Collective Excitations in a Trapped Bose-Einstein Condensate with Weak Disorder

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Gianmaria Falco

(Universität zu Köln)

Prof. Robert Graham

Axel Pelster

Weak disorder: Bogoliubov theory for uniform gas.

K. Huang and H. F. Meng, PRL **69**, 644 (1992)

The action of the theory:

$$S[\psi^*,\psi;U] = \int_0^{\hbar\beta} d\tau \left(\int d\mathbf{r}\ \psi^*(\mathbf{r},\tau) \left[\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2\nabla^2}{2m} - \mu + U(\mathbf{r}) \right] \psi(\mathbf{r},\tau) \right.$$
$$\left. + \frac{1}{2}\, g \int d\mathbf{r}\ \psi^*(\mathbf{r},\tau)\psi^*(\mathbf{r},\tau)\psi(\mathbf{r},\tau)\psi(\mathbf{r},\tau) \right) \qquad g = 4\pi\hbar^2 a/m$$

Ensemble averages assumed:

$$\langle U(\mathbf{r})\rangle = 0,$$

$$\langle U(\mathbf{r})U(\mathbf{r}')\rangle = R\left(\mathbf{r}-\mathbf{r}'\right) = R_0 \frac{\exp\left\{-\frac{(\mathbf{r}-\mathbf{r}')^2}{2\xi^2}\right\}}{(2\pi\xi^2)^{3/2}}$$

Delta correlated disorder: $\xi \to 0$

Perturbative approach: uniform gas at $T = 0$

Total density: $n = n_0 + n_g + n_{\mathrm{R}}$

interaction: $n_g = \dfrac{8}{3\sqrt{\pi}} \sqrt{a\, n}^{\,3}$

disorder:

$$n_{\mathrm{R}} = R_0 \frac{m^2}{8\ \pi^{3/2}\ \hbar^4} \sqrt{\frac{n}{a}} f(4\pi n a \xi^2)$$

Galilei boost $\longrightarrow$ Superfluid density:

Normal component due to the disorder:

$$n_n = n - n_s = \frac{4}{3} n_{\mathrm{R}} \quad \rightarrow \quad n_0 > n_s$$

Perturbative approach: conditions of validity

Thermodynamics

Dilute gas: $n^{1/3}a \ll 1$

Weak disorder approximation:

$$n_{\mathrm{R}} \ll n \Leftrightarrow R_0' \equiv c_0 \frac{\xi_{\mathrm{heal}}}{\mathcal{L}} \ll 1$$

T. Nattermann & V. Pokrovskii PRL 100, 060402 (2008)

Larkin length $\mathcal{L} \equiv \dfrac{\hbar^4}{m^2 R_0}$ **healing length** $\xi_{\mathrm{heal}} \equiv 1/\sqrt{8\pi n a}$

Dynamics

Self-averaging condition for the low-energy modes:

$$\xi \ll \xi_{\mathrm{heal}}$$

Hydrodynamic equations

$\xi \ll \xi_{\text{heal}} \rightarrow$ **"Two-fluids" collisionless hydrodynamic equations at $T = 0$:**

$$\frac{\partial}{\partial t} n + \nabla \left(\mathbf{v}_s n_s + \mathbf{v}_n n_n \right) = 0$$
$$m \frac{\partial}{\partial t} \mathbf{v}_s + \nabla \left(\mu + \frac{1}{2} m \mathbf{v}_s^2 \right) = 0 \qquad \text{("Josephson equation")}$$

Pinning of normal component in linear regime, $\underline{\mathbf{v}_n = 0}$

$\longrightarrow$ **"Fourth sound" linearized hydrodynamic equation:**

$$m \frac{\partial^2}{\partial t^2} \delta n - \nabla \left[n_s \nabla \left(\frac{\partial \mu}{\partial n} \delta n \right) \right] = 0$$

Disorder :

$(\partial \mu / \partial n)^{-1} \rightarrow$ **change of the macroscopic compressibility**

$n_s \neq n \rightarrow$ **disorder-induced superfluid depletion**

Huang-Meng theory in the trap: local density approximation

Harmonic isotropic trap: $V(\mathbf{r}) = m\omega_{\rm HO}^2 r^2/2$

Large N Thomas-Fermi limit: $Na/a_{\rm HO} \gg 1$ $\quad a_{\rm HO} = \sqrt{\hbar/m\omega_{\rm HO}}$

Thomas-Fermi approximation $R_{\rm TF}$: $R_{\rm TF} \gg a_{\rm HO}$ $\quad \frac{1}{2}\omega_{\rm HO}^2 R_{\rm TF}^2 = \mu$

