

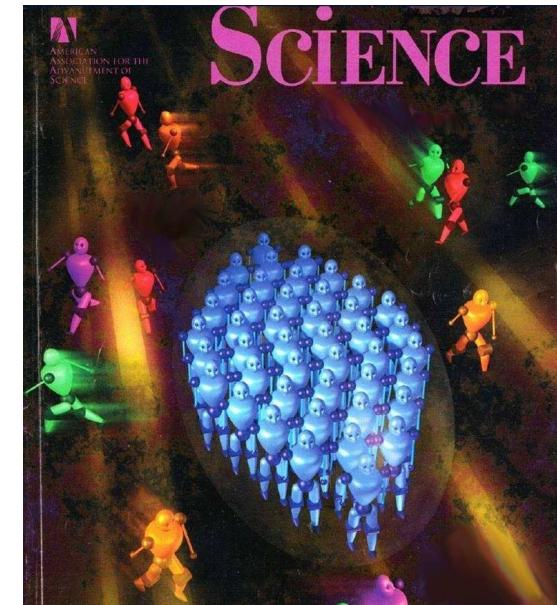
Ultracold Quantum Gases – A Fascinating Playground for Basic Research in Physics

Axel Pelster

Freie Universität Berlin



- 1. Bose-Einstein Condensation**
- 2. Dipolar Bose-Einstein Condensates**
- 3. On the Dirty Boson Problem**
- 4. Bosons in Optical Lattices**
- 5. Conclusion**



1.1 Identical Quantum Particles

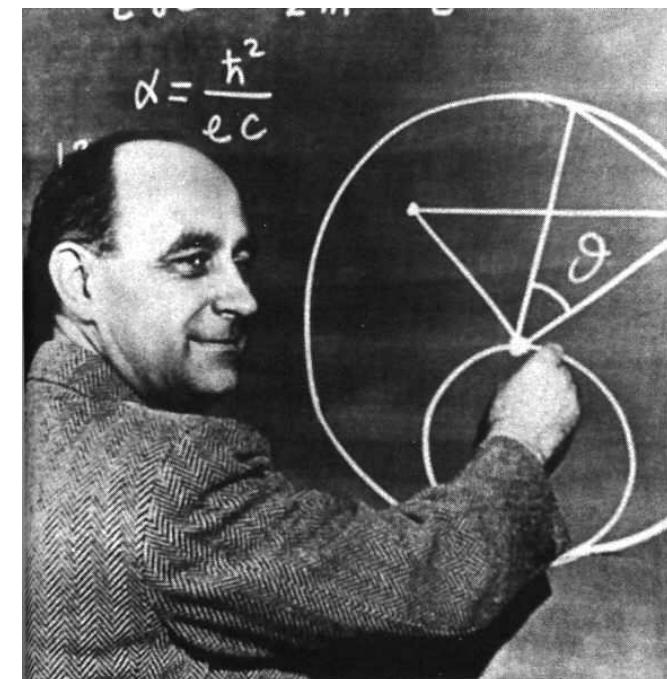
Bosons:

- integer spin
- symmetric wave function

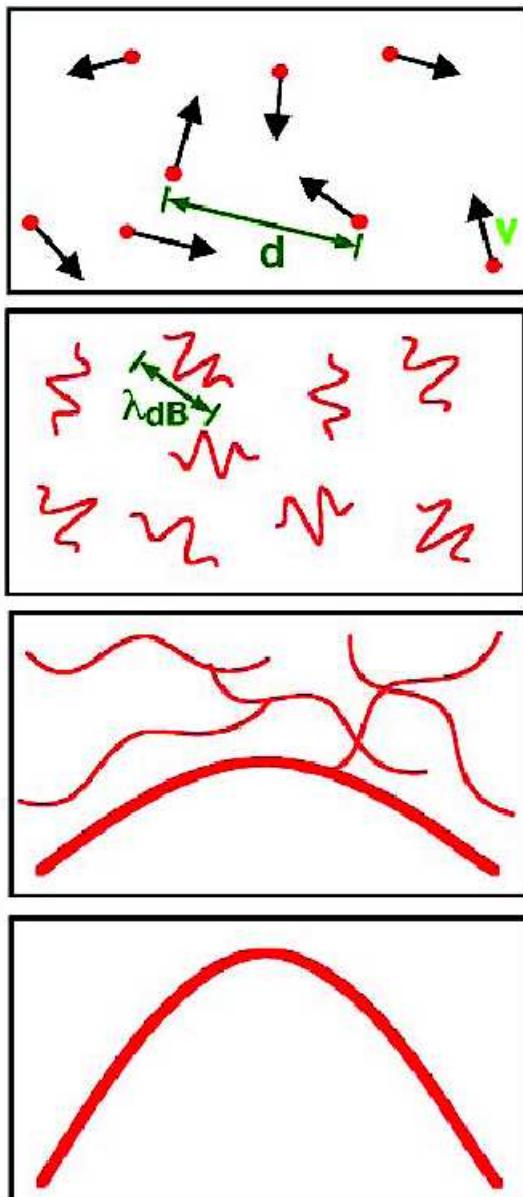


Fermions:

- half-integer spin
- anti-symmetric wave function



1.2 What is Bose-Einstein Condensation?



High Temperature T:

thermal velocity v
density d^{-3}

"Billiard balls"

Low Temperature T:

De Broglie wavelength
 $\lambda_{dB} = \hbar/mv \propto T^{-1/2}$

"Wave packets"

$T=T_{crit}$:
Bose-Einstein Condensation

$\lambda_{dB} \approx d$

"Matter wave overlap"

$T=0$:
Pure Bose condensate

"Giant matter wave"

$$\bullet \lambda_{dB} = \frac{\hbar}{\sqrt{2Mk_B T}}$$

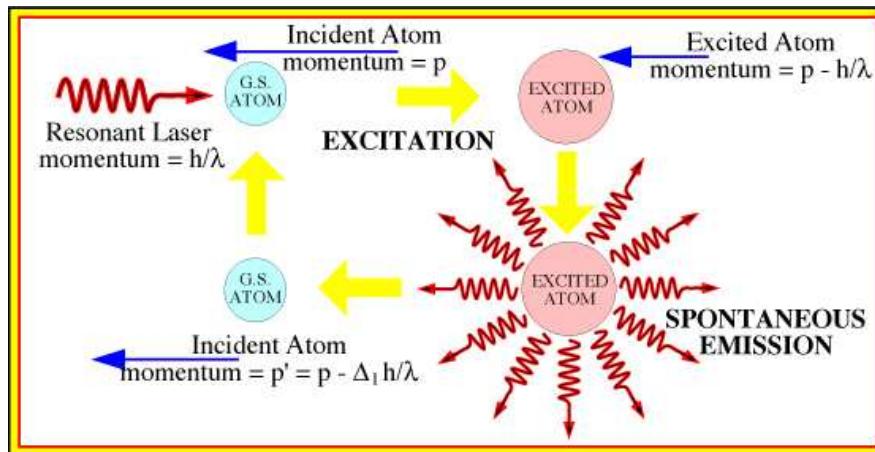
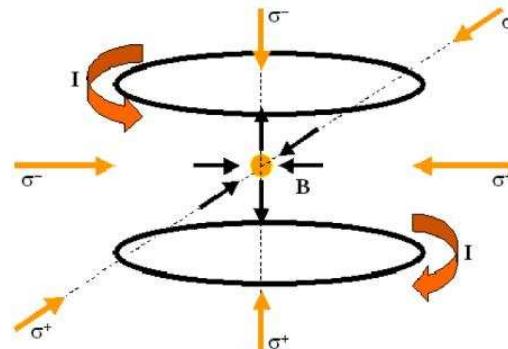
$$\bullet n = \frac{1}{d^3}$$

