[\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2}{2M}\boldsymbol{\Delta} + \boldsymbol{U}(\mathbf{x}) + \boldsymbol{V}(\mathbf{x}) - \boldsymbol{\mu} \right] \psi(\mathbf{x},\tau) + \frac{\boldsymbol{g}}{2}\psi^*(\mathbf{x},\tau)^2\psi(\mathbf{x},\tau)^2 \right\}$$

Properties:

- harmonic trap potential: $\boldsymbol{U}(\mathbf{x}) = \frac{M}{2}\sum_{i=1}^{3}\omega_i^2 x_i^2$
- disorder potential: $\boldsymbol{V}(\mathbf{x})$
- chemical potential: $\boldsymbol{\mu}$
- repulsive interaction: $\boldsymbol{g} = \frac{4\pi\hbar^2 a}{M}$
- periodic Bose fields: $\psi(\mathbf{x},\tau+\hbar\beta) = \psi(\mathbf{x},\tau)$

2.2 Random Potential

Disorder Ensemble Average:

$$\overline{\bullet} = \int \mathcal{D}V \bullet P[V]\,, \qquad \int \mathcal{D}V\, P[V] = 1$$

Assumption:

$$\overline{V(\mathbf{x}_1)} = 0\,, \qquad \overline{V(\mathbf{x}_1)V(\mathbf{x}_2)} = R^{(2)}(\mathbf{x}_1 - \mathbf{x}_2)$$

Characteristic Functional:

$$\overline{\exp\left\{ i \int d^D x\, j(\mathbf{x}) V(\mathbf{x}) \right\}}$$

$$= \exp\left\{ \sum_{n=2}^{\infty} \frac{i^n}{n!} \int d^D x_1 \cdots \int d^D x_n\, R^{(n)}(\mathbf{x}_1, \ldots, \mathbf{x}_n)\, j(\mathbf{x}_1) \cdots j(\mathbf{x}_n) \right\}$$

2.3 Grand-Canonical Potential

Aim:

$$\Omega = -\frac{1}{\beta}\overline{\ln \mathcal{Z}}$$

$$\mathcal{Z} = \oint D\psi^* \oint D\psi \, e^{-\mathcal{A}[\psi^*,\psi]/\hbar}$$

Problem:

$$\overline{\ln \mathcal{Z}} \neq \ln \overline{\mathcal{Z}}$$

Solution: Replica Trick

$$\Omega = -\frac{1}{\beta} \lim_{N\to 0} \frac{\overline{\mathcal{Z}^N} - 1}{N}$$

Parisi, J. Phys. (France) **51**, 1595 (1990)

Mezard and Parisi, J. Phys. I (France) **1**, 809 (1991)

Dotsenko, *Introduction to the Replica Theory of Disordered Statistical Systems* (2001)

2.4 Replica Trick

Disorder Averaged Partition Function:

$$\overline{\mathcal{Z}^N} = \overline{\left\{ \prod_{\alpha'=1}^{N} \oint D\psi^*_{\alpha'} \oint D\psi_{\alpha'} \right\} e^{-\sum_{\alpha=1}^{N} \mathcal{A}([\psi^*_\alpha,\psi_\alpha])/\hbar}} = \left\{ \prod_{\alpha=1}^{N} \oint D\psi^*_\alpha \oint D\psi_\alpha \right\} e^{-\mathcal{A}^{(N)}/\hbar}$$

Replicated Action:

$$\mathcal{A}^{(N)} = \int_0^{\hbar\beta} d\tau \int d^D x \sum_{\alpha=1}^{N} \left\{ \psi^*_\alpha(\mathbf{x},\tau) \left[\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2}{2M}\mathbf{\Delta} + U(\mathbf{x}) - \mu \right] \psi_\alpha(\mathbf{x},\tau) \right.$$

$$\left. + \frac{g}{2} \left|\psi_\alpha(\mathbf{x},\tau)\right|^4 \right\} + \sum_{n=2}^{\infty} \frac{1}{n!} \left(\frac{-1}{\hbar} \right)^{n-1} \int_0^{\hbar\beta} d\tau_1 \cdots \int_0^{\hbar\beta} d\tau_n \int d^D x_1 \cdots \int d^D x_n$$

$$\times \sum_{\alpha_1=1}^{N} \cdots \sum_{\alpha_n=1}^{N} R^{(n)}(\mathbf{x}_1,\ldots,\mathbf{x}_n) \left|\psi_{\alpha_1}(\mathbf{x}_1,\tau_1)\right|^2 \cdots \left|\psi_{\alpha_n}(\mathbf{x}_n,\tau_n)\right|^2$$

$\Longrightarrow$ **Disorder amounts to attractive interaction for $n = 2$**

Graham and Pelster, IJBC **19**, 2745 (2009)

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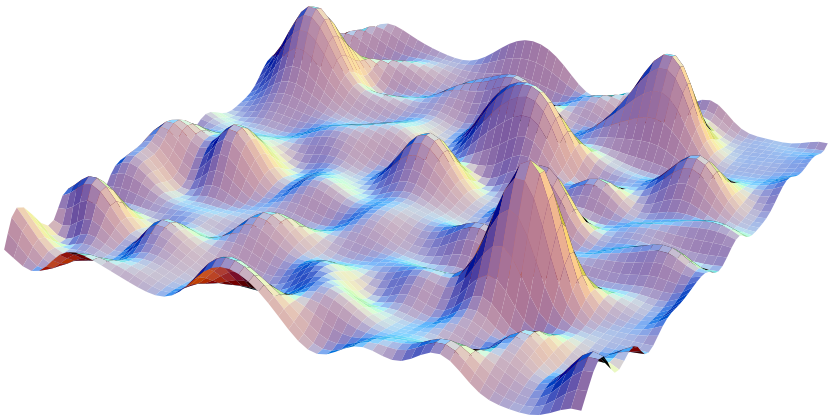


