The thermodynamic properties of the Bose-Einstein condensates using the recursive canonical approach

Jonata S. Soares - USP - SP - Physics department

Prof: Dr Arnaldo Gammal and Dr Axel Pelster

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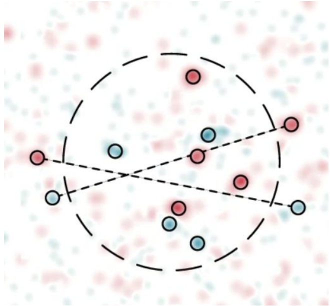
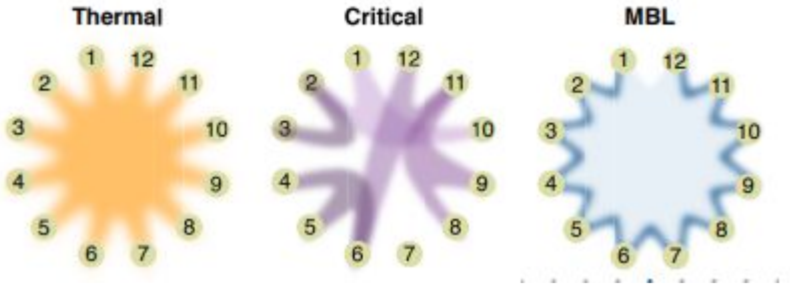
- Connections between grand-canonical ensemble and canonical ensemble
- Non-interacting canonical theory
- Weakly interacting canonical theory - first order perturbation theory

Motivation

Ideal Bose Gas revisited → R.M. Ziff, G.E. Uhlenbeck, and M. Kac, The Ideal Bose-Einstein Gas, Revisited, Phys. Reports 32, 169 (1977)



Holten, M., Bayha, L., Subramanian, K. *et al.* Observation of Cooper pairs in a mesoscopic two-dimensional Fermi gas. *Nature* 606, 287–291 (2022)



Rispoli, M., Lukin, A., Schittko, R. *et al.* Quantum critical behaviour at the many-body localization transition. *Nature* 573, 385–389 (2019)

Connections between grand-canonical and canonical ensemble

- Our idea is to calculate the thermodynamic properties of a confined Bose gas for a fixed number of particles in a

- To build a canonical theory, let us start with grand-canonical theory because there are mathematical connections between these two distributions

Connections between grand-canonical and canonical ensemble

We know that: $$\Xi(\beta, z) = \mathrm{Tr}\left(e^{-\beta(\hat{H}-\mu\hat{N})}\right)$$

$$\hat{H} = \sum_{\mathbf{k}} E_{\mathbf{k}}\, a^\dagger_{\mathbf{k}} a_{\mathbf{k}}$$

$$\mathrm{Tr}(\hat{A}) = \prod_{\mathbf{k}} \sum_{n_{\mathbf{k}}} \langle n_{\mathbf{k}}|\hat{A}|n_{\mathbf{k}}\rangle$$

$$\hat{N} = \sum_{\mathbf{k}} a^\dagger_{\mathbf{k}} a_{\mathbf{k}}$$

Connections between grand-canonical and canonical ensemble

Therefore, $$\Xi(\beta, z) = \sum_{N=0}^{\infty} Z_N(\beta)\, z^N$$

$$Z_N(\beta) = \frac{1}{N!} \frac{\partial^N \Xi(\beta, z)}{\partial z^N}\bigg|_{z=0}$$

$$\mathcal{F} = -\frac{1}{\beta} \ln \Xi(\beta, z)$$

- Non-interacting canonical theory

Important observation: the non-interacting quantities will be represented by (0) superscript

$$\log \Xi^{(0)}(\beta, z) = -\sum_{\mathbf{k}} \log\left(1 - e^{-\beta(E_{\mathbf{k}}-\mu)}\right) \longrightarrow \Xi^{(0)}(\beta, z) = \exp\left(\sum_{n=1}^{\infty} \frac{z^n}{n} \sum_{\mathbf{k}} e^{-n\beta E_{\mathbf{k}}}\right)$$

Final GC result:

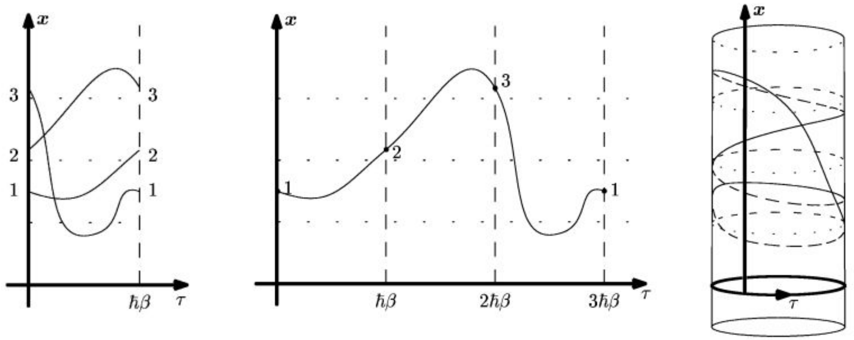
Final recursive formula:

$$Z_N^{(0)}(\beta) = \frac{1}{N!} \frac{\partial^N \Xi^{(0)}(\beta, z)}{\partial z^N}\bigg|_{z=0} \longrightarrow Z_N^{(0)}(\beta) = \frac{1}{N} \sum_{n=1}^{N} Z_1(n\beta) Z_{N-n}^{(0)}(\beta)$$

Starting point: $Z_0^{(0)}(\beta) \equiv 1$ (Vacuum)

- Non-interacting canonical theory

where $$Z_1(3\beta) = \sum_{\mathbf{k}} e^{-3\beta E_{\mathbf{k}}}$$



general : $$Z_1(n\beta) \equiv \sum_{\mathbf{k}} e^{-n\beta E_{\mathbf{k}}}$$

With $Z_N^B(\beta)$

Heat capacity $$C_N^B(T) = k_B\, T \frac{d^2}{dT^2}\left(T \ln Z_N^B(T)\right)$$

- ## Non-interacting canonical theory

$$Z_1(n\beta) = \gamma_1^n(\beta) + \xi_n(\beta)$$

$$\gamma_1^n(\beta) \equiv e^{-n\beta E_G} \qquad \xi_n(\beta) \equiv \sum_{\mathbf{k}\neq 0} e^{-n\beta E_{\mathbf{k}}}$$

$$P_C(n,\beta) = \gamma_1^n(\beta)\,\frac{Z^B_{N-n}(\beta)}{Z^B_N(\beta)} - \gamma_1^{n+1}(\beta)\,\frac{Z^B_{N-n-1}(\beta)}{Z^B_N(\beta)}$$

$$\langle n^m \rangle = \sum_{n=1}^{N} n^m\, P_C(n,\beta)$$

$$\frac{\langle N_0 \rangle}{N} = \frac{1}{N}\sum_{n=1}^{N} \gamma_1^n(\beta)\frac{Z^B_{N-n}(\beta)}{Z^B_N(\beta)}$$

Condensed fraction

$$\frac{\Delta N_0}{N} = \frac{1}{N}\sqrt{\sum_{n=1}^{N}(2n-1)\,\gamma_1^n(\beta)\frac{Z^B_{N-n}(\beta)}{Z^B_N(\beta)} - \left[\sum_{n=1}^{N}\gamma_1^n(\beta)\frac{Z^B_{N-n}(\beta)}{Z^B_N(\beta)}\right]^2}$$