$$\bullet \frac{\lambda_{dB}}{d} \approx 1$$

$$\bullet T_c \approx \frac{\hbar^2 n^{3/2}}{2Mk_B}$$

1.3 Cooling Techniques

magneto-optical trap

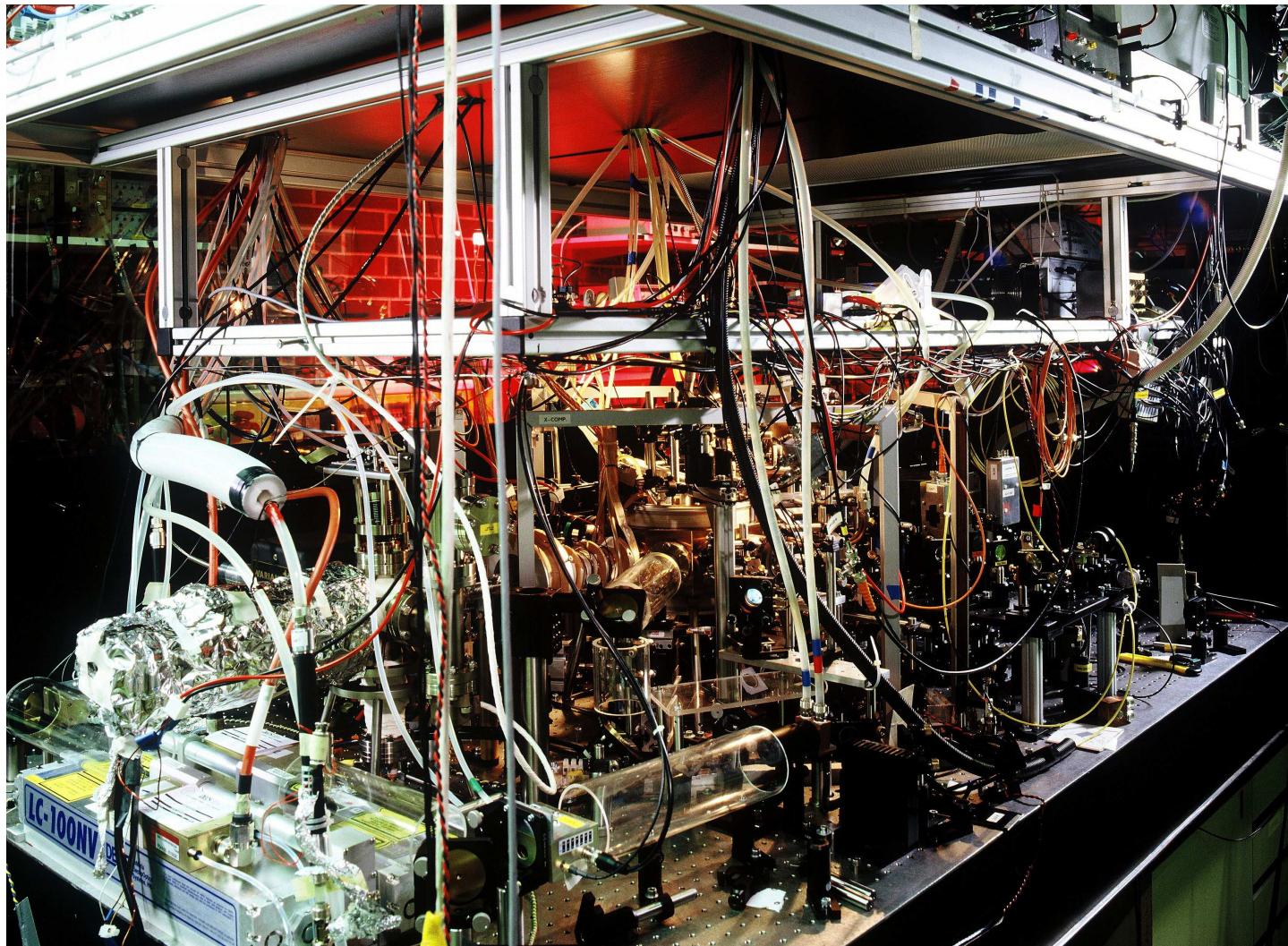


laser cooling



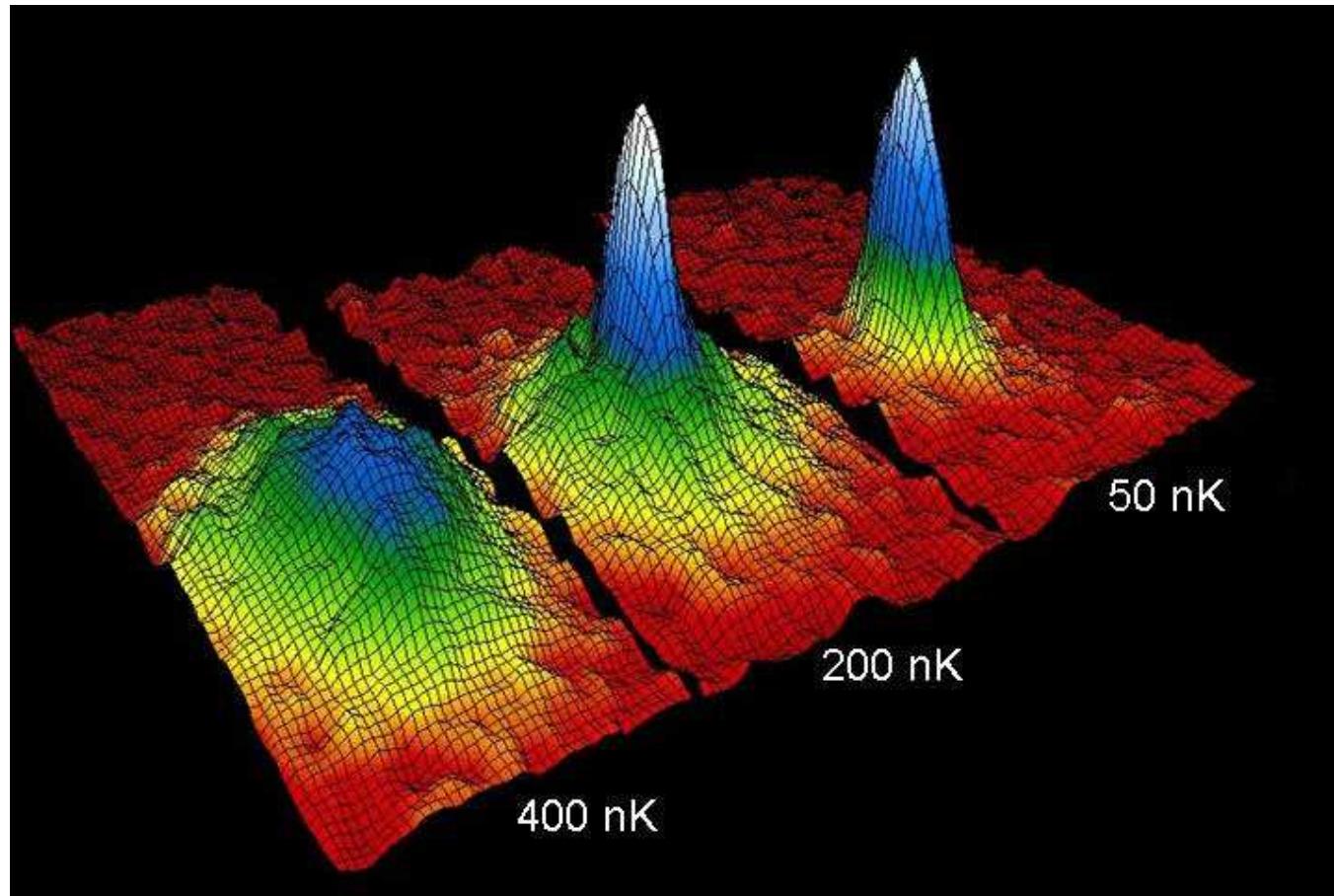
evaporative cooling

1.4 Experimental Apparatus



Costs about 1.000.000 EUR

1.5 Time-of-Flight Absorption Pictures



JILA (1995): $^{87}_{37}\text{Rb}$, $N=20\,000$, $\omega_1 = \omega_2 = \omega_3/\sqrt{8} = 2\pi \times 120 \text{ Hz}$