$\Downarrow$

local density approximation $\xi_{\rm heal} \ll R_{\rm TF}$

Disorder in the equation of state:

beyond mean-field corrections

$$\mu_{local}(n) = g\ n(\mathbf{r}) \left(1 + \frac{32}{3}\sqrt{\frac{n(\mathbf{r})\,a^3}{\pi}}\right) + 6\ g\ n_{\rm R}(\mathbf{r})$$

$\downarrow$

Beliaev corrections

L. Pitaevskii & S. Stringari PRL 81, 4541 (1998)
A. Altmayer et al. PRL 98, 040401 (2007)

Hydrodynamic equation in the trap

Ex : delta-correlated disorder $\xi = 0$**,**

$$m\frac{\partial^2}{\partial t^2}\delta n\left(\mathbf{r}\right) - \nabla\left[g\ n_{\mathrm{TF}}\left(\mathbf{r}\right)\nabla\delta n\left(\mathbf{r}\right)\right] =$$

$$-\nabla^2\left[3\ g\ n_{\mathrm{R}}\left(\mathbf{r}\right)\ \ \delta n\left(\mathbf{r}\right)\right] - \nabla\left[\frac{4n_{\mathrm{R}}\left(\mathbf{r}\right)}{3}\nabla\left[g\ \delta n\left(\mathbf{r}\right)\right]\right]$$

$R_0 = 0 \rightarrow$ **Stringari's hydrodynamics:** $\omega_0\left(n_r, l\right) = \omega_{\mathrm{HO}}\left(2n_r^2 + 2n_r l + 3n_r + l\right)^{1/2}$

Surface modes: $n_r = 0,\ \omega_l = \sqrt{l}\ \omega_{\mathrm{HO}},\quad \delta n\left(\mathbf{r}\right) = r^l Y_{lm},\quad \nabla^2\delta n\left(\mathbf{r}\right) = 0$

$\rightarrow$ **No shift due to the beyond-mean-field effects in the atomic interaction**

Results for delta-correlated disorder ($\xi = 0$)

Falco, Pelster, and Graham, Phys. Rev. A 76, 013624 (2007)

Monopole mode, $n_r = 1$, $l = 0$

$$\frac{\delta\omega_M}{\omega_M} = -\frac{469\pi}{768} R_0'(\mathbf{0})$$

Dipole mode, $n_r = 0$, $l = 1$ **(generalized Kohn's theorem** $\rightarrow \omega_{\mathrm{D}} = \omega_{\mathrm{HO}}$**)**

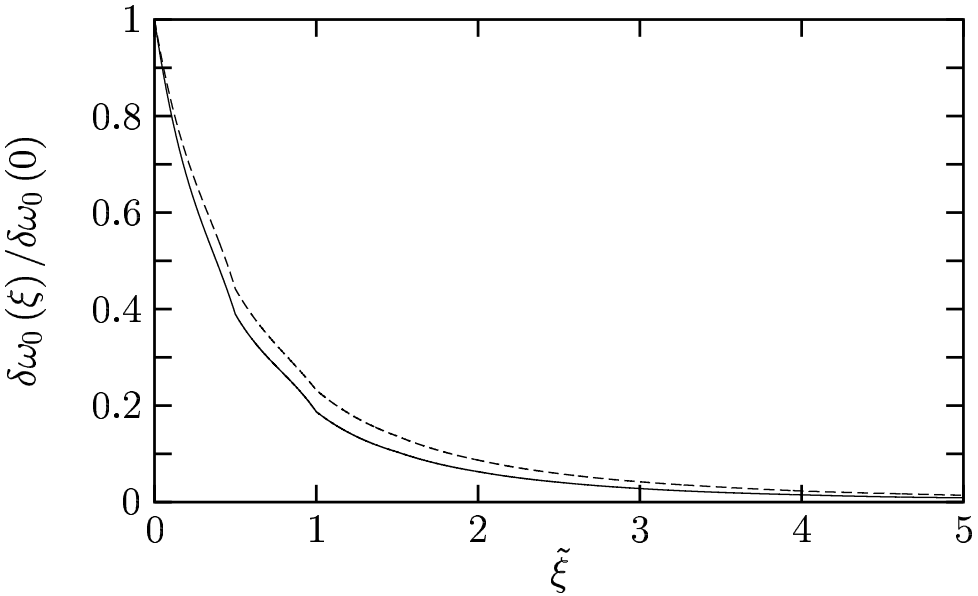
$$\frac{\delta\omega_{\mathrm{D}}}{\omega_{\mathrm{D}}} = -\frac{5\pi}{16} R_0'(\mathbf{0})$$