Ground-state fluctuation

- Non-interacting canonical theory - Results

Harmonic trap

$$V(\mathbf{x}) = \frac{M}{2}\sum_{j=1}^{3}\omega_j^2 x_j^2$$

$$\downarrow$$

$$E_{\mathbf{k}} = E_G + \hbar\sum_{j=1}^{3}\omega_j n_j$$

$$n_j \in \mathbb{N}_0$$

Finite box

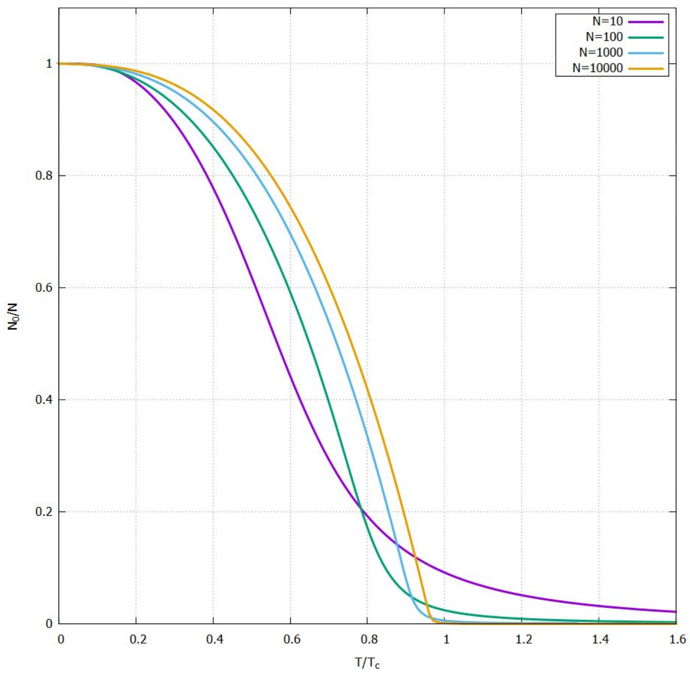
$$V(\mathbf{x}) = 0 \qquad 0 \leq |x_j| \leq L$$

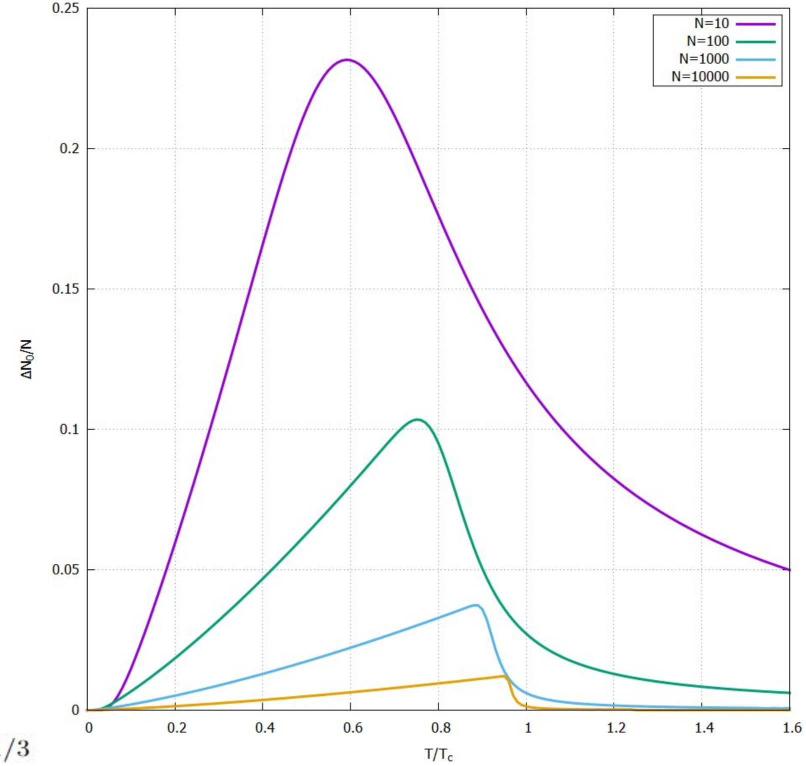
$$\downarrow$$

$$E_{\mathbf{k}} = \frac{\hbar^2}{2M}\sum_{j=1}^{3}k_j^2 \qquad k_j = \frac{n_j\pi}{L}$$

$$n_j \in \mathbb{N}$$

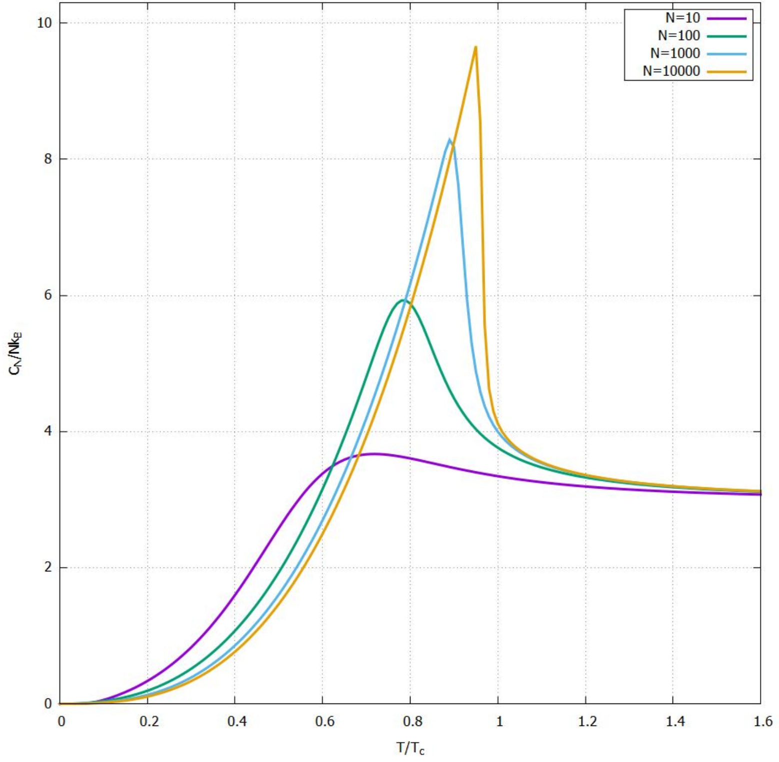
- Non-interacting canonical theory - Results - Harmonic trap





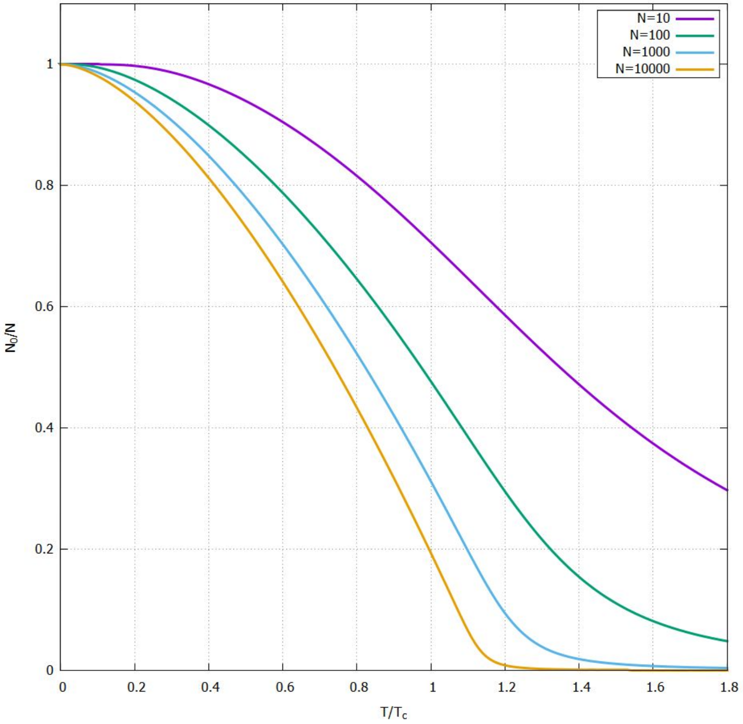
$$T_c = \frac{\hbar\tilde{\omega}}{k_B}\left(\frac{N}{\zeta(3)}\right)^{1/3}$$