1.6 Periodic Table of Chemical Elements

Periode	□ = Hauptgruppen		□ = Nebengruppen		□ = Edelgase		Schale																																						
	I	II	IIIa	IVa	Va	VIa	VIIa	VIIIa	Ia	Ib	III	IV	V	VI	VII	VIII																													
1	1.008 1 H Wasserstoff															4.003 2 He Helium	K																												
2	6.941 3 Lithium	9.012 4 Beryllium									10.811 5 Bor	12.011 6 Kohlenstoff	14.007 7 Stickstoff	15.999 8 Sauerstoff	18.998 9 Fluor	20.180 10 Neon	L																												
3	22.990 11 Natrium	24.305 12 Magnesium									26.982 13 Aluminium	28.086 14 Silicium	30.974 15 Phosphor	32.066 16 Schwefel	35.453 17 Chlor	39.948 18 Argon	M																												
4	39.098 19 Kalium	40.078 20 Calcium	44.956 21 Scandium	47.88 22 Titan	50.942 23 Vanadin	51.996 24 Chrom	54.938 25 Mangan	55.847 26 Eisen	58.933 27 Kobalt	58.69 28 Nickel	63.546 29 Kupfer	65.39 30 Zink	69.723 31 Gallium	72.61 32 Germanium	74.922 33 Arsen	78.96 34 Selen	79.904 35 Brom	83.8 36 Krypton	N																										
5	85.468 37 Rubidium	87.62 38 Strontium	88.906 39 Yttrium	91.224 40 Zirkonium	92.906 41 Niob	95.94 42 Molybdän	98.906 43 Technetium	101.07 44 Rutherfordium	102.906 45 Rhodium	106.42 46 Palladium	107.868 47 Silber	112.411 48 Cadmium	114.82 49 Indium	118.71 50 Zinn	121.75 51 Antimon	127.6 52 Tellur	126.904 53 Iod	131.29 54 Xenon	O																										
6	132.905 55 Cäsium	137.327 56 Barium	138.906 57 Lanthan	178.49 72 Hafnium	180.948 73 Tantal	183.85 74 Wolfram	186.207 75 Rhenium	190.2 76 Osmium	192.22 77 Iridium	195.08 78 Platin	196.967 79 Gold	200.59 80 Quecksilber	204.383 81 Thallium	207.2 82 Blei	208.98 83 Bismut	208.982 84 Polonium	209.987 85 Astat	222.018 86 Radon	P																										
7	223.02 87 Francium	226.025 88 Radium	227.028 89 Actinium	261.109 104 Rutherfordium	262.114 105 Hahnium	263.118 106 Seaborgium	262.123 107 Nilsbohrium	ca. 265 108 Hassium	ca. 268 109 Meitnerium	ca. 269 110 Damastium	ca. 272 111 Roentgenium	ca. 277 112 ?	ca. 289 114 ?	ca. 289 116 ?	ca. 289 118 ?	ca. 293 118 ?		Q																											
	Lanthanide																																												
	Aggregatzustand unter Normalbedingungen:																																												
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Th	232.04 90 Thorium	Pa	231 91 Protactinium	U	238.03 92 Uran	Np	237 93 Neptunium	Pu	244 94 Plutonium	Am	243 95 Americium	Cm	247 96 Curium	Bk	247 97 Berkellium	Cf	251 98 Californium	Es	254 99 Einsteinium	Fm	257 100 Fermium	Md	258 101 Mendelevium	No	259 102 Nobelium	Lr	260 103 Lawrencium																		

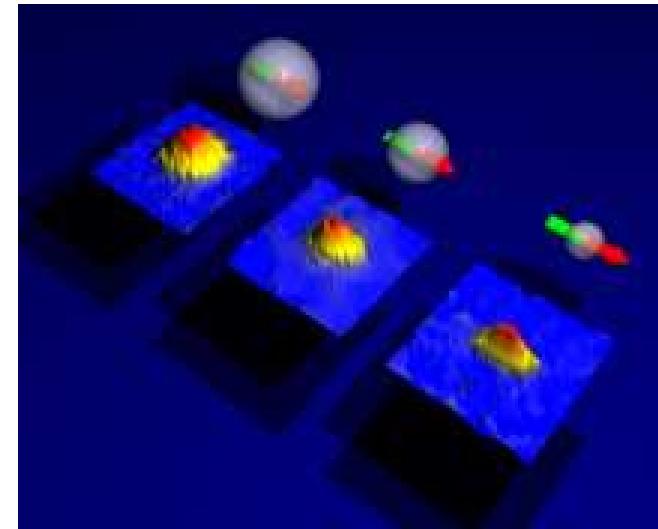
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2.1 Magnetic versus Electric Dipolar Systems

- **Magnetic systems:**

- Interaction strength: $C_{dd}^{\mathcal{B}} = \mu_0 m^2$, with $m \sim 1$ to $10 \mu_B$
- Realized samples

Boson: ^{52}Cr Griesmaier *et al.*, PRL **94**, 160401 (2005)

Boson: ^{87}Rb Vengalattore *et al.*, PRL **100**, 170403 (2008)

Fermion: ^{53}Cr Chicireanu *et al.*, PRA **73**, 053406 (2006)

Both: Dy Lu *et al.*, PRL **104**, 063001 (2010); PRL **107**, 190401 (2011)

- Observed effects: magnetostriiction (Cr), Bose-nova explosion (Cr)

- **Electric systems:**

- Interaction strength: $C_{dd}^{\mathcal{E}} = 4\pi d^2$, with $d \sim 1$ Debye
- Realized samples

Fermion: $^{40}\text{K}^{87}\text{Rb}$ Ospelkaus *et al.*, Science **32**, 231 (2008)

Boson: $^{41}\text{K}^{87}\text{Rb}$ Aikawa *et al.*, NJP **11**, 055035 (2009)

- Observed effects: thermalization ($^{40}\text{K}^{87}\text{Rb}$)

- **Ratio:** $C_{dd}^{\mathcal{B}}/C_{dd}^{\mathcal{E}} \approx 10^{-4}$

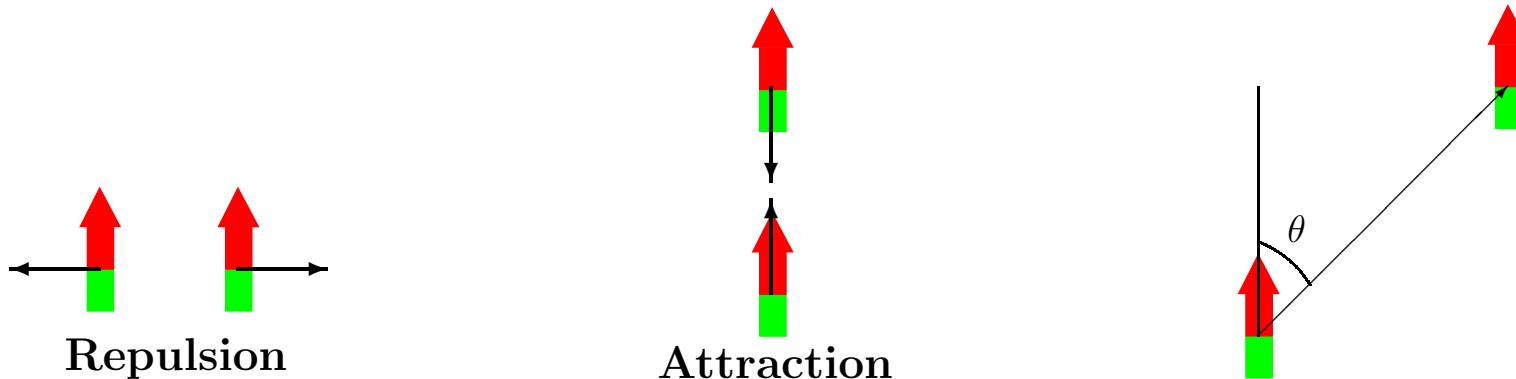
2.2 Trapping and Interaction Potentials

- **Harmonic trap:**

$$U_{\text{trap}}(\mathbf{x}) = \frac{M}{2} \omega^2 (x^2 + y^2 + \lambda^2 z^2)$$

