$\begin{array}{l} \mbox{Introduction} \\ \mbox{Theory} \\ \mbox{Anisotropic Trap} \ \mathcal{T} = 0 \\ \mbox{Isotropic Trap} \ \mathcal{T} > 0 \\ \mbox{Outlook} \end{array}$

Hartree-Fock Theory of a Harmonically Trapped Dirty Bose-Einstein Condensate

Tama Khellil



January 5, 2015 New Year Seminar

Tama Khellil Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 1/28

Outline

1 Introduction

2 Theory

- Model
- Replica Trick
- Self-Consistency Equations
- 3 Anisotropic Trap T = 0
 - Self-Consistency Equations
 - Densities
 - Thomas-Fermi Radii
- 4 Isotropic Trap T > 0
 - Self-Consistency Equations
 - Densities
 - Thomas-Fermi Radii





→ < ∃ →</p>

3

Introduction Theory

Anisotropic Trap T = 0Isotropic Trap T > 0Outlook

Introduction

- Superfluid Helium in Porous Media: (persistence of superfluidity) Reppy et al., PRL 51, 666 (1983)
- Laser Speckles: (controlled randomness) Inguscio et al., PRL **95**, 070401 (2005) Aspect et al., PRL **95**, 170409 (2005)
- Wire Traps: (undesired randomness) Schmiedmayer et al., PRA **76**, 063621 (2007) Fortágh and Zimmermann, RMP **79**,235 (2007)

• Localized Atomic Species:

Gavish and Castin, PRL **95**, 020401 (2005) (theoretical suggestion) Schneble et al., PRL **107**, 145306 (2011) (experimental realization)

• Incommensurate Lattices: (quasi-disorder) Lewenstein et al., PRL **91**, 080403 (2003) Ertmer et al., PRL **95**, 170411 (2005)

ヘロト ヘ河ト ヘヨト ヘヨト

 $\begin{array}{c} \mbox{Introduction} \\ \mbox{Theory} \\ \mbox{Anisotropic Trap } \mathcal{T} = 0 \\ \mbox{Isotropic Trap } \mathcal{T} > 0 \\ \mbox{Outlook} \end{array}$

Model Replica Trick Self-Consistency Equations

Model System

• Action of a Bose Gas

$$\mathcal{A} = \int_{0}^{\hbar\beta} d\tau \int d\mathbf{r} \left\{ \psi^{*}\left(\mathbf{r},\tau\right) \left[\hbar \frac{\partial}{\partial \tau} - \frac{\hbar^{2}}{2M} \Delta + V\left(\mathbf{r}\right) + \frac{U(\mathbf{r})}{U(\mathbf{r})} - \mu \right] \psi\left(\mathbf{r},\tau\right) + \frac{g}{2} \left| \psi\left(\mathbf{r},\tau\right) \right|^{4} \right\}$$

- Properties
 - trap potential V(r)
 - disorder potential U(r)
 - chemical potential μ
 - repulsive interaction $g = \frac{4\pi\hbar^2 a}{M}$
 - periodic Bose fields $\psi(\mathbf{r}, \tau + \hbar\beta) = \psi(\mathbf{r}, \tau)$

・ 同 ト ・ ヨ ト ・ ヨ

Model Replica Trick Self-Consistency Equations

Random Potential

Disorder Ensemble Average

$$\overline{\bullet} = \int \mathcal{D}U \bullet P[U], \qquad \int \mathcal{D}U = \prod_{\mathbf{r}} \int_{-\infty}^{+\infty} dU(\mathbf{r}), \qquad \int \mathcal{D}U P[U] = 1$$

Assumption

 $\overline{U(\mathbf{r})} = 0, \qquad \overline{U(\mathbf{r}_1)U(\mathbf{r}_2)} = R^{(2)}(\mathbf{r}_1 - \mathbf{r}_2)$

Characteristic Functional

$$\overline{\exp\left\{i\int d\mathbf{r}J(\mathbf{r})U(\mathbf{r})\right\}} = \exp\left\{\sum_{n=2}^{\infty}\frac{i^n}{n!}\int d\mathbf{r}_1\cdots\int d\mathbf{r}_n R^{(n)}(\mathbf{r}_1,\ldots,\mathbf{r}_n)J(\mathbf{r}_1)\cdots J(\mathbf{r}_n)\right\}$$