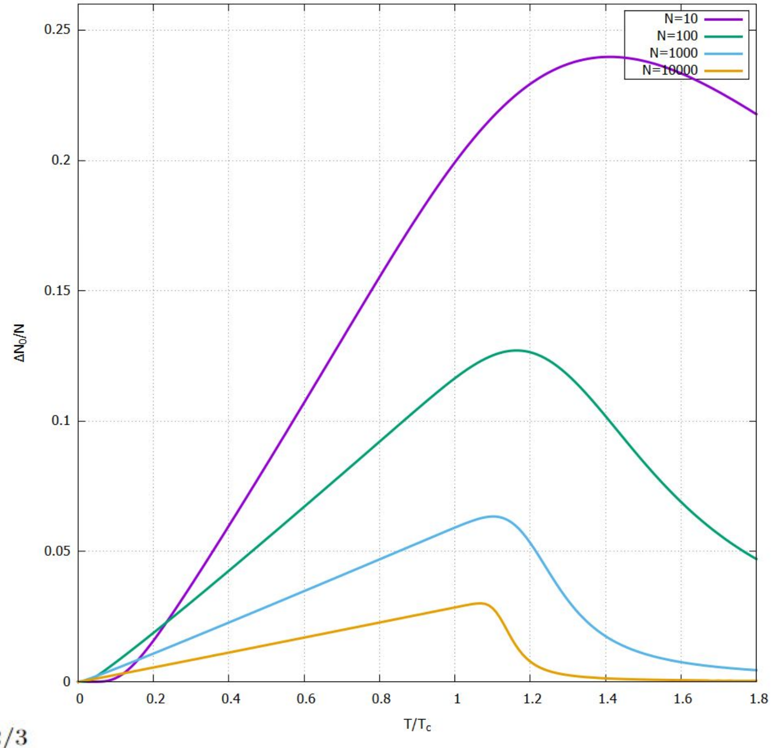
- Non-interacting canonical theory - Results - Harmonic trap



$$T_c = \frac{\hbar\tilde{\omega}}{k_B}\left(\frac{N}{\zeta(3)}\right)^{1/3}$$

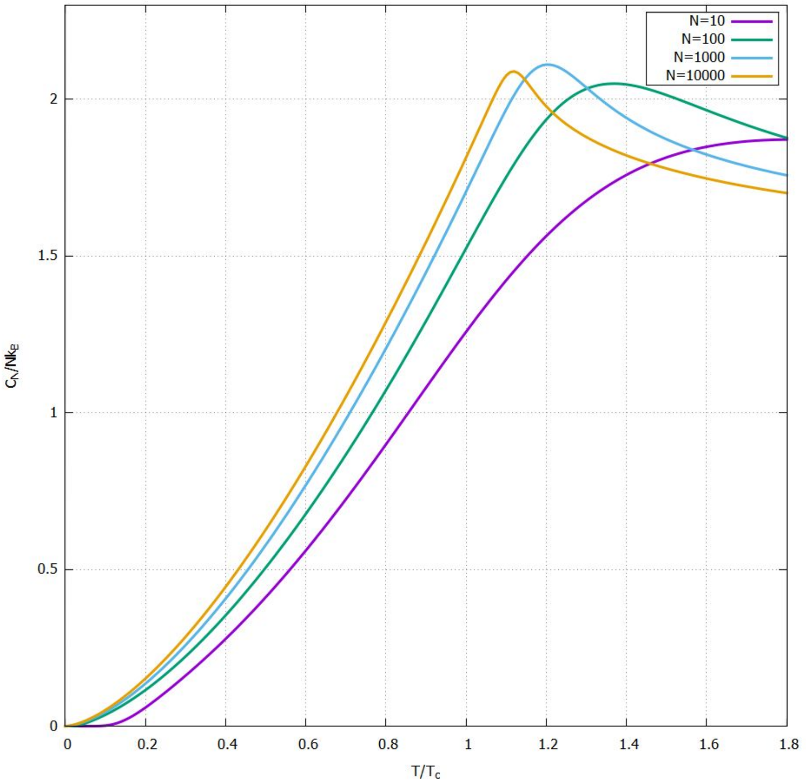
- Non-interacting canonical theory - Results - Finite box





$$T_c = \frac{2\pi\hbar^2}{Mk_B}\left(\frac{N}{V\,\zeta(3/2)}\right)^{2/3}$$

- Non-interacting canonical theory - Results - Finite box



$$T_c = \frac{2\pi\hbar^2}{Mk_B}\left(\frac{N}{V\,\zeta(3/2)}\right)^{2/3}$$

And when we include the interactions - what is happening?

- Interacting canonical theory

Let us start with the grand-canonical representation of the partition function

$$\Xi = \oint D\psi^* \, D\psi \, e^{-\left(A^{(0)}[\psi^*,\psi]+A^{(\mathrm{int})}[\psi^*,\psi]\right)/\hbar}$$

$$\downarrow$$

$$\Xi = \Xi^{(0)} \left(1 - \frac{1}{2\hbar}\int_0^{\hbar\beta} d\tau \int d^3x \, d^3x' \, V^{(\mathrm{int})}(\mathbf{x}-\mathbf{x}')\langle \psi^*(\mathbf{x},\tau)\,\psi(\mathbf{x},\tau)\,\psi^*(\mathbf{x}',\tau)\psi(\mathbf{x}',\tau)\rangle_{(0)}\right)$$

- Interacting canonical theory

By Wick's theorem,

$$\langle\psi^*(\mathbf{x},\tau)\,\psi(\mathbf{x},\tau)\,\psi^*(\mathbf{x}',\tau)\psi(\mathbf{x}',\tau)\rangle_{(0)} = \langle\psi^*(\mathbf{x},\tau)\,\psi(\mathbf{x},\tau)\rangle_{(0)}\langle\psi^*(\mathbf{x}',\tau)\,\psi(\mathbf{x}',\tau)\rangle_{(0)}+\langle\psi^*(\mathbf{x},\tau)\,\psi(\mathbf{x}',\tau)\rangle_{(0)}\langle\psi^*(\mathbf{x}',\tau)\,\psi(\mathbf{x},\tau)\rangle_{(0)}$$

Green's function:

$$\langle\psi(\mathbf{x},\tau)\,\psi^*(\mathbf{x}',\tau)\rangle_{(0)} = G^{(0)}(\mathbf{x},\tau;\mathbf{x}',\tau) = \lim_{\tau'\downarrow\tau} G^{(0)}(\mathbf{x},\tau;\mathbf{x}',\tau') = \sum_{\mathbf{k}} \psi_{\mathbf{k}}(\mathbf{x})\psi^*_{\mathbf{k}}(\mathbf{x}')\frac{1}{e^{\beta(E_{\mathbf{k}}-\mu)}-1}$$