3.1 Bogoliubov Theory of Dirty Bosons

Assumptions:

homogeneous Bose gas: $U(\mathbf{x}) = 0$

δ-correlated disorder: $R(\mathbf{x}) = R\,\delta(\mathbf{x})$

Condensate Depletion:

$$n_0 = n - \frac{8}{3\sqrt{\pi}}\sqrt{a\,n}^3 - \frac{M^2 R}{8\pi^{3/2}\hbar^4}\sqrt{\frac{n}{a}}$$

Superfluid Depletion:

$$n_s = n - n_n = n - \frac{4}{3}\frac{M^2 R}{8\pi^{3/2}\hbar^4}\sqrt{\frac{n}{a}}$$

Huang and Meng, PRL **69**, 644 (1992)

Pelster, BEC Lecture Notes in German (2004)

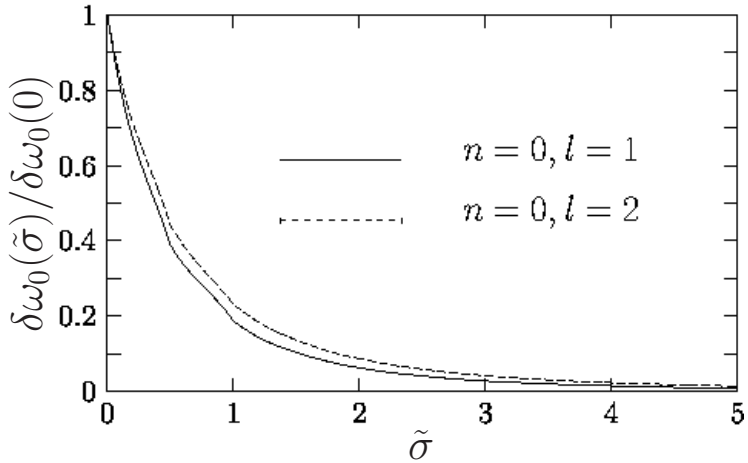
Falco, Pelster, and Graham, PRA **75**, 063619 (2007)

3.2 Collective Excitations

Typical values:

Lye et al., PRL **95**, 070401 (2005)

$$\left.\begin{array}{rcl} \sigma & = & 10\ \mu\text{m} \\ R_{\text{TF}} & = & 100\ \mu\text{m} \\ l_{\text{HO}} & = & 10\ \mu\text{m} \end{array}\right\} \quad \tilde{\sigma} = \frac{\sigma R_{\text{TF}}}{l_{\text{HO}}^2\sqrt{2}} \approx 7$$



$\Longrightarrow$ **Disorder effect vanishes in laser speckle experiment**

Improvement:

laser speckle setup with correlation length $\sigma = 1\ \mu$m

Aspect et al., NJP **8**, 165 (2006)

$\Longrightarrow$ **Disorder effect should be measurable**

Falco, Pelster, and Graham, PRA **76**, 013624 (2007)

3.3 Superfluid Density as Tensor

- **Linear response theory:** $p_i = VM\left(n_{\mathrm{n}ij}v_{\mathrm{n}j} + n_{\mathrm{s}ij}v_{\mathrm{s}j}\right) + \ldots$

  M. Ueda, *Fundamentals and New Frontiers of Bose-Einstein Condensation* (2010)

- **Dipolar interaction at zero temperature:**
  $\Longrightarrow$ **no anisotropic superfluidity**

  Lima and Pelster, PRA **84**, 041604(R) (2011); PRA **86**, 063609 (2012)

- **Dipolar interaction at finite temperature:**
  $\Longrightarrow$ **Directional dependence of first and second sound velocity**

  Ghabour and Pelster, PRA **90**, 063636 (2014)

- **Dipolar interaction and isotropic disorder at zero temperature:**

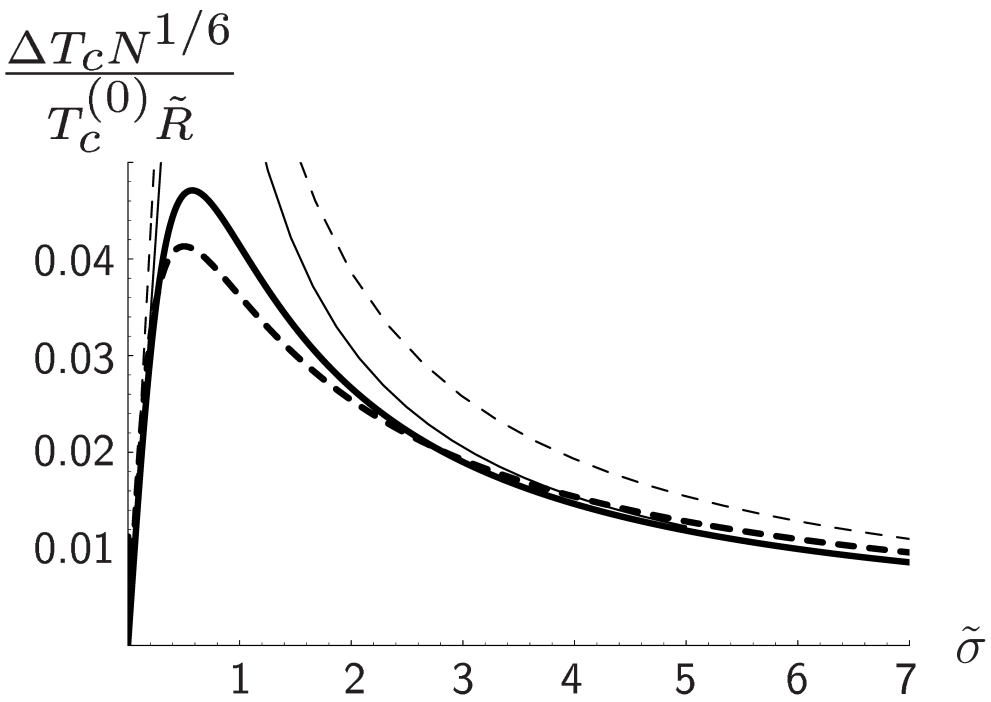
  Krumnow and Pelster, PRA **84**, 021608(R) (2011)