(日)

Model Replica Trick Self-Consistency Equations

Grand-Canonical Potential

Aim

$$\begin{split} \mathcal{F} &= -\frac{1}{\beta}\overline{\ln \mathcal{Z}} \\ \mathcal{Z} &= \oint \mathcal{D}\psi^* \mathcal{D}\psi e^{-\mathcal{A}\left[\psi^*,\psi\right]/\hbar} \end{split}$$

Problem

$$\overline{\ln \mathcal{Z}} \neq \ln \overline{\mathcal{Z}}$$

• Solution: Replica Trick

$$\mathcal{F} = -rac{1}{eta} \lim_{\mathcal{N} o 0} rac{\mathcal{Z}^{\mathcal{N}} - 1}{\mathcal{N}}$$

G. Parisi, J. Phys. France **51**, 1595 (1990) M. Mezard and G. Parisi, J. Phys. I France **1**, 809 (1991)

イロト イポト イヨト イヨ

Model Replica Trick Self-Consistency Equations

Replica Trick

• Disorder Averaged Partition Function

$$\overline{\mathcal{Z}^{\mathscr{N}}} = \overline{\left\{\prod_{\alpha'=1}^{\mathscr{N}} \oint \mathcal{D}\psi_{\alpha'}^* \mathcal{D}\psi_{\alpha'}\right\} e^{-\sum_{\alpha=1}^{\mathscr{N}} \mathcal{A}[\psi_{\alpha}^*, \psi_{\alpha}]/\hbar}} = \oint \left\{\prod_{\alpha=1}^{\mathscr{N}} \mathcal{D}\psi_{\alpha}^* \mathcal{D}\psi_{\alpha}\right\} e^{-\mathcal{A}^{(\mathscr{N})}/\hbar}$$

Replicated Action

$$\mathcal{A}^{(\mathcal{N})} = \int_{0}^{\hbar\beta} d\tau \int d\mathbf{r} \sum_{\alpha=1}^{\mathcal{N}} \left\{ \psi_{\alpha}^{*}\left(\mathbf{r},\tau\right) \left[\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^{2}}{2M}\Delta + V(\mathbf{r}) - \mu\right] \psi_{\alpha}\left(\mathbf{r},\tau\right) \right. \\ \left. + \frac{g}{2} \left|\psi_{\alpha}\left(\mathbf{r},\tau\right)\right|^{4} \right\} + \sum_{n=2}^{\infty} \frac{1}{n!} \left(\frac{-1}{\hbar}\right)^{n-1} \int_{0}^{\hbar\beta} d\tau_{1} \cdots \int_{0}^{\hbar\beta} d\tau_{n} \int d\mathbf{r}_{1} \cdots \int d\mathbf{r}_{n} \\ \left. \times \sum_{\alpha_{1}=1}^{\mathcal{N}} \cdots \sum_{\alpha_{n}=1}^{\mathcal{N}} R^{(n)}(\mathbf{r}_{1},\ldots,\mathbf{r}_{n}) \left|\psi_{\alpha_{1}}\left(\mathbf{r}_{1},\tau_{1}\right)\right|^{2} \cdots \left|\psi_{\alpha_{n}}\left(\mathbf{r}_{n},\tau_{n}\right)\right|^{2}$$

Tama Khellil Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 7/28

イロト イボト イヨト イヨト

 $\begin{array}{c} \mbox{Introduction} \\ \mbox{Theory} \\ \mbox{Anisotropic Trap } \mathcal{T} = 0 \\ \mbox{Isotropic Trap } \mathcal{T} > 0 \\ \mbox{Outlook} \end{array}$

Model Replica Trick Self-Consistency Equations

Assumptions

- Bogoliubov background method $\psi_{\alpha}(\mathbf{r},\tau) = \Psi_{\alpha}(\mathbf{r},\tau) + \delta\psi_{\alpha}(\mathbf{r},\tau)$
- Hartree-Fock theory
- In the replica limit $\mathscr{N}\to 0$ higher-order disorder cumulants are negligible: only $R^{(2)}(\mathbf{r})$ contributes
- Disorder amounts to attractive interaction for n = 2
- Semiclassical approximations due to V(r)
- Replica symmetry