- **Interaction potential:**

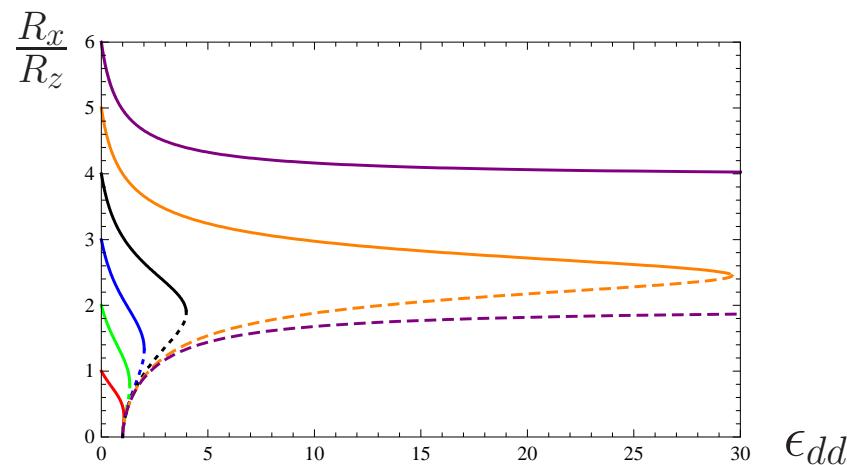
$$V_{\text{int}}(\mathbf{x} - \mathbf{x}') = g \left[\delta(\mathbf{x} - \mathbf{x}') + \frac{3\epsilon_{dd}}{4\pi|\mathbf{x} - \mathbf{x}'|^3} (1 - 3\cos^2 \theta) \right], \quad \epsilon_{dd} = \frac{C_{dd}}{3g}$$



2.3 Mean-Field Results ($T=0$)

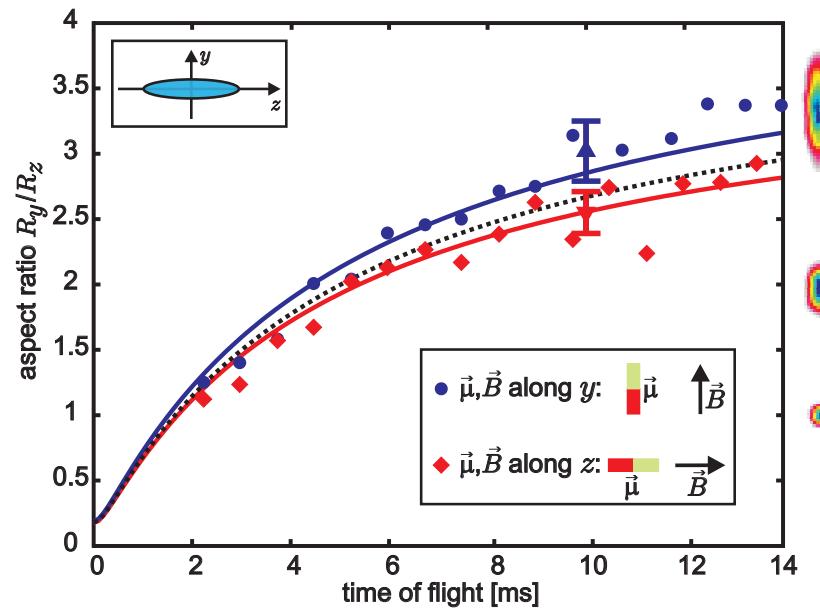
- **Aspect Ratio:**

Eberlein *et al.*,
PRA **71**, 033618 (2005)



- **Time-of-Flight:**

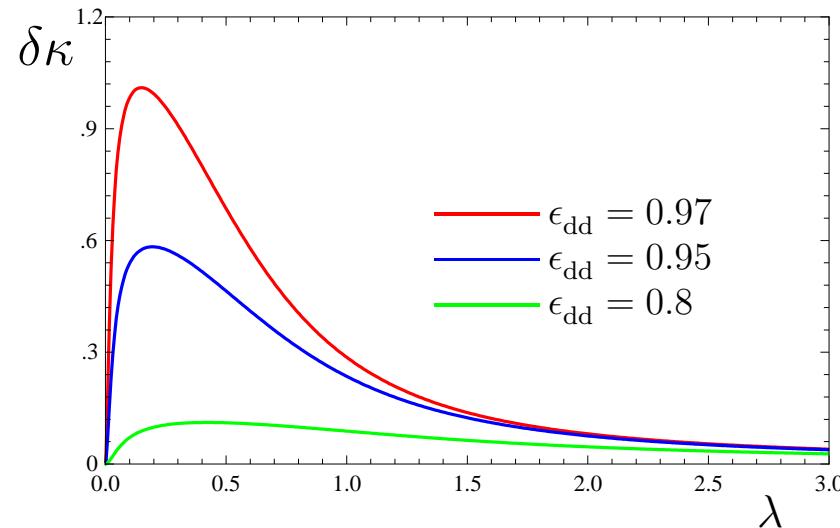
Stuhler *et al.*,
PRL **95**, 150406 (2005)



2.4 Beyond Mean-Field Results (T=0)

- **Aspect Ratio:**

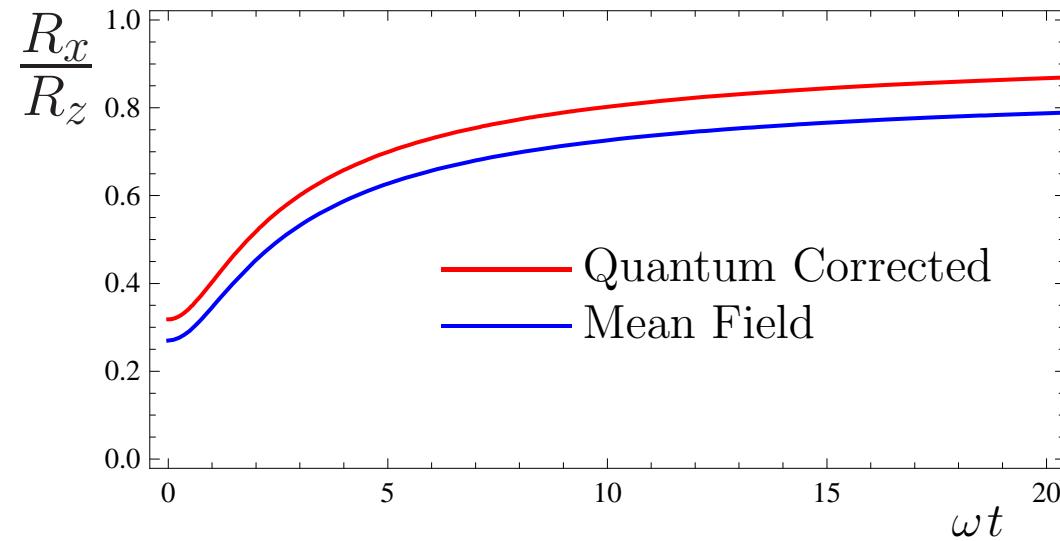
$$\frac{R_x}{R_z} = \kappa_{\text{MF}} (1 + \delta\kappa)$$