  Nikolić, Balaž, and Pelster, PRA **88**, 013624 (2013)

- **Condensate depletion larger than parallel superfluid depletion:**
  $\Longrightarrow$ **Finite localization time**

  Graham and Pelster, IJBC **19**, 2745 (2009)

3.4 Shift of Condensation Temperature



solid: Gaussian

dashed: Lorentzian

Length Scale: $$l_{\rm HO} = \sqrt{\frac{\hbar}{M\omega_{\rm g}}} \quad , \qquad \omega_{\rm g} = (\omega_1\omega_2\omega_3)^{1/3}$$

Dimensionless Units: $$\tilde{\sigma} = \frac{\sigma N^{1/6}}{l_{\rm HO}} \quad , \qquad \tilde{R} = \frac{R}{\left(\frac{\hbar^2}{M l_{\rm HO}^2}\right)^2 l_{\rm HO}^3}$$

Timmer, Pelster, and Graham, EPL **76**, 760 (2006)

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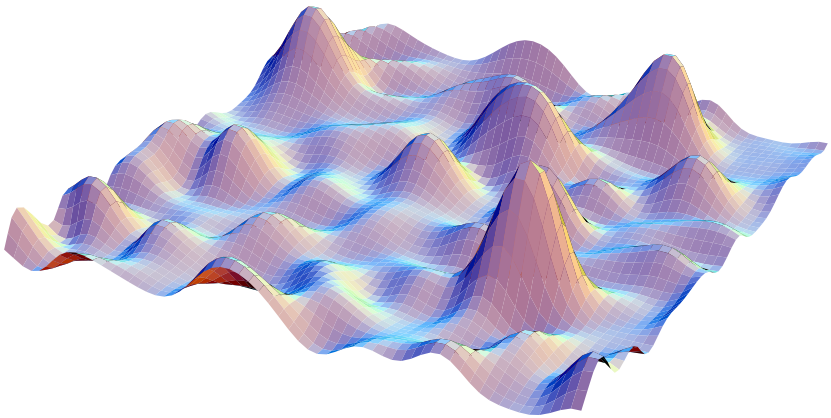


4.1 Order Parameters

Definition:

$$\lim_{|\mathbf{x}-\mathbf{x}'|\to\infty} \overline{\langle\psi(\mathbf{x},\tau)\psi^*(\mathbf{x}',\tau)\rangle} = n_0$$

$$\lim_{|\mathbf{x}-\mathbf{x}'|\to\infty} \overline{|\langle\psi(\mathbf{x},\tau)\psi^*(\mathbf{x}',\tau)\rangle|^2} = (n_0+q)^2$$

Graham and Pelster, IJBC **19**, 2745 (2009)

Notes About q:

- **Quantifies number of bosons localized in minima of disorder**
- **Similar to Edwards-Anderson order parameter of spin-glass**

  Edwards and Anderson, J. Phys. F **5**, 965 (1975)

  Fischer and Hertz, *Spin Glasses* (1991)

Phase classification: $n = n_0 + q + n_{\rm th}$

thermal gas	Bose-glass	superfluid
$q = n_0 = 0$	$q > 0, n_0 = 0$	$q > 0, n_0 > 0$

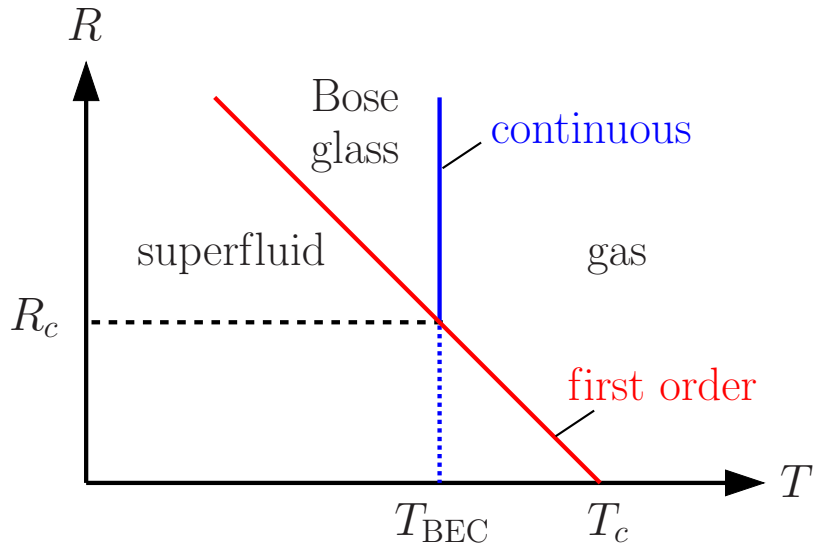
4.2 Homogeneous Dirty Bose-Einstein-Condensate

Hartree-Fock Mean-Field Theory:

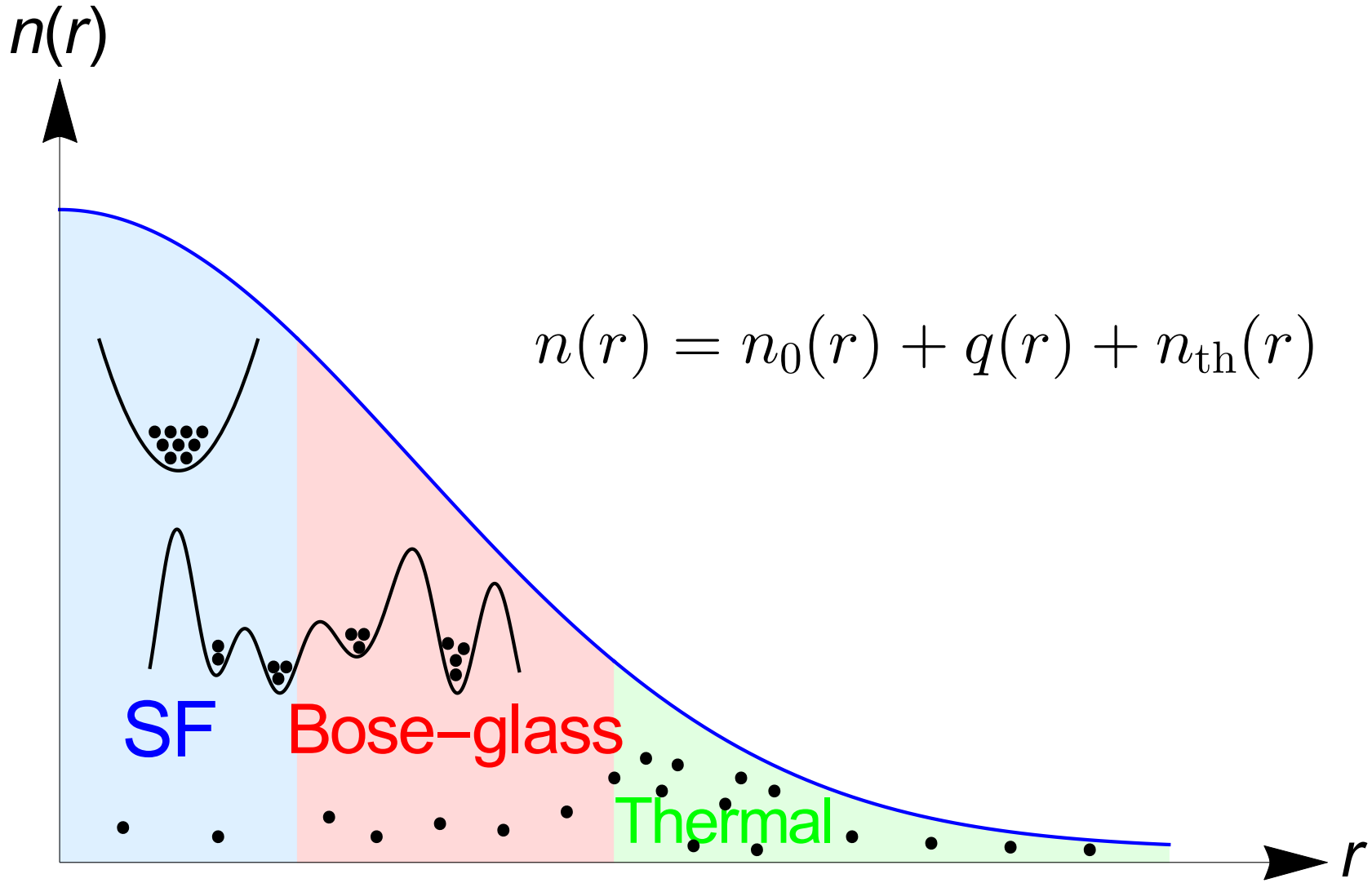
- **Delta-correlated disorder** $R(\mathbf{x}-\mathbf{x}') = R\,\delta(\mathbf{x}-\mathbf{x}')$
- **Replica-symmetric solution**
- **Self-consistency equations for** n, n_0, **and** q
- **Bose-glass phase** $q > 0, n_0 = 0$**:**
  localized short-lived excitations with gapless density of states
  Fisher, Weichman, Grinstein, and Fisher, PRB **40** 546 (1989)

Phase Diagram

Graham and Pelster,
IJBC **19**, 2745 (2009)
Khellil and Pelster,
JSM 093108 (2017)



4.3 Trapped Dirty Bose-Einstein-Condensate



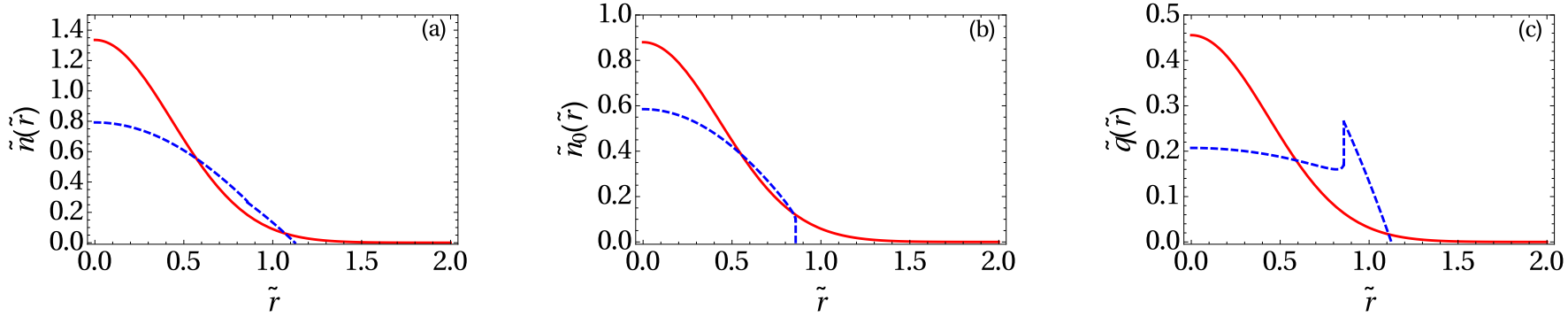
$$n(r) = n_0(r) + q(r) + n_{\mathrm{th}}(r)$$