Bose-Einstein distribution: $$\frac{1}{e^{\beta(E_{\mathbf{k}}-\mu)}-1} = \sum_{n=1}^{\infty} e^{-n\beta(E_{\mathbf{k}}-\mu)} = \sum_{n=1}^{\infty} z^n\, e^{-n\beta E_{\mathbf{k}}}$$

- Interacting canonical theory

$$G^{(0)}(\mathbf{x},\tau;\mathbf{x}',\tau) = \sum_{n=1}^{\infty} z^n \left(\mathbf{x}, n\hbar\beta; \mathbf{x}', 0\right)_{(0)} \qquad z \equiv e^{\beta\mu}$$ (Fugacity)

One-particle Schrödinger propagator: $\left(\mathbf{x}, n\hbar\beta; \mathbf{x}', 0\right)_{(0)} = \sum_{\mathbf{k}} \psi^*_{\mathbf{k}}(\mathbf{x}')\, \psi_{\mathbf{k}}(\mathbf{x})\, e^{-n\beta E_{\mathbf{k}}}$

$$\Xi^{(1)}(\beta, z) = -\frac{\beta}{2} \int d^3x\, d^3x'\, V^{(int)}(\mathbf{x}-\mathbf{x}')\left([H]+[F]\right)$$

$$[H] \equiv \sum_{k,l=1}^{\infty} z^{k+l} \left(\mathbf{x}, \mathbf{k}\hbar\beta; \mathbf{x}, \mathbf{0}\right)_{(0)} \left(\mathbf{x}', \mathbf{l}\hbar\beta; \mathbf{x}', \mathbf{0}\right)_{(0)} \Xi^{(0)}(\beta, z)$$

$$[F] \equiv \sum_{k,l=1}^{\infty} z^{k+l} \left(\mathbf{x}, \mathbf{k}\hbar\beta | \mathbf{x}', \mathbf{0}\right)_{(0)} \left(\mathbf{x}', \mathbf{l}\hbar\beta | \mathbf{x}, \mathbf{0}\right)_{(0)} \Xi^{(0)}(\beta, z)$$

- Interacting canonical theory

$$\Xi_H^{(1)}(\beta, z) = -\frac{\beta}{2} \sum_{N=0}^{\infty} \sum_{k,l=1}^{\infty} Z_N^{(0)}(\beta) z^{N+k+l} \int d^3x\, d^3x'\, V^{(int)}(\mathbf{x} - \mathbf{x}')\, (\mathbf{x}, \mathbf{k\hbar}\beta; \mathbf{x}, \mathbf{0})_{(\mathbf{0})}\, (\mathbf{x}', \mathbf{l\hbar}\beta; \mathbf{x}', \mathbf{0})_{(\mathbf{0})}$$

Doing a transformation $N' \equiv N + k + l$

$$Z_N^{(H)}(\beta) = -\frac{1}{2\hbar} \sum_{k=2}^{N} \sum_{l=1}^{k-1} I_{l,k-l}^{(D)}(\beta)\, Z_{N-k}^{(0)}(\beta)$$

The same for the Fock term

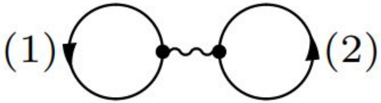
$$Z_N^{(F)}(\beta) = -\frac{1}{2\hbar} \sum_{k=2}^{N} \sum_{l=1}^{k-1} I_{l,k-l}^{(E)}(\beta)\, Z_{N-k}^{(0)}(\beta)$$

$$I_{l,k-l}^{(D)}(\beta) \equiv \hbar\beta \int d^3x\, d^3x'\, V^{(\text{int})}(\mathbf{x} - \mathbf{x}')\, (\mathbf{x}, l\hbar\beta; \mathbf{x}, 0)_{(0)}\, (\mathbf{x}', (k-l)\hbar\beta; \mathbf{x}', 0)_{(0)}$$

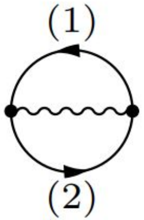
$$I_{l,k-l}^{(E)}(\beta) \equiv \hbar\beta \int d^3x\, d^3x'\, V^{(\text{int})}(\mathbf{x} - \mathbf{x}')\, (\mathbf{x}, n\hbar\beta; \mathbf{x}', 0)_{(0)}\, (\mathbf{x}', (k-l)\hbar\beta; \mathbf{x}, 0)_{(0)}$$

- Interacting canonical theory

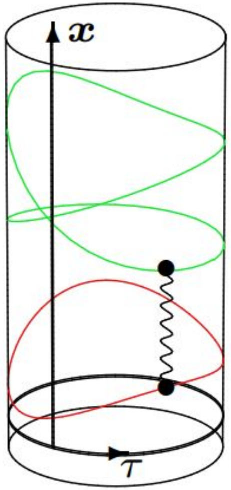
$$I_{1,2}^{(D)} \equiv$$



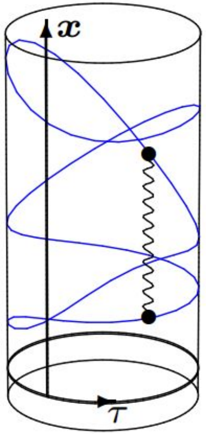
$$I_{1,2}^{(E)} \equiv$$



$$I_{1,2}^{(D)}(\beta) \equiv$$



$$I_{1,2}^{(E)}(\beta) \equiv$$



- Interacting canonical theory

Therefore, the N-particle partition function is given by

$$Z_N^B(\beta) = Z_N^{(0)}(\beta) - \frac{1}{2\hbar}\sum_{k=2}^{N}\sum_{l=1}^{k-1}\left[I^{(D)}_{l,k-l}(\beta) + I^{(E)}_{l,k-l}(\beta)\right] Z^{(0)}_{N-k}(\beta)$$

Next step - To write a recursive formula

$$Z^{(0)}_{N-n}(\beta) = Z^B_{N-n}(\beta) + \frac{1}{2\hbar}\sum_{k=2}^{N}\sum_{l=1}^{n-1}\left(I^{(D)}_{l,n-l}(\beta) + I^{(E)}_{l,n-l}(\beta)\right) Z^{(0)}_{N-k-n}(\beta)$$

Final formula: $$Z_N^B(\beta) = \frac{1}{N}\sum_{n=1}^{N}\left[Z_1(n\beta) - \frac{n}{2\hbar}\sum_{l=1}^{n-1}\left(I^{(D)}_{l,n-l}(\beta) + I^{(E)}_{l,n-l}(\beta)\right)\right] Z^B_{N-n}(\beta)$$

- Interacting canonical theory

Problem:

$$I_{l,n-l}^{(D),(E)}(\beta \to \infty) >> Z_1(n\beta \to \infty) \Rightarrow Z_N^B(\beta \to \infty) < 0$$

Solution:

$$Z_1(n\beta) \to \tilde{Z}_1(n\beta) = \sum_{\mathbf{k}} e^{-n\beta E_{\mathbf{k}}^{(n)}}$$

$$E_{\mathbf{k}}^{(n)} = E_{\mathbf{k}} + \text{cyclic interacting contribution}$$