- **Time-of-Flight:**

expected for Dy

$$\epsilon_{dd} = 0.9$$



Lima and Pelster, PRA **84**, 041604(R) (2011); arXiv:1111.0900

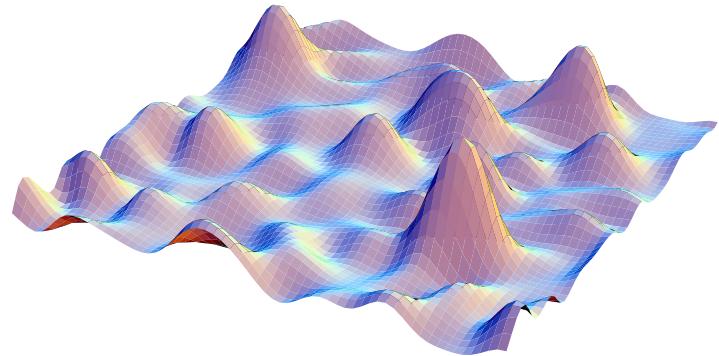
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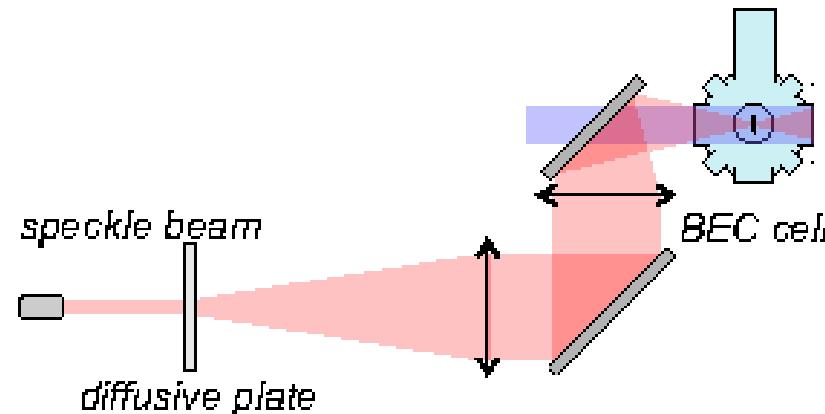


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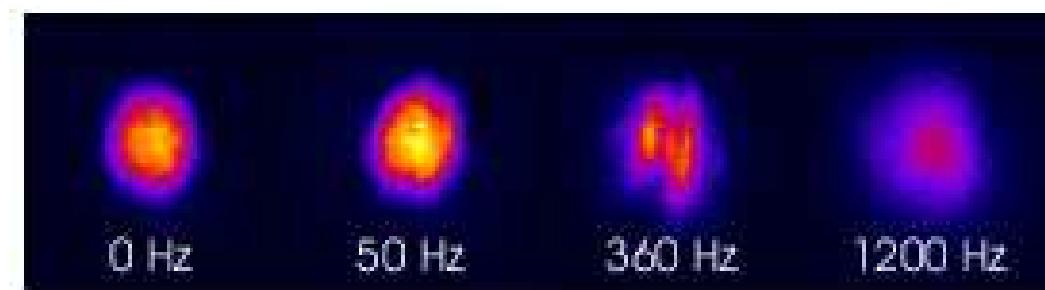


3.1 Laser Speckles: Controlled Randomness

Experimental Set-Up:

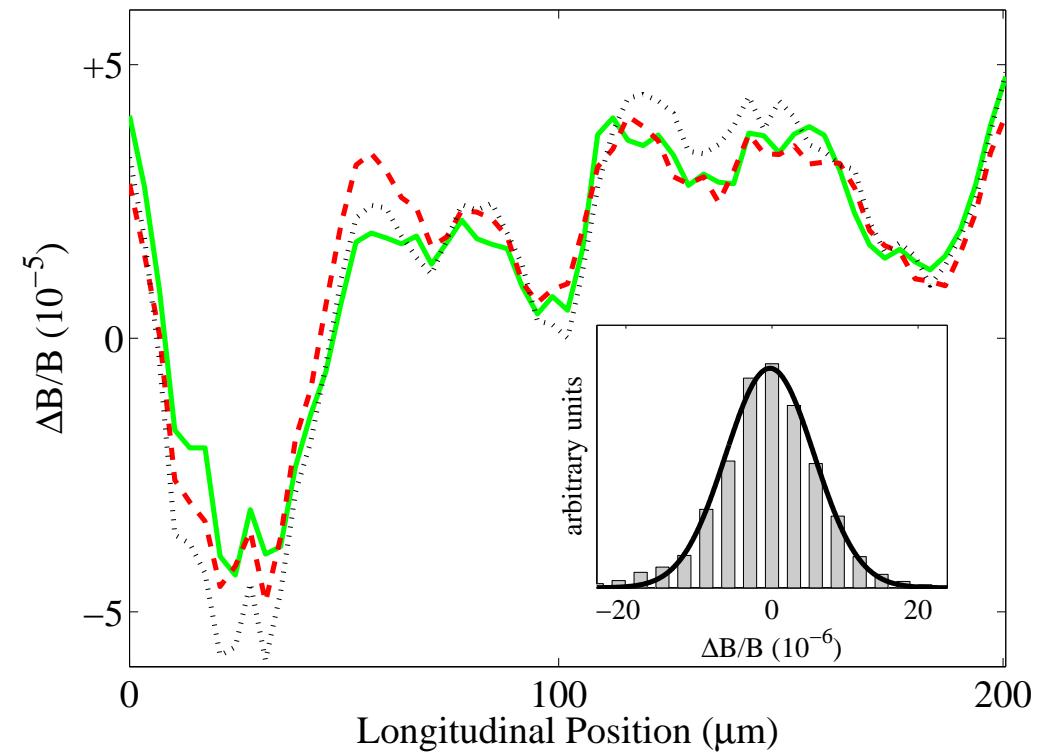
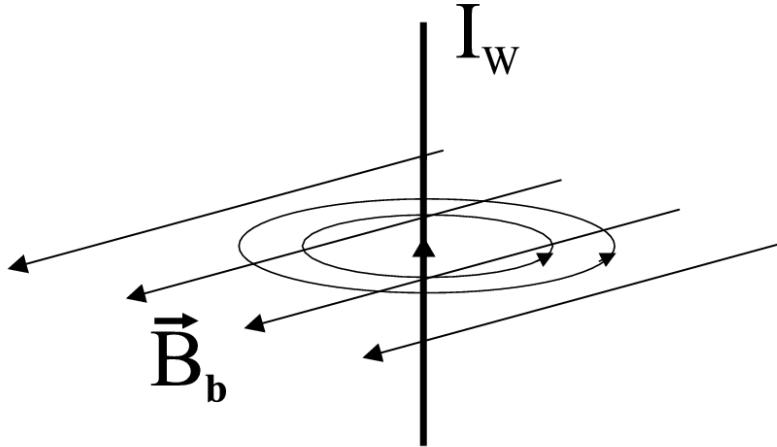


Fragmentation:



Lye *et al.*, PRL 95, 070401 (2005)