Khellil, Balaž, and Pelster, NJP **18**, 063003 (2016)

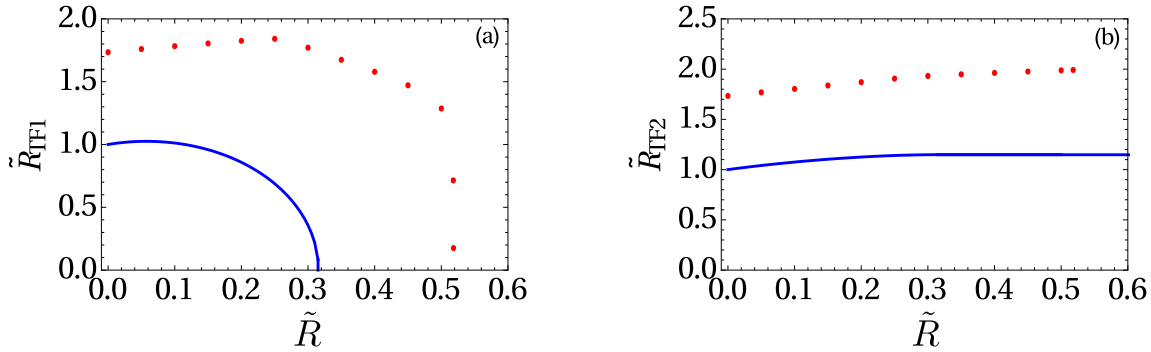
Khellil and Pelster, JSM 063301 (2016); JSM 093108 (2017)

4.4 Isotropic 3D Trap, Delta-Correlated Disorder, $T = 0$

- **Density profiles**: $\tilde{R} = 0.2$



- **Thomas-Fermi radii:**



Red $\Longrightarrow$ Variational
Blue $\Longrightarrow$ Analytical

Khellil and Pelster, JSM 093108 (2017)

Nattermann and Pokrovsky, PRL **100**, 060402 (2008): $\tilde{R}_c = 0.115$

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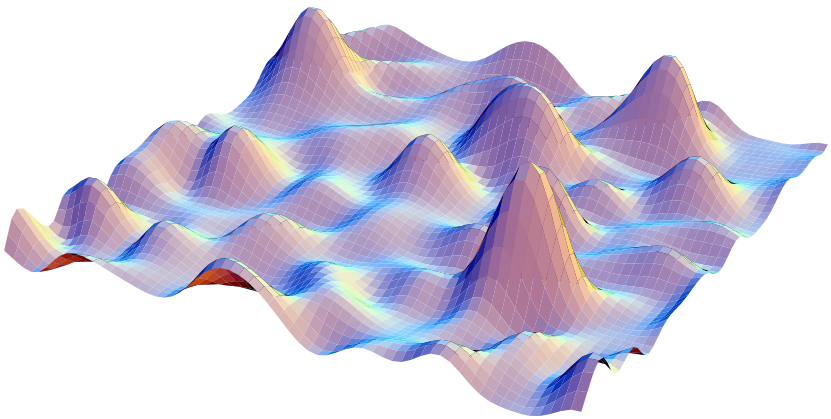
   News From Kaiserslautern



5.1 Dynamics of Bosons in Random Potentials

Initial Value Problem:

$$
\begin{aligned}
i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{x},t) &= \left\{-\frac{\hbar^2}{2M}\Delta + V(\mathbf{x},t) - \mu_0 + g\left|\Psi(\mathbf{x},t)\right|^2\right\}\Psi(\mathbf{x},t) \\
\Psi(\mathbf{x},0) &= \sqrt{n}\,, \qquad \mu_0 = gn
\end{aligned}
$$

Smooth quench: $f(t) = 1 - e^{-t/\tau}$

$$
V(\mathbf{x},t) = \begin{cases} 0 & t \le 0 \\ f(t)V(\mathbf{x}) & t > 0 \end{cases} \qquad \overline{V(\mathbf{x})} = 0, \quad \overline{V(\mathbf{x})V(\mathbf{x}')} = R(\mathbf{x}-\mathbf{x}')
$$

Perturbative Results in Second Order:

- **Condensate Density:** $n_0(t) = \left|\overline{\Psi(\mathbf{x},t)}\right|^2$
- **Particle Density:** $n = \overline{\left|\Psi(\mathbf{x},t)\right|^2}$
- **Condensate Depletion:** $q(t) = n - n_0(t)$

5.2 Initial Results: Delta Correlated Disorder

Evolution of condensate depletion:

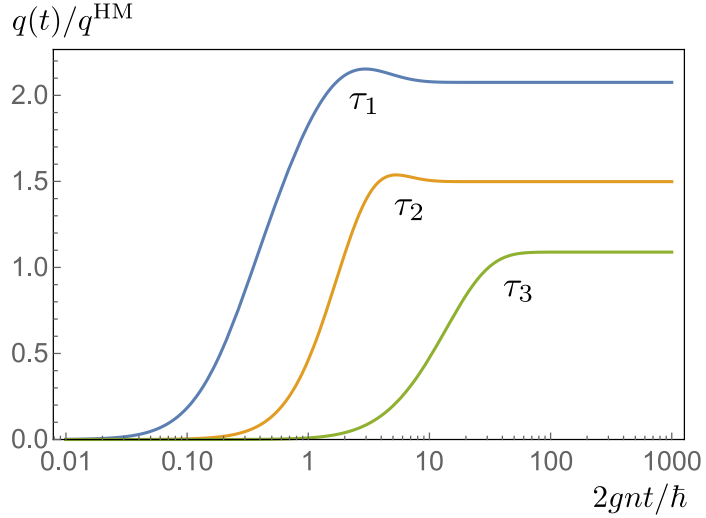
$\tau_1 < \tau_2 < \tau_3$

- Huang-Meng depletion:

$$q^{\mathrm{HM}} = \frac{M^2 R}{8\pi^{3/2}\hbar^4}\sqrt{\frac{n}{a}}$$

- Time scale:

$$\frac{\xi}{c} = \frac{\hbar}{gn}$$



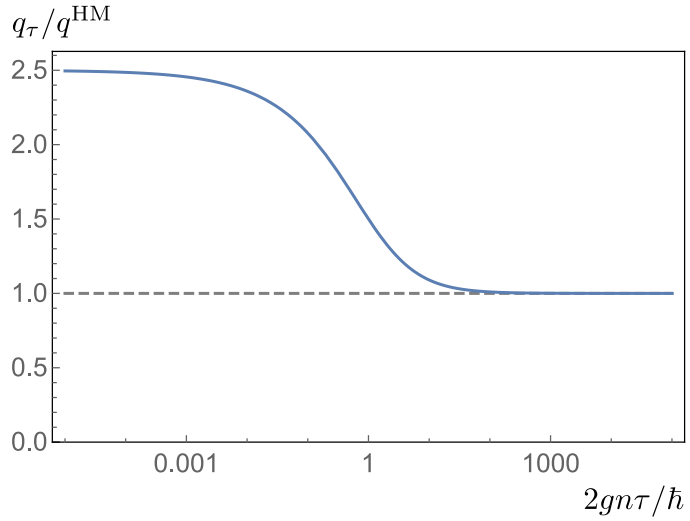
Disorder rise time influences steady state:

- Sudden quench of disorder:

$$\lim_{\tau\to 0} q_\tau = \frac{5}{2} q^{\mathrm{HM}}$$

- Adiabatic switching on disorder:

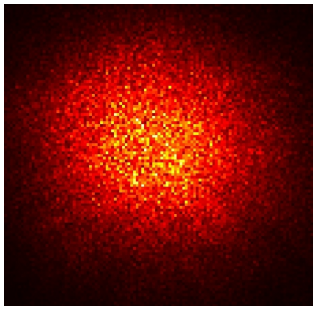
$$\lim_{\tau\to\infty} q_\tau = q^{\mathrm{HM}}$$



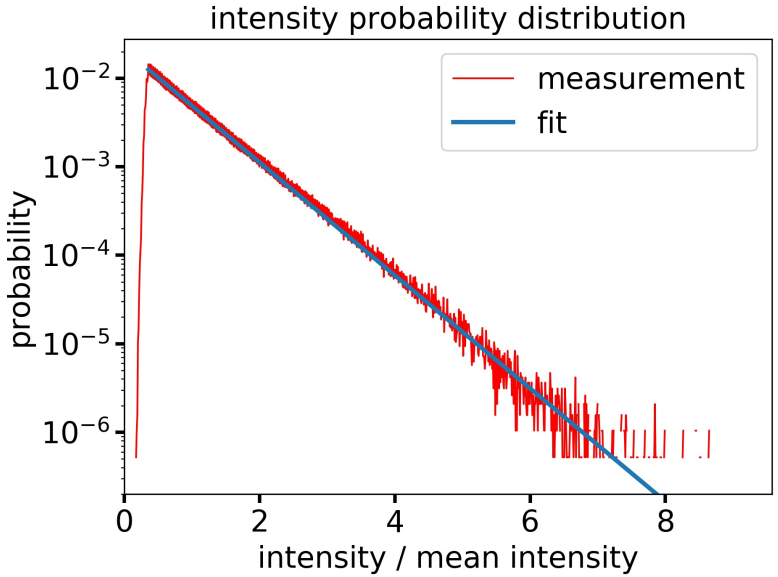
Radonjić and Pelster (unpublished)

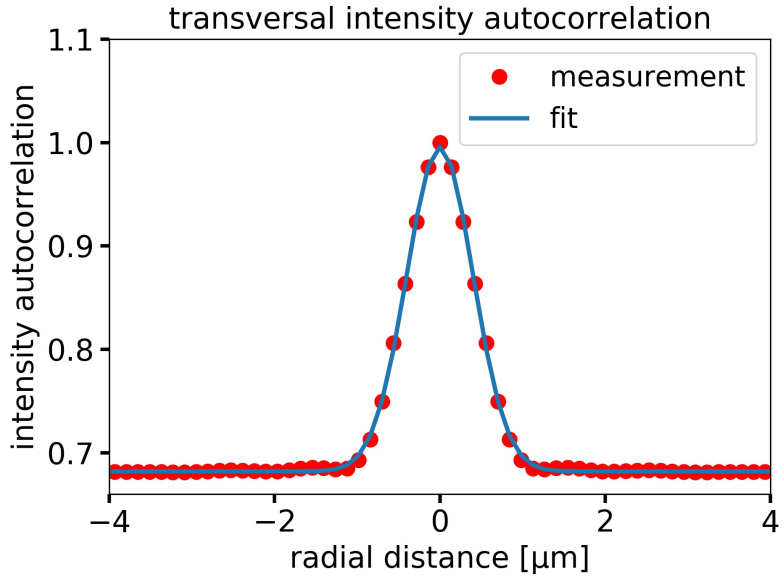
5.3 Optical Speckle Disorder

- Electric field $\mathbf{E}(\mathbf{x})$:
  Gaussian distributed
- Intensity $I(\mathbf{x}) \sim |\mathbf{E}(\mathbf{x})|^2$:
  non-Gaussian distributed