- Interacting canonical theory

Fully interacting Green's function:

$$G(\mathbf{x},\tau;\mathbf{x}',\tau') = \frac{1}{\Xi}\oint D\psi^*\, D\psi\;\psi(\mathbf{x},\tau)\psi^*(\mathbf{x}',\tau')\,e^{-A[\psi^*,\psi]/\hbar}$$

$$\downarrow$$

$$G(\mathbf{x},\tau;\mathbf{x}',\tau') = \langle\psi(\mathbf{x},\tau)\psi^*(\mathbf{x}',\tau')\rangle_{(0)} - \frac{1}{\hbar}\langle\psi(\mathbf{x},\tau)\psi^*(\mathbf{x}',\tau')A^{(int)}[\psi^*,\psi]\rangle_{(0)} + \frac{1}{\hbar}\langle A^{(int)}[\psi^*,\psi]\rangle_{(0)}\langle\psi(\mathbf{x},\tau)\psi^*(\mathbf{x}',\tau')\rangle_{(0)}$$

$$\downarrow$$

Dyson's equation:

$$G(\mathbf{x},\tau;\mathbf{x}',\tau') = G^{(0)}(\mathbf{x},\tau;\mathbf{x}',\tau') + \int_0^{\hbar\beta} d\tau\,d\tau' \int d^3x''\,d^3x'''\,G^{(0)}(\mathbf{x},\tau;\mathbf{x}'',\tau'')\,\Sigma(\mathbf{x}'',\tau'';\mathbf{x}''',\tau''')\,G(\mathbf{x}''',\tau''';\mathbf{x}',\tau')$$

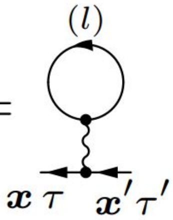
- Interacting canonical theory

Self-energy: $$\Sigma(\mathbf{x},\tau;\mathbf{x}',\tau') = \Sigma^{(D)}(\mathbf{x},\tau;\mathbf{x}',\tau') + \Sigma^{(E)}(\mathbf{x},\tau;\mathbf{x}',\tau')$$

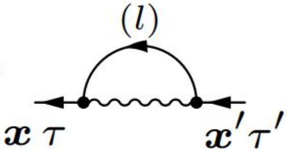
$$\Sigma^{(D)}(\mathbf{x},\tau;\mathbf{x}',\tau') = -\frac{1}{\hbar}\,\delta(\mathbf{x}-\mathbf{x}')\,\delta(\tau-\tau')\int d^3x''\,V^{(int)}(\mathbf{x}-\mathbf{x}'')\,G^{(0)}(\mathbf{x}'',\tau;\mathbf{x}'',\tau)$$

$$\Sigma^{(E)}(\mathbf{x},\tau;\mathbf{x}',\tau') = -\frac{1}{\hbar}\,\delta(\tau-\tau')V^{(int)}(\mathbf{x}-\mathbf{x}')\,G^{(0)}(\mathbf{x}',\tau;\mathbf{x}',\tau)$$

$$\Sigma_l^{(D)}(\boldsymbol{x},\tau;\boldsymbol{x}',\tau') =$$



$$\Sigma_l^{(E)}(\boldsymbol{x},\tau;\boldsymbol{x}',\tau') =$$



- Interacting canonical theory

Non-interacting

$$G^{(0)}(\mathbf{p},\omega_m;\mathbf{X}) = \frac{\hbar}{-i\hbar\omega_m + \frac{\mathbf{p}^2}{2M} + V(\mathbf{X}) - \hat{\mu}}$$

$$\hat{H}_0 = \frac{\mathbf{p}^2}{2M} + V(\mathbf{X})$$

$$E_{\mathbf{k}}$$

Interacting

$$G(\mathbf{p};\omega_m,\mathbf{X}) = \frac{\hbar}{-i\hbar\omega_m + \frac{\mathbf{p}^2}{2M} + V(\mathbf{X}) - \hat{\mu} - \hbar\Sigma(\mathbf{p};\omega_m,\mathbf{X})}$$

$$\hat{H} = \frac{\mathbf{p}^2}{2M} + V(\mathbf{X}) - \hbar\Sigma(\mathbf{p};\mathbf{X})$$

$$E_{\mathbf{k}}^{(n)} = E_{\mathbf{k}} - \hbar\sigma_n(\mathbf{k})$$

- Interacting canonical theory

$$\Sigma^{(D)}(\mathbf{x},\tau;\mathbf{x}',\tau') = \sum_{n=0}^{\infty} \sigma_n^{(D)}(\mathbf{x},\tau;\mathbf{x}',\tau') z^n \qquad \Sigma^{(E)}(\mathbf{x},\tau;\mathbf{x}',\tau') = \sum_{n=0}^{\infty} \sigma_n^{(E)}(\mathbf{x},\tau;\mathbf{x}',\tau') z^n$$

$$\sigma_n^{(D)}(\mathbf{x},\tau;\mathbf{x}',\tau') = -\frac{1}{\hbar}\delta(\mathbf{x}-\mathbf{x}')\delta(\tau-\tau') \int d^3x''\, V^{(int)}(\mathbf{x}-\mathbf{x}'')\,(\mathbf{x}'', n\hbar\beta; \mathbf{x}'', 0)_{(0)}$$

$$\sigma_n^{(E)}(\mathbf{x},\tau;\mathbf{x}',\tau') = -\frac{1}{\hbar}\delta(\tau-\tau') V^{(int)}(\mathbf{x}-\mathbf{x}')\,(\mathbf{x}, n\hbar\beta; \mathbf{x}', 0)_{(0)}$$

are known as the canonical representation of the self-energy.

- Interacting canonical theory

Fourier-Matsubara transformation:

$$\Sigma(\mathbf{k}, i\omega_m) = \int_0^{\hbar\beta} d(\tau - \tau') \int d^3x\, d^3x'\, \Sigma(\mathbf{x}, \tau; \mathbf{x}', \tau')\psi_{\mathbf{k}}(\mathbf{x})\psi^*_{\mathbf{k}}(\mathbf{x}')\frac{e^{-i\omega_m(\tau-\tau')}}{\hbar\beta}$$

$$\Sigma^{(D)}(\mathbf{k}, i\omega_m) = \sum_{n=1}^{\infty} \sigma_n^{(D)}(\mathbf{k}, i\omega_m) z^n$$

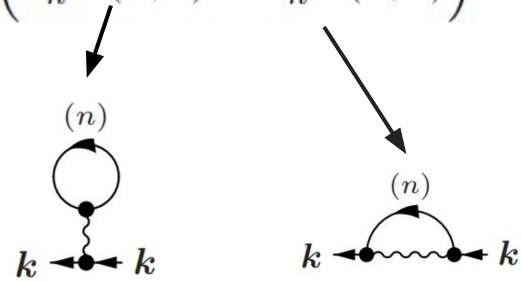
$$\Sigma^{(E)}(\mathbf{k}, i\omega_m) = \sum_{n=1}^{\infty} \sigma_n^{(E)}(\mathbf{k}, i\omega_m) z^n$$

$$\sigma_n^{(D)}(\mathbf{k}, i\omega_m) = \int_0^{\hbar\beta} d(\tau - \tau') \int d^3x\, d^3x'\, \psi_{\mathbf{k}}(\mathbf{x})\psi^*_{\mathbf{k}}(\mathbf{x}')\sigma_n^{(D)}(\mathbf{x}, \tau; \mathbf{x}', \tau')\,\frac{e^{-i\omega_m(\tau-\tau')}}{\hbar\beta}$$

$$\sigma_n^{(E)}(\mathbf{k}, i\omega_m) = \int_0^{\hbar\beta} d(\tau - \tau') \int d^3x\, d^3x'\, \psi_{\mathbf{k}}(\mathbf{x})\psi^*_{\mathbf{k}}(\mathbf{x}')\sigma_n^{(E)}(\mathbf{x}, \tau; \mathbf{x}', \tau')\,\frac{e^{-i\omega_m(\tau-\tau')}}{\hbar\beta}$$