3.2 Wire Trap: Undesired Randomness



Distance: $d = 10 \mu\text{m}$

Wire Width: $100 \mu\text{m}$

Magnetic Field: 10 G, 20 G, 30 G

Deviation: $\Delta B/B \approx 10^{-4}$

Krüger *et al.*, PRA **76**, 063621 (2007)

Fortagh and Zimmermann, RMP **79**, 235 (2007)

3.3 Bogoliubov Theory of Dirty Bosons

Assumptions:

homogeneous Bose gas: $U(\mathbf{x}) = 0$

δ -correlated disorder: $R(\mathbf{x}) = R \delta(\mathbf{x})$

Condensate Depletion:

$$n_0 = n - \frac{8}{3\sqrt{\pi}} \sqrt{a n_0}^3 - \frac{M^2 R}{8\pi^{3/2} \hbar^4} \sqrt{\frac{n_0}{a}}$$

Superfluid Depletion:

$$n_s = n - n_n = n - \frac{4}{3} \frac{M^2 R}{8\pi^{3/2} \hbar^4} \sqrt{\frac{n_0}{a}}$$

Huang and Meng, PRL **69**, 644 (1992)

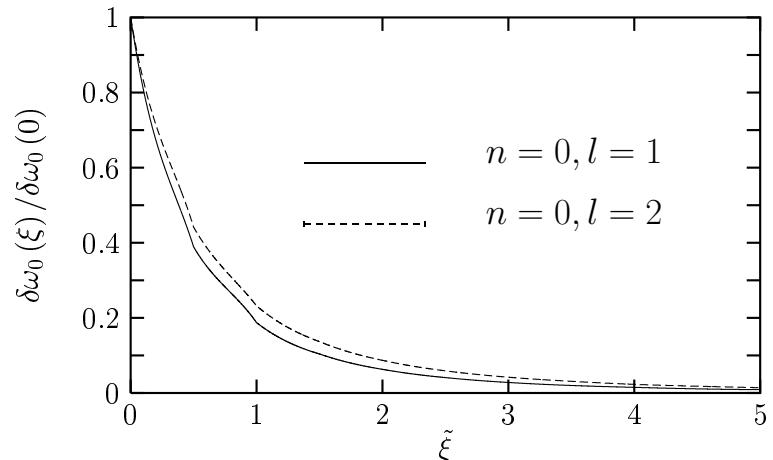
Falco, Pelster, and Graham, PRA **75**, 063619 (2007)

3.4 Collective Excitations

Typical Values:

Lye *et al.*, PRL **95**, 070401 (2005)

$$\left. \begin{array}{l} \xi = 10 \text{ } \mu\text{m} \\ R_{\text{TF}} = 100 \text{ } \mu\text{m} \\ l_{\text{HO}} = 10 \text{ } \mu\text{m} \end{array} \right\} \tilde{\xi} = \frac{\xi R_{\text{TF}}}{l_{\text{HO}}^2 \sqrt{2}} \approx 7$$



⇒ **Disorder effect vanishes in laser speckle experiment**

Improvement:

laser speckle setup with correlation length $\xi = 1 \text{ } \mu\text{m}$

Aspect *et al.*, NJP **8**, 165 (2006)

⇒ **Disorder effect should be measurable**

Falco, Pelster, and Graham, PRA **76**, 013624 (2007)

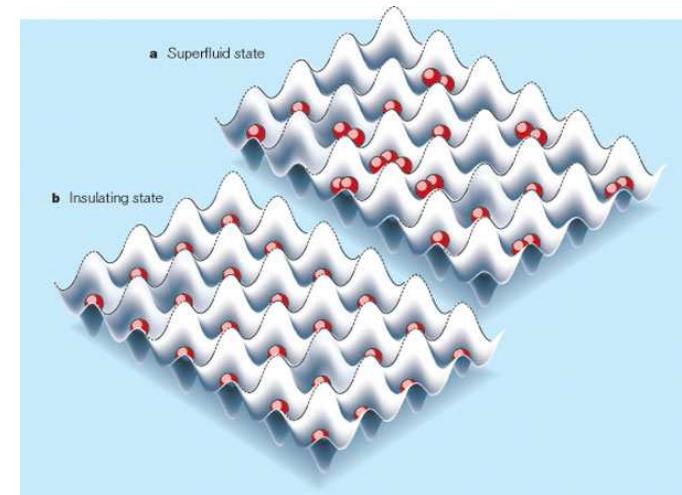
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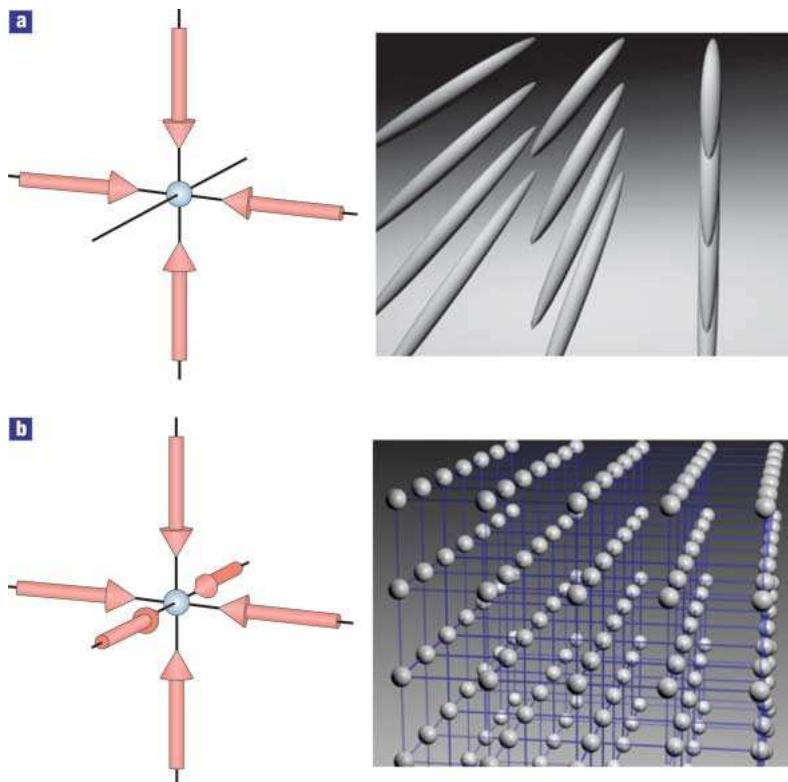


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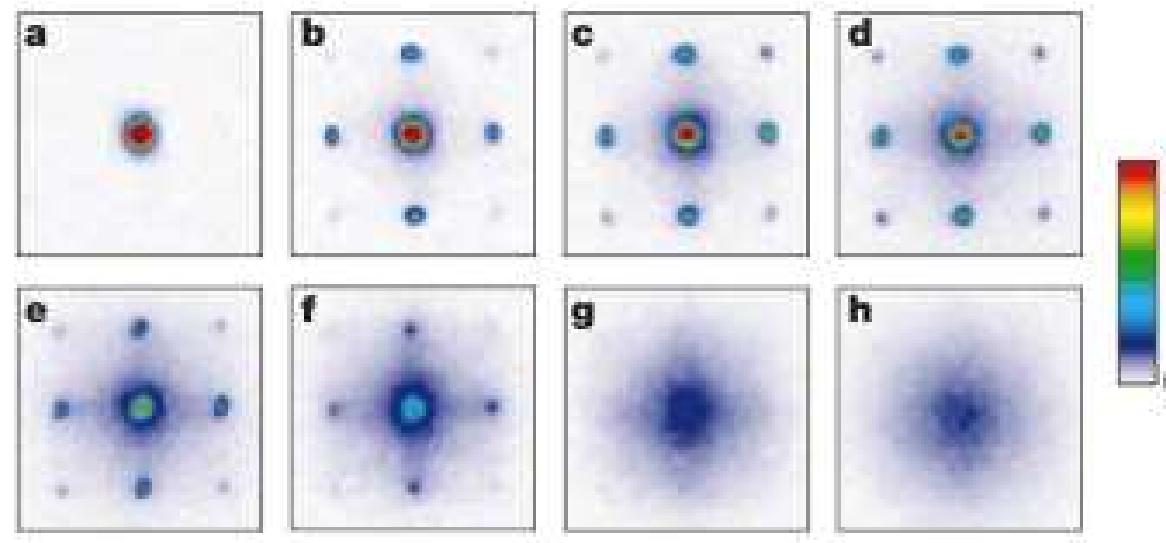
4.1 Optical Lattice