Quadrupole mode $n_r = 0$, $l = 2$, $m = 2$

$$\frac{\delta\omega_{\mathrm{Q}}}{\omega_{\mathrm{Q}}} = -\frac{35\pi}{96} R_0'(\mathbf{0})$$

Effects of finite disorder correlation length ($\xi \neq 0$)

Relative shift



Scaling variable $\tilde{\xi} = \xi R_{\mathrm{TF}}/a_{\mathrm{HO}}^2\sqrt{2}$

Self-averaging $\tilde{\xi} \ll 1 \Leftrightarrow \xi \ll \xi_{\mathrm{heal}}$

Typical Values:

Inguscio *et al.*, **PRL 95, 070401 (2005)**

$$\left.\begin{aligned} \xi &= 10\ \mu\mathrm{m} \\ R_{\mathrm{TF}} &= 100\ \mu\mathrm{m} \\ a_{\mathrm{HO}} &= 10\ \mu\mathrm{m} \end{aligned}\right\} \quad \tilde{\xi} = \frac{\xi R_{\mathrm{TF}}}{a_{\mathrm{HO}}^2\sqrt{2}} \approx 7$$

Improvement

laser speckle setup with

$$\xi = 1\ \mu\mathrm{m}$$

Aspect *et al.*, **New J. Phys. 8, 165 (2006)**

Anisotropy effects ($\xi = 0$)

Anisotropic trap: $V(\mathbf{x}) = m\left(\omega_\perp^2 r_\perp^2 + \omega_z^2 z^2\right)/2$

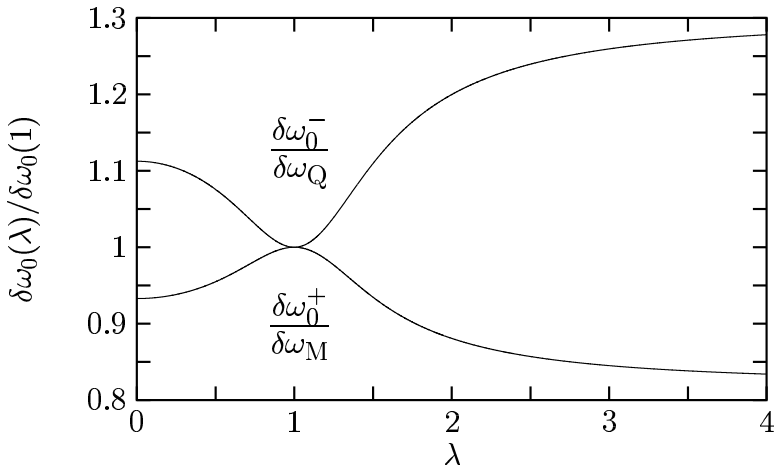
Anisotropy factor: $\lambda = \omega_z/\omega_\perp$

quadrupole mode with $m = 0$ couples to monopole mode

$$[\omega_0^\pm(m=0)]^2 = \omega_\perp^2(2 + \tfrac{3}{2}\lambda^2 \pm \tfrac{1}{2}\sqrt{9\lambda^4 - 16\lambda^2 + 16})$$

$$\lambda = 1 \Rightarrow \quad \omega_0^+ = \omega_{\mathrm{M}},\ \omega_0^- = \omega_{\mathrm{Q}}$$

Disorder corrections:



$$\frac{\delta\omega_0^\pm}{\omega_0^\pm} = -\pi R_0' \frac{7(\pm 72 \pm 9\lambda^2 + 107\sqrt{16 - 16\lambda^2 + 9\lambda^4})}{1536\sqrt{16 - 16\lambda^2 + 9\lambda^4}}$$

Hydrodynamic theory can be extended to the "$1D$ mean-field" regime

Conclusions and Perspectives

- **Collective frequencies can be measured with great accuracy in trapped BECs. Beyond-mean-field effects can be measured.**

  A. Altmeyer *et al.*, PRL 98, 040401 (2007).

- **Due to the Kohn's theorem the effects of the disorder should be clearly distinguishible from the beyond mean-field corrections due to the normal interactions.**

- **Realistic test for the predictions of the Huang and Meng theory.**

  J. Fortagh & C. Zimmermann, Magnetic Microtraps for Ultracold Atoms, Rev. Mod. Phys. 79, 235 (2007).