- Interacting canonical theory

New energy eigenvalues: $E_{\mathbf{k}}^{(n)} = E_{\mathbf{k}} - \hbar\,\sigma_n(\mathbf{k},0) = E_{\mathbf{k}} - \hbar\left(\sigma_n^{(D)}(\mathbf{k},0) + \sigma_n^{(E)}(\mathbf{k},0)\right)$



Therefore: $\tilde{Z}_1(n\beta) = \sum_{\mathbf{k}} e^{-n\beta E_{\mathbf{k}}^{(n)}}$

Final interacting recursive formula:

$$Z_N^B(\beta) = \frac{1}{N}\sum_{n=1}^{N}\left[\tilde{Z}_1(n\beta) - \frac{n}{2\hbar}\sum_{l=1}^{n-1}\left(I_{l,n-l}^{(D)}(\beta) + I_{l,n-l}^{(E)}(\beta)\right)\right] Z_{N-n}^B(\beta)$$

$$Z_0^B(\beta) \equiv 1$$

- Interacting canonical theory - results - contact interaction

$$V^{(int)}(\mathbf{x}-\mathbf{x}') = g\,\delta(\mathbf{x}-\mathbf{x}')$$, where $$g = \frac{4\pi\hbar^2 a_s}{M}$$

$$I^{(D,E)}_{l,n-l}(\beta) = g\hbar\beta \int d^3x\,(\mathbf{x}, l\hbar\beta; \mathbf{x}, 0)_{(0)}\;(\mathbf{x}, (n-l)\hbar\beta; \mathbf{x}, 0)_{(0)}$$

$$\sigma^{(D,E)}_n(\mathbf{k}) = -\frac{g}{\hbar}\int d^3x\,\psi_{\mathbf{k}}(\mathbf{x})\psi^*_{\mathbf{k}}(\mathbf{x})\,(\mathbf{x}, n\hbar\beta; \mathbf{x}, 0)_{(0)}$$

- Interacting canonical theory - results - Dilute gas in a finite box

Wave function:

$$\psi_{\mathbf{k}}(\mathbf{x}) = \left(\frac{2}{L}\right)^{3/2} \prod_{j=1}^{3} \sin\left(\frac{\pi k_j x_j}{L}\right)$$

Propagator:

$$(\mathbf{x}, k\hbar\beta; \mathbf{x}', 0)_{(0)} = \frac{1}{L^3} \prod_{j=1}^{3} \sum_{k_j=1}^{\infty} \left[1 - \cos\left(\frac{2\pi k_j x_j}{L}\right)\right] e^{-k\beta\hbar^2\pi^2 k_j^2/2ML^2}$$

- Interacting canonical theory - results - Dilute gas in a finite box

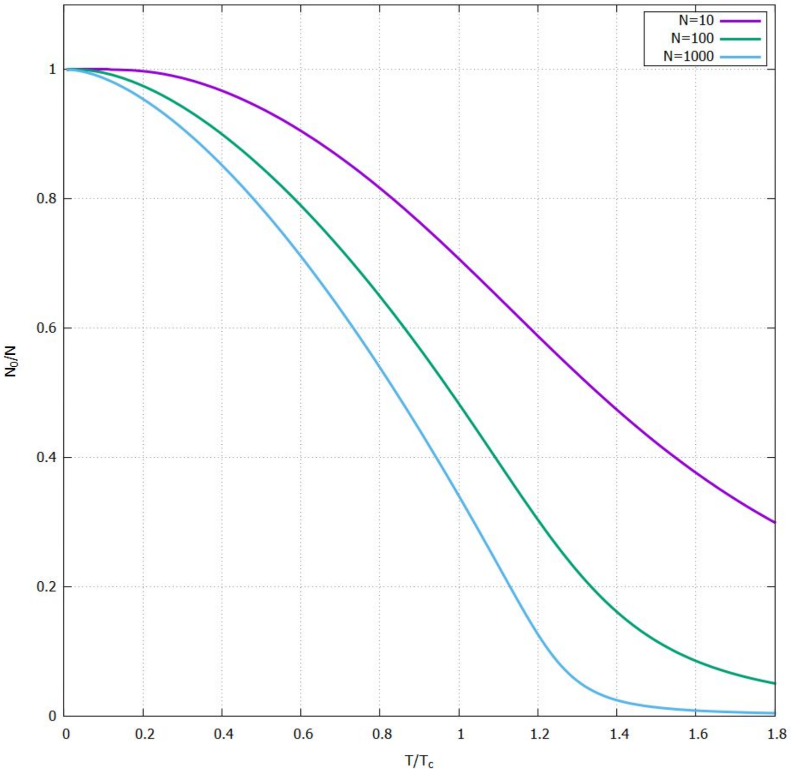
Hartree-Fock integral:

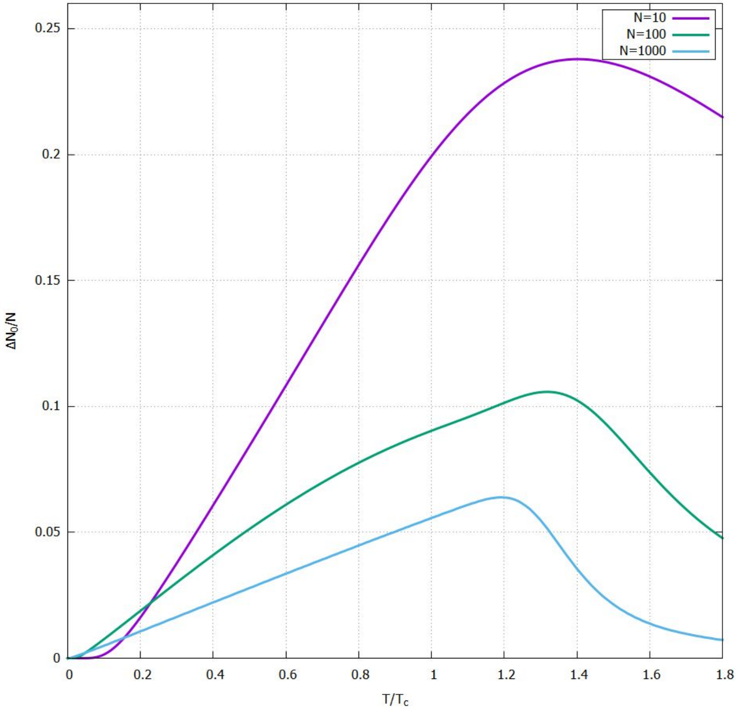
$$I_{l,n-l}^{(D,E)}(\beta) = \frac{g\hbar\beta}{L^3} \prod_{j=1}^{3} \left[\frac{1}{2} g_j(n\beta) + g_j(l\beta) g_j((n-l)\beta) \right] \qquad g_j(m\beta) = \sum_{s_j=1}^{\infty} e^{-m\beta E_{s_j}}$$