- Counter-propagating laser beams create periodic potential
- Different possible topologies at 1D, 2D, and 3D
- Hopping and interactions are highly controllable



4.2 Time-of-Flight Absorption Pictures

- **Superfluid phase:**
delocalization in space, localization in Fourier space
- **Mott phase:**
localization in space, delocalization in Fourier space



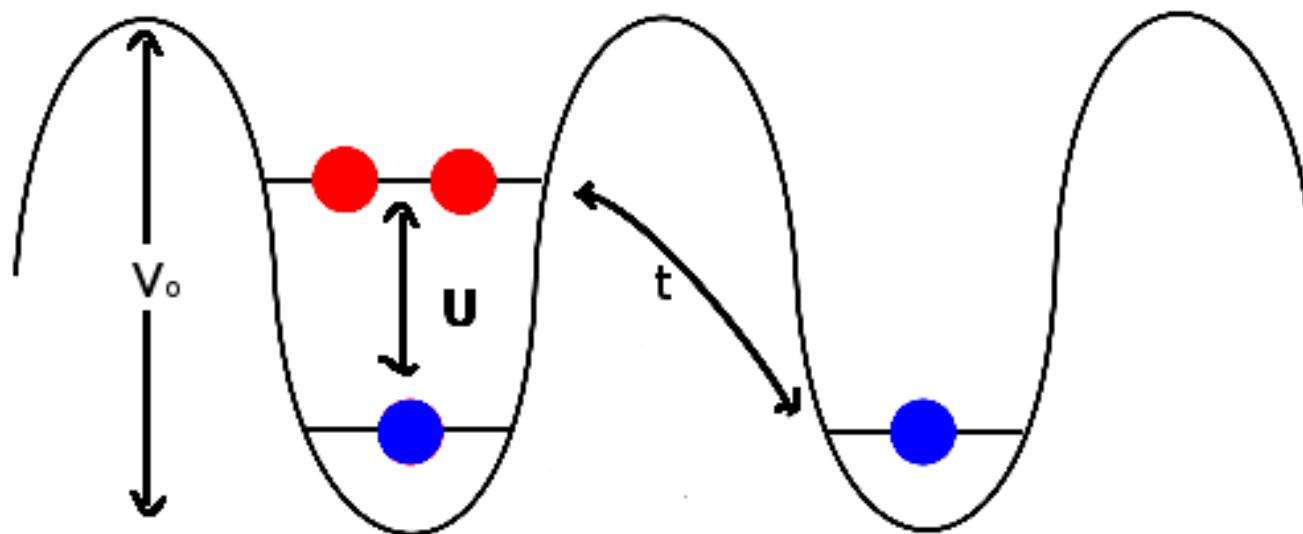
Greiner, Mandel, Esslinger, Hänsch, and Bloch, Nature **415**, 39 (2002)

4.3 Theoretical Description

Bose-Hubbard Hamiltonian:

$$\hat{H}_{\text{BH}} = -t \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \left[\frac{U}{2} \hat{n}_i (\hat{n}_i - 1) - \mu \hat{n}_i \right], \quad \hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$$

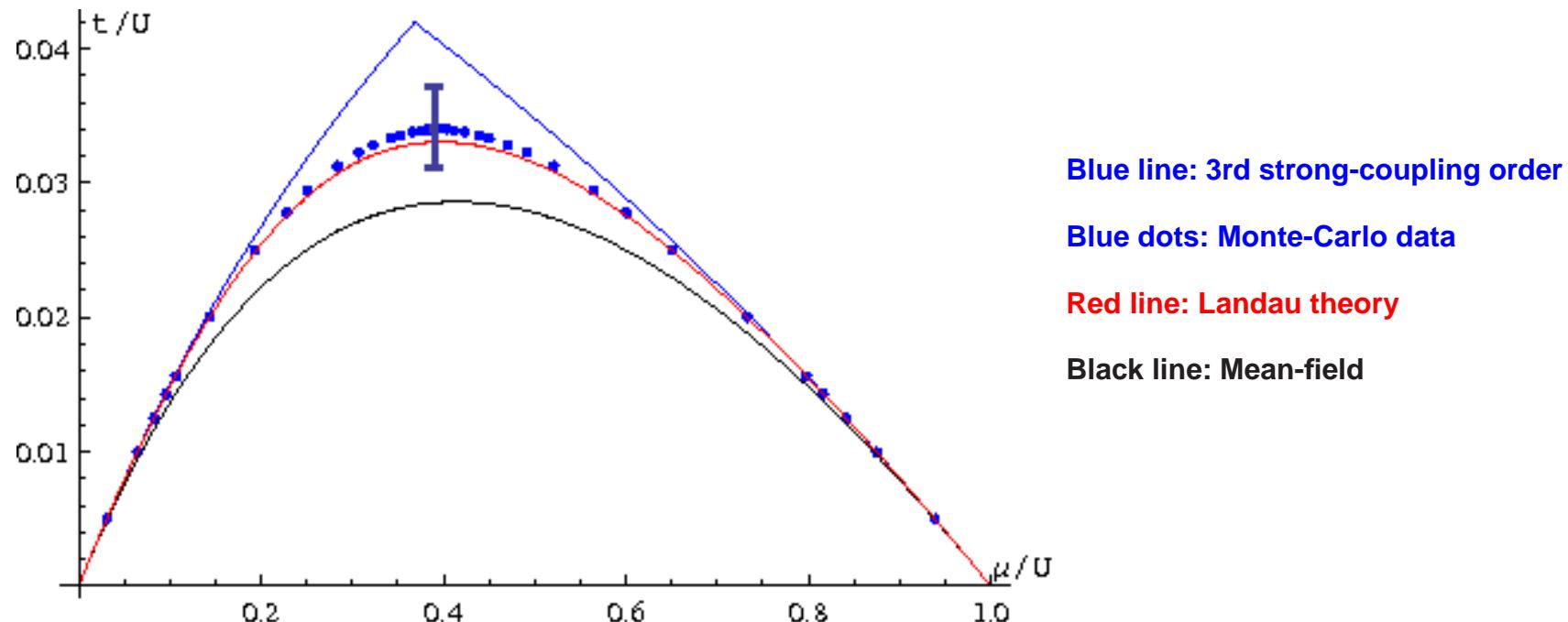
System Parameters:



Fisher *et al.*, PRB 40, 546 (1989)

4.4 Landau Theory

Quantum Phase Diagram ($T=0$):



Santos and Pelster, PRA **79**, 013614 (2009)

Extension to Higher Orders:

Teichmann, Hinrichs, Holthaus, and Eckardt, PRB **79**, 100503(R) (2009)