- **Speckle characterization measurements:**





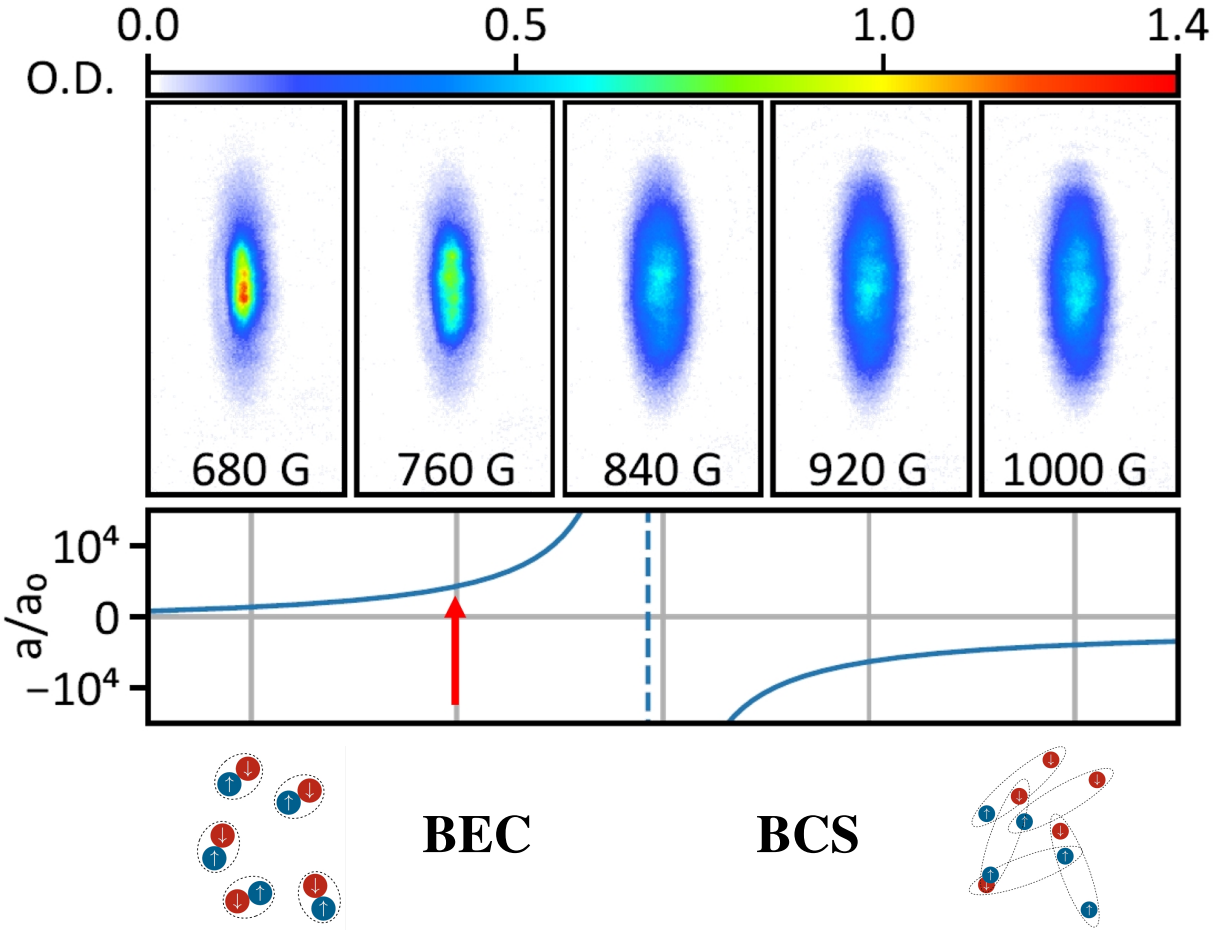
$$P(I) = \overline{\delta\left(I - I(\mathbf{x})\right)}$$

$$\overline{I(\mathbf{x})I(\mathbf{x}')} = R(\mathbf{x} - \mathbf{x}')$$

Nagler, Gänger, Phieler, and Widera (unpublished)

5.4 BEC-BCS Crossover

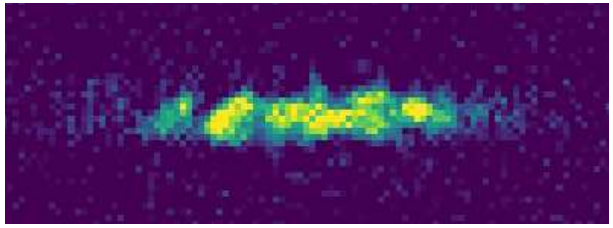
^{6}Li, $N \sim 10^5, T < 100$ nK: confinement in combined potential of optical dipole and magnetic trap with aspect ratio ~ 7



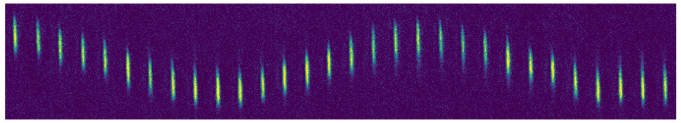
Gänger, Phieler, Nagler, and Widera, Rev. Sci. Instr. **89**, 093105 (2018)

5.5 Molecular BEC in Disorder

Fragmentation:

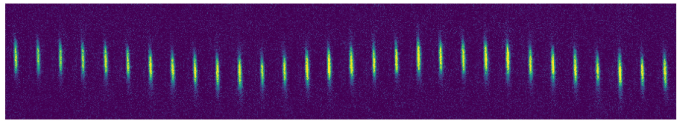


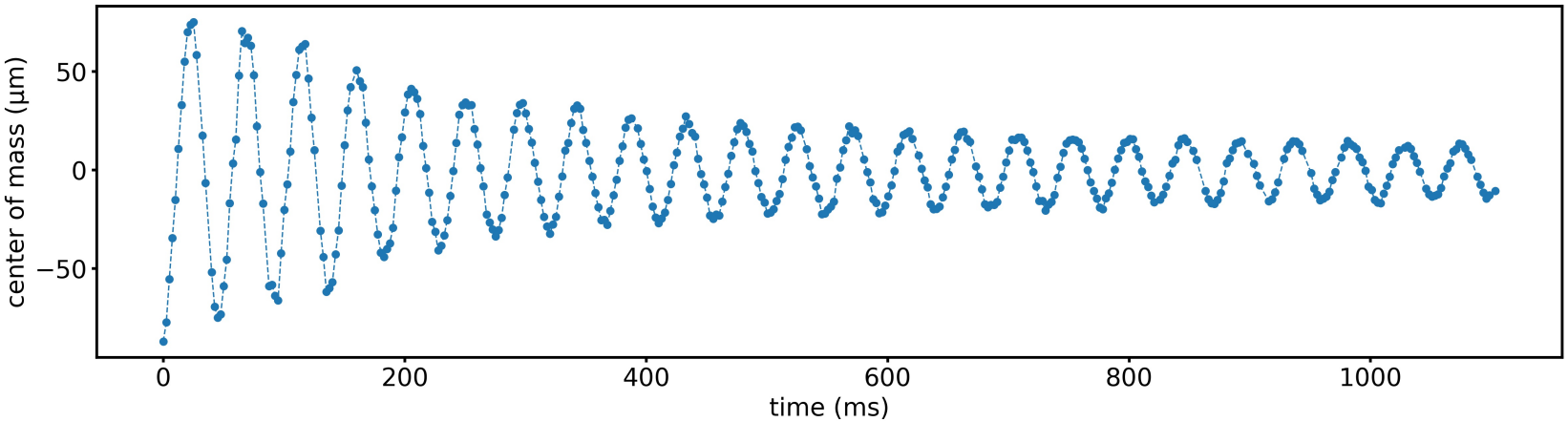
Center of mass oscillations:



several

10 periods



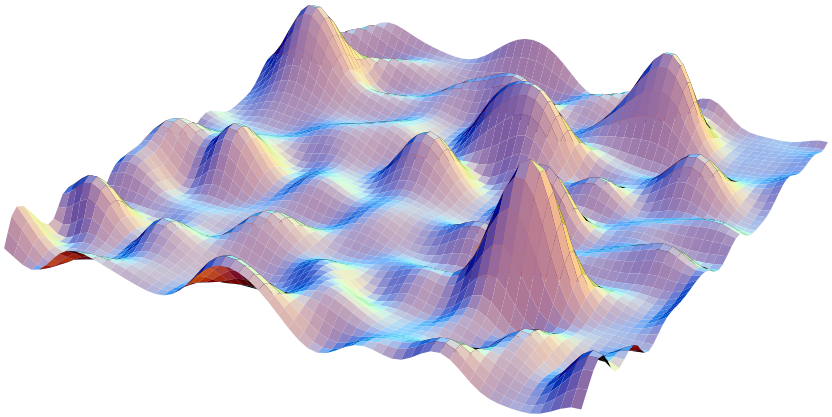


Nagler, Gänger, Phieler, and Widera (unpublished)

On the Dirty Boson Problem

Axel Pelster

1. Experimental Realizations
2. Theoretical Description
3. Perturbative Results
4. Non-Perturbative Results
5. Outlook
6. Announcements



6.1 School Anyon Physics of Ultracold Atomic Gases

VolkswagenStiftung

organized by Bakhodir Abdullaev and Axel Pelster

Technische Universität Kaiserslautern, Germany

December 10 – 14, 2018

Speakers:

Mikhail Baranov (Innsbruck, Austria), Sebastian Greschner (Geneva, Switzerland), Anne Nielsen (Dresden, Germany), Belén Paredes (Munich, Germany), Thore Posske (Hamburg, Germany), Philipp Preiss (Heidelberg, Germany), Christof Weitenberg (Hamburg, Germany)

Additional Talks:

Colloquium: Sabine Hossenfelder (Frankfurt, Germany),

Theoretical Colloquium: Wolfgang Ketterle (Boston, USA),

Laser and Quantum Optics Seminar: Joachim Brand (Auckland, New Zealand)

`http://www-user.rhrk.uni-kl.de/~apelster/Anyon3/index.html`

6.2 Bad Honnef Physics School on Methods of Path Integration in Modern Physics

organized by Stefan Kirchner and Axel Pelster

Bad Honnef (Germany); August 25 – 31, 2019

Speakers and Topics:

Lawrence Schulman (Potsdam, USA): *Quantum mechanics, semiclassics*
Andreas Wipf (Jena, Germany): *Statistical field theory, Matsubara*
Carlos Sá de Melo (Atlanta, USA): *BEC-BCS Crossover, Hubbard-Stratonovich*
Jean Zinn-Justin (Paris, France): *Quantum field theory, large-N technique, instantons*
Victor Dotsenko (Paris, France): *Random matrix theory, supersymmetry, replica trick*
Steve Simon* (Oxford, UK): *Wilson loops spin, topology, holonomy group*
Wolfhard Janke (Leipzig, Germany): *Quantum Monte Carlo*

Hagen Kleinert (Berlin, Germany): *Vortices and GIMPs*

*to be confirmed

`http://www.dpg-physik.de/dpg/pbh/aktuelles/S619.html?lang=en&`