Self-energy:

$$\sigma_n^{(D,E)}(\mathbf{k}) = -\frac{g}{\hbar} \prod_{j=1}^{3} \left[\frac{1}{2} e^{-n\beta E_{k_j}} + g_j(n\beta) \right]$$

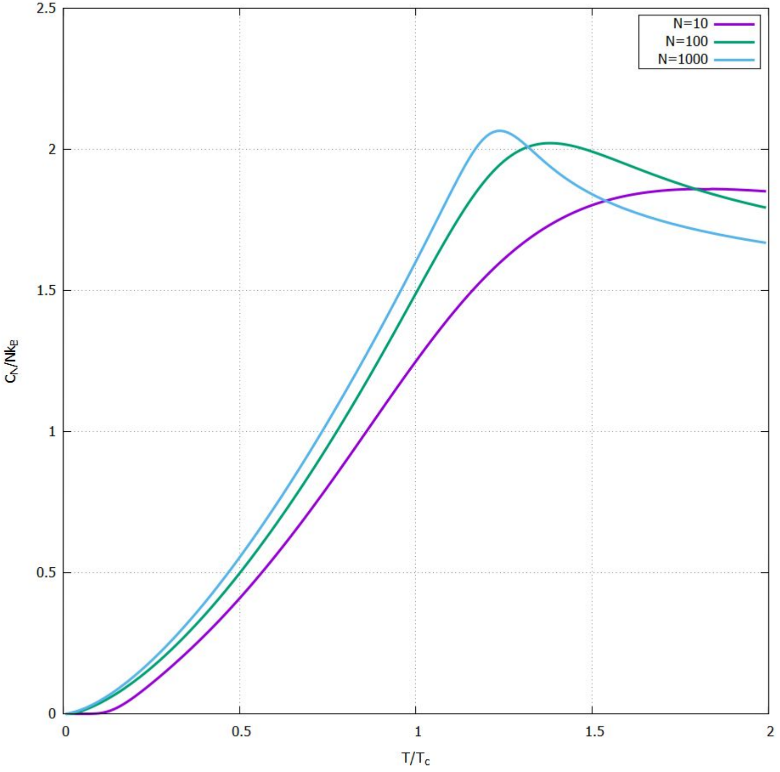
- Interacting canonical theory - results - Dilute gas in a finite box

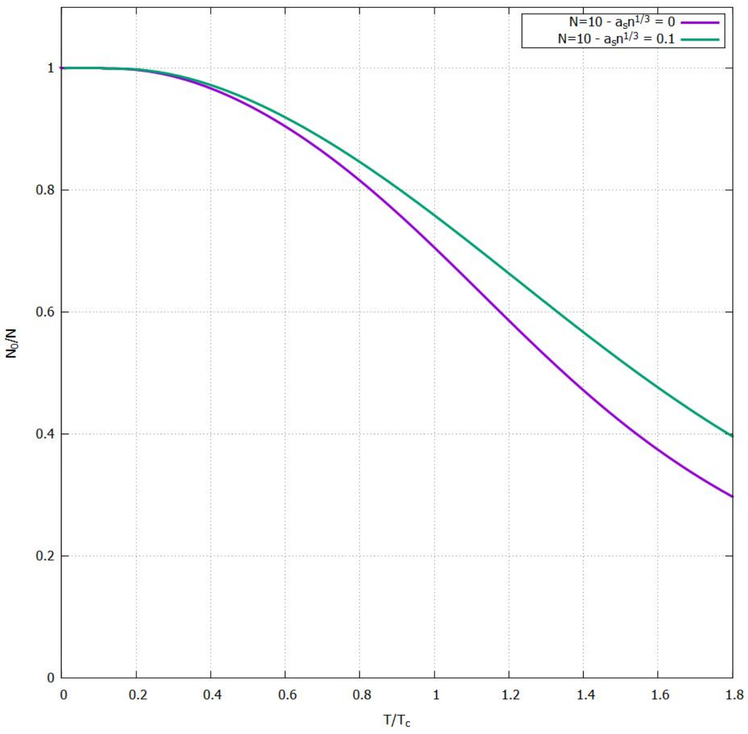




$$T_c = \frac{2\pi\hbar^2}{Mk_B}\left(\frac{N}{V\,\zeta(3/2)}\right)^{2/3}$$

- Interacting canonical theory - results - Dilute gas in a finite box





$$T_c = \frac{2\pi\hbar^2}{Mk_B}\left(\frac{N}{V\,\zeta(3/2)}\right)^{2/3}$$

- Interacting canonical theory - results - Dilute gas in a harmonic trap

Wave function:

$$\psi_{\mathbf{k}}(\mathbf{x}) = \prod_{j=1}^{3} \left(\frac{M\omega_j}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^{k_j}\, k_j!}} e^{-M\omega_j x_j^2/2\hbar} H_{k_j}\left(\sqrt{\frac{M\omega_j}{\hbar}} x_j\right)$$

Propagator:

$$(\mathbf{x}, k\hbar\beta; \mathbf{x}', 0)_{(0)} = \left[\frac{M\omega}{2\pi\hbar \sinh(k\beta\hbar\omega)}\right]^{3/2} \exp\left[-\frac{M\omega}{2\hbar\sinh(k\beta\hbar\omega)} \left[\left(\mathbf{x}^2 + \mathbf{x}'^2\right)\cosh(k\beta\hbar\omega) - 2\left(\mathbf{x}\cdot\mathbf{x}'\right)\right]\right]$$

- Interacting canonical theory - results - Dilute gas in a harmonic trap

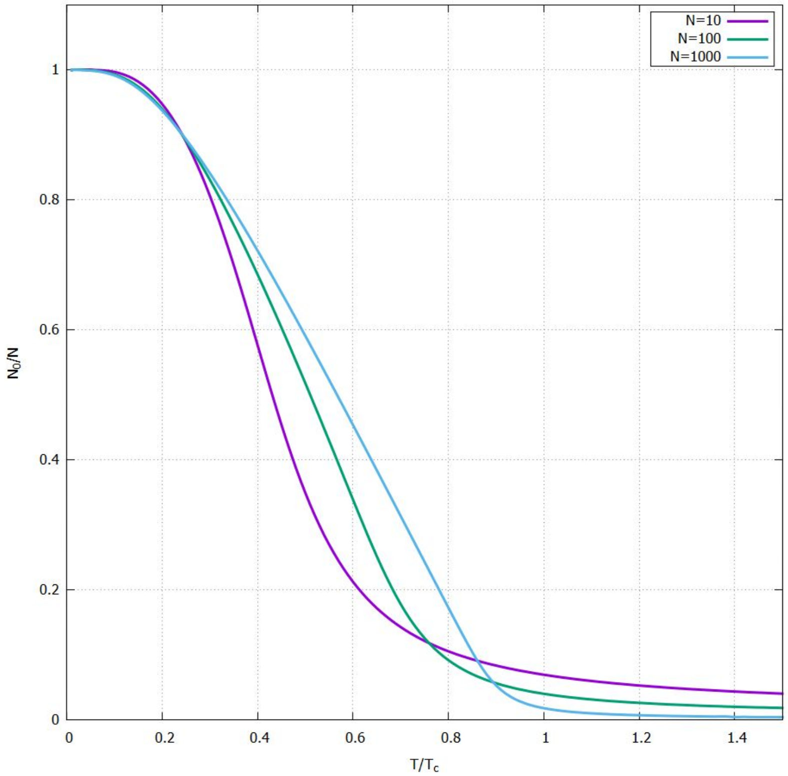
Hartree-Fock integral:

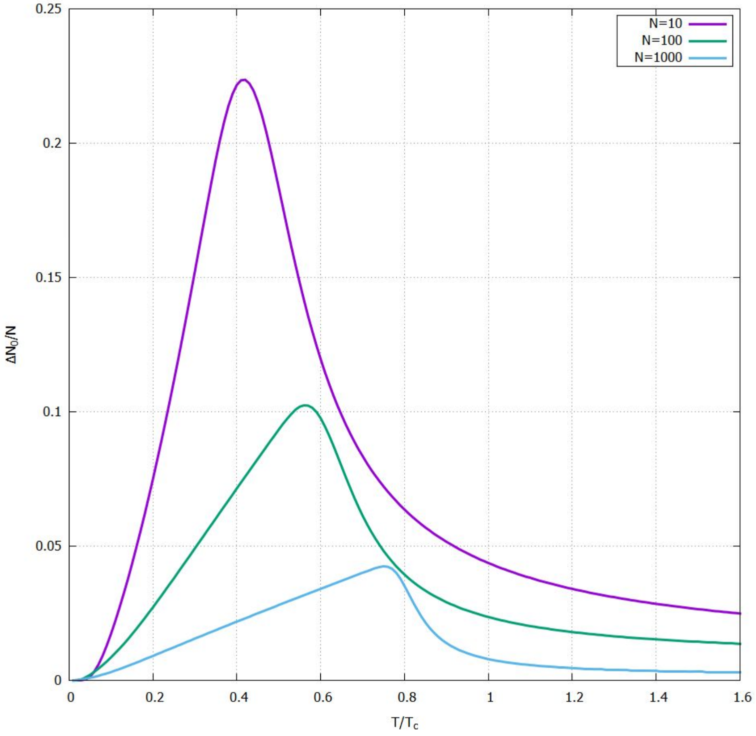
$$I_{l,n-l}^{(D,E)}(\beta) = g\hbar\beta \left[\frac{M\tilde{\omega}}{2\pi\hbar}\right]^{3/2} \left[Z_1(n\beta)Z_1(l\beta)Z_1((n-l)\beta)\right]^{1/2}$$

Self-energy:

$$\sigma_n^{(D,E)}(\mathbf{k}) = -\frac{g}{\hbar}\prod_{j=1}^{3}\left[\frac{M\omega_j}{\pi\hbar}\right]^{1/2} \frac{k_j!}{(e^{n\beta\hbar\omega_j} - e^{-n\beta\hbar\omega_j})^{1/2}} \sum_{\nu=0}^{k_j}\left(\frac{e^{n\beta\hbar\omega_j}-1}{e^{n\beta\hbar\omega_j}}\right)^{k_j-\nu} \frac{(2(k_j-\nu))!}{(-4)^{k_j-\nu}\nu!\left[(k_j-\nu)!\right]^3}$$

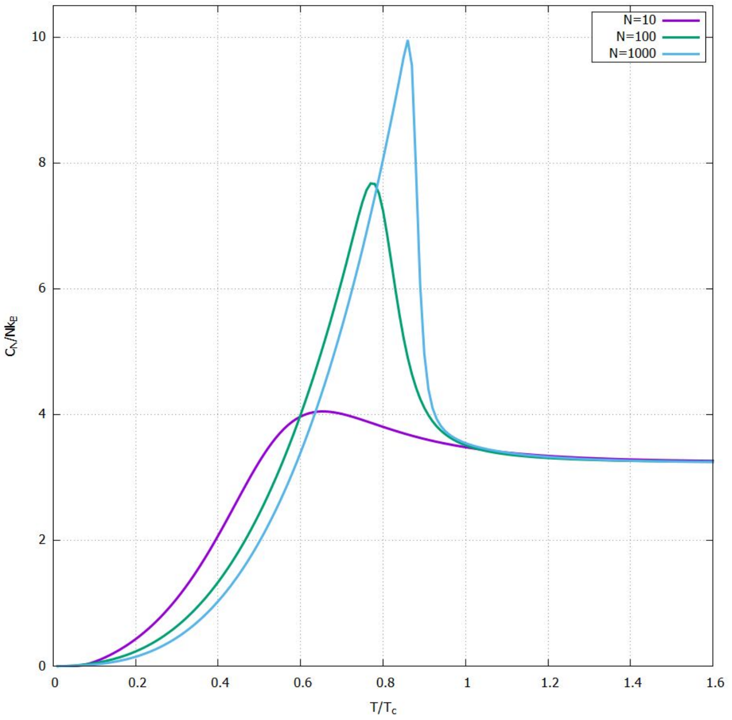
- Interacting canonical theory - results - Dilute gas in a harmonic trap

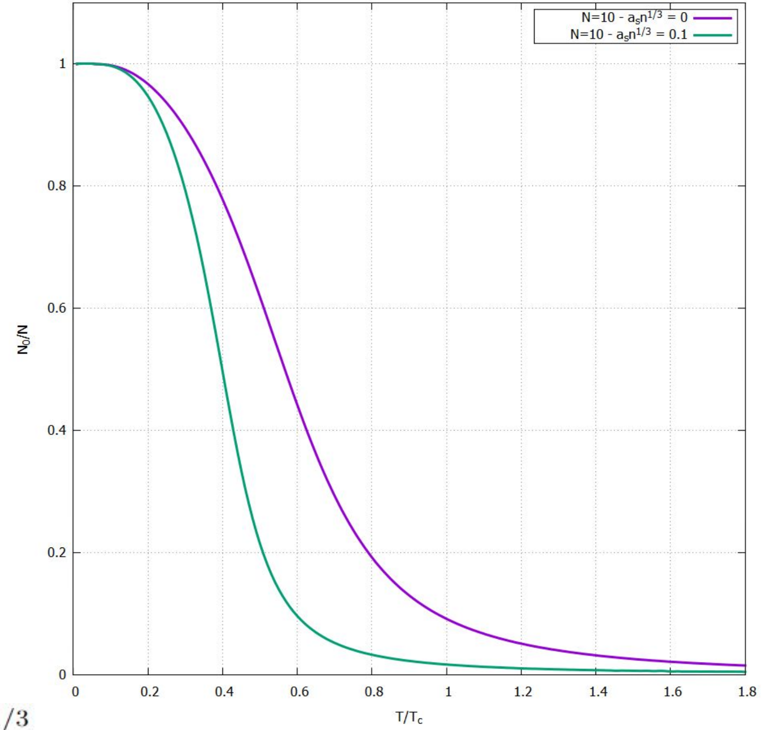




$$T_c = \frac{\hbar \tilde{\omega}}{k_B} \left(\frac{N}{\zeta(3)} \right)^{1/3}$$

- Interacting canonical theory - results - Dilute gas in a harmonic trap





$$T_c = \frac{\hbar\tilde{\omega}}{k_B}\left(\frac{N}{\zeta(3)}\right)^{1/3}$$

- Conclusions

- The canonical ensemble is a good method to describe the thermodynamic properties of confined quantum gases

- The canonical plots qualitative agree with the grand-canonical ones

- The main difficulty of this method is its computation work, specially for the interacting cases

- Acknowledgments

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Thank you for your attention!