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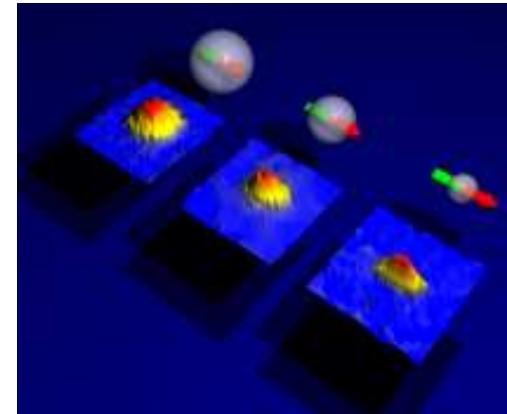
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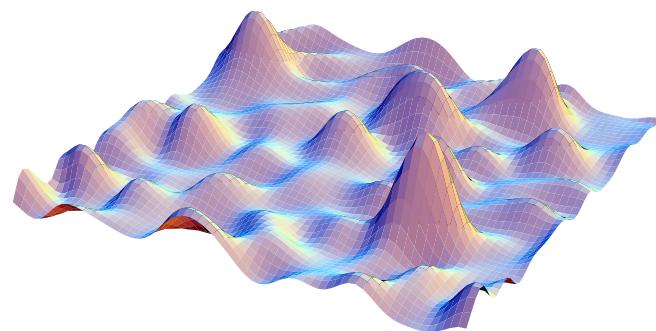
5.1 Summary and Outlook



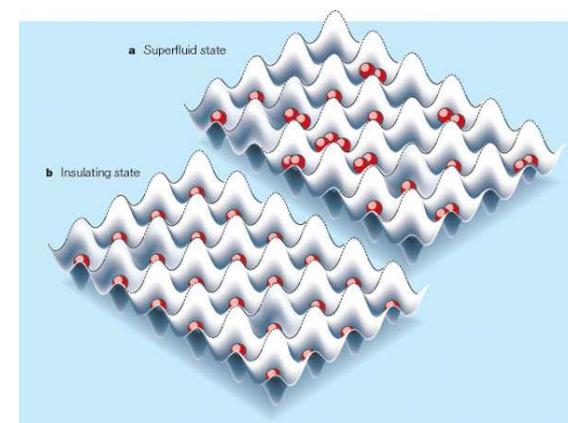
quantum simulation



anisotropic superfluidity



strong disorder



driven optical lattices

5.2 Announcements

**1) School Low-Dimensional Quantum Many-Body Systems
of Collaborative Research Center SFB/TR 49**

organized by Andreas Kreisel and Axel Pelster

Hotel Deutscher Hof, Trier

August 16 – 21, 2012

2) 514th Wilhelm and Else Heraeus Seminar

Quo Vadis BEC? IV

organized by Carlos Sá de Melo and Axel Pelster

Physikzentrum, Bad Honnef

August 21 – 25, 2012

<http://users.physik.fu-berlin.de/~pelster>

5.3 Acknowledgement

Postdocs:

- Aristeu Lima (CAPES)
- Ednilson Santos (FAPESP)

PhD students:

- Javed Akram (DAAD)
- Hamid Al-Jibbouri (DAAD)
- Mahmoud Ghabour
- Dennis Hinrichs (Oldenburg)
- Tama Khellil (DAAD)
- Mohamed Mobarek (Egyp. Gov.)
- Tao Wang (CSC)

Diploma students:

- Max Lewandowski (Potsdam)
- Tobias Rexin (Potsdam)
- Falk Wächtler (Potsdam)

Bachelor students:

- Tomasz Checinski (Bielefeld)
- Christian Krumnow (FU Berlin)
- Johannes Lohmann (FU Berlin)
- Moritz von Hase (FU Berlin)
- Carolin Wille (FU Berlin)
- Nikolas Zöller (FU Berlin)

Volkswagen: Bakhodir Abdullaev, Abdulla Rakhimov (Tashkent)

DAAD: Antun Balaz, Vladimir Lukovic, Branko Nikolic (Belgrade)

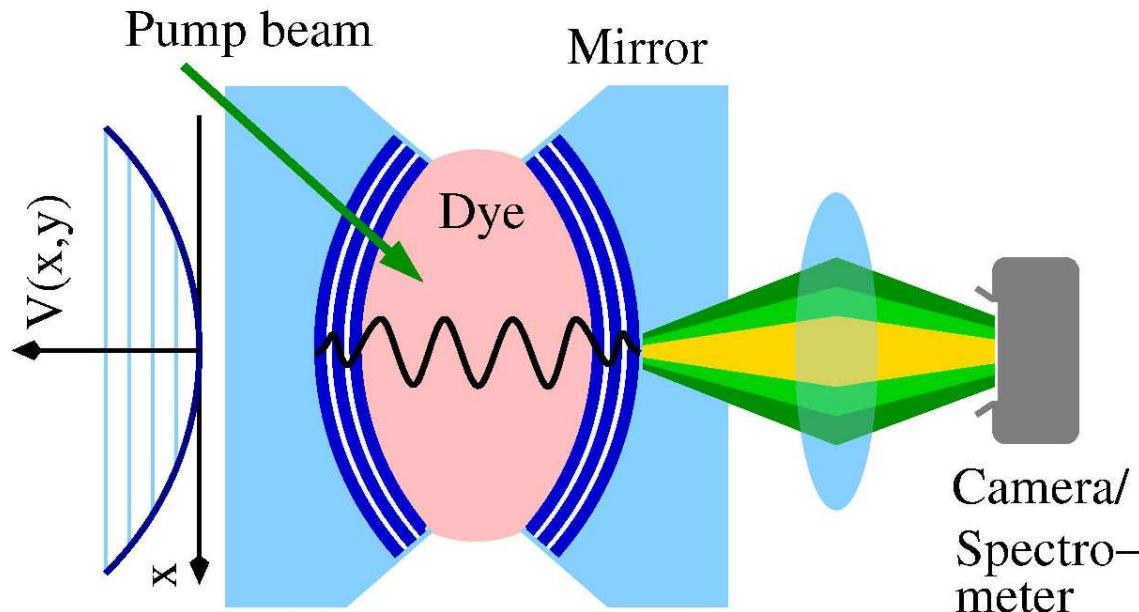
Mentors: Robert Graham (Duisburg-Essen), **Martin Holthaus (Oldenburg)**, Hagen Kleinert (FU Berlin)

5.4 Many Thanks!

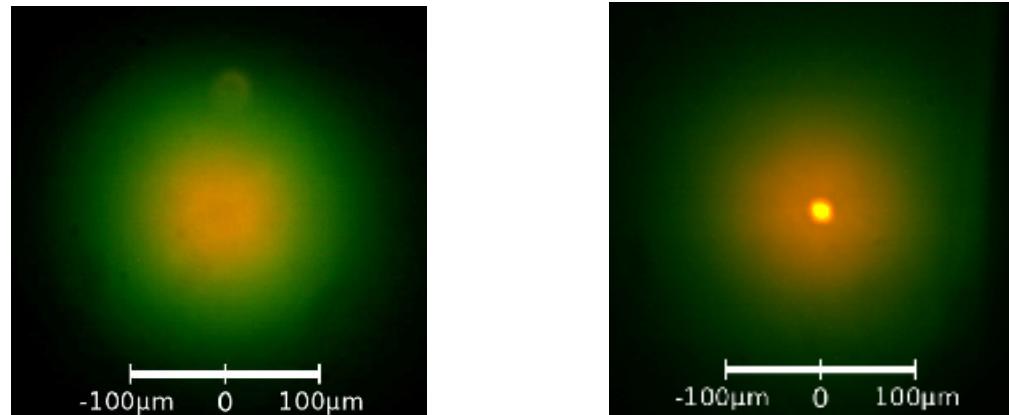


5.5 Bose-Einstein Condensation of Light

Set-Up



Result



Klaers, Schmitt, Vewinger, and Weitz, Nature **468**, 545 (2010)

Pelster, Physik-Journal **10**, Nr. 1, 20 (2011)