 $\Rightarrow \begin{cases} \Psi_{\alpha}(\mathbf{r},\tau) = \sqrt{n_{0}(\mathbf{r})} \\ \left\langle \delta\psi_{\alpha}\left(\mathbf{r},\tau\right)\delta\psi_{\alpha'}\left(\mathbf{r}',\tau'\right)\right\rangle = \mathcal{Q}\left(\mathbf{r}-\mathbf{r}',\frac{\mathbf{r}+\mathbf{r}'}{2},\tau-\tau'\right)\delta_{\alpha\alpha'} + q\left(\frac{\mathbf{r}+\mathbf{r}'}{2},\tau-\tau'\right) \\ n\left(\mathbf{r}\right) = \Psi_{\alpha}(\mathbf{r},\tau)\Psi_{\alpha}^{*}(\mathbf{r},\tau) + \left\langle \delta\psi_{\alpha}\left(\mathbf{r},\tau\right)\delta\psi_{\alpha}\left(\mathbf{r},\tau\right)\right\rangle \end{cases}$

Remark: Homogeneous case worked out in Ref.

R. Graham and A. Pelster, Int. J. Bif. Chaos 19, 2745 (2009)

< ロ > < 同 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ >

 $\begin{array}{c} \mbox{Introduction}\\ \mbox{Theory}\\ \mbox{Anisotropic Trap } \mathcal{T}=0\\ \mbox{Isotropic Trap } \mathcal{T}>0\\ \mbox{Outlook} \end{array}$

Model Replica Trick Self-Consistency Equations

Self-Consistency Equations

$$n(\mathbf{r}) = n_0(\mathbf{r}) + q(\mathbf{r}) + n_{\mathrm{th}}(\mathbf{r})$$

$$\left\{-gn_0(\mathbf{r}) + \left[\sqrt{-\mu + d^2 + 2gn(\mathbf{r}) + V(\mathbf{r})} + d\right]^2 - \frac{\hbar^2}{2M}\Delta\right\}\sqrt{n_0(\mathbf{r})} = 0$$

$$q(\mathbf{r}) = \frac{dn_0(\mathbf{r})}{\sqrt{-\mu + d^2 + 2gn(\mathbf{r}) + V(\mathbf{r})}}$$
$$n_{\rm th}(\mathbf{r}) = \left(\frac{M}{2\pi\beta\hbar^2}\right)^{3/2} \varsigma_{3/2} \left(e^{\beta \left[\mu - d^2 - 2gn(\mathbf{r}) - V(\mathbf{r})\right]}\right)$$
$$N = \int n(\mathbf{r})d\mathbf{r}$$

where $\varsigma_{\nu}(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^{\nu}}, \ R^{(2)}(\mathbf{r}_1 - \mathbf{r}_2) = D\delta(\mathbf{r}_1 - \mathbf{r}_2), \ d = \sqrt{\pi}D\left(\frac{M}{2\pi\hbar^2}\right)^{3/2}$

< ロ > < 同 > < 三 > < 三 >

Self-Consistency Equations Densities Thomas-Fermi Radii

Assumptions

•
$$V(\mathbf{r}) = M \left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2 \right) / 2$$

- Thomas-Fermi approximation
- *T* = 0
- Length scales $I_i = \sqrt{\frac{\hbar}{M\omega_i}}, \ i = x, y, z$

• Energy scale
$$\bar{\mu} = \frac{15^{2/5}}{2} \left(\frac{aN}{(l_x l_y l_z)^{1/3}}\right)^{2/5} \hbar \left(\omega_x \omega_y \omega_z\right)^{1/3}$$

• Trap aspect ratios
$$k=\omega_y/\omega_x,\,\lambda=\omega_z/\omega_x$$

Dimensionless quantities

$$\tilde{x} = \sqrt{\frac{M\omega_x^2}{2\bar{\mu}}}x, \qquad \tilde{n}(\tilde{x}, \tilde{y}, \tilde{z}) = \frac{gn(x, y, z)}{\bar{\mu}}, \qquad \tilde{\mu} = \frac{\mu - d^2}{\bar{\mu}}, \qquad \tilde{d} = \frac{d}{\sqrt{\bar{\mu}}}$$

$$= 1 + d^2 + \frac{1}{2} + \frac{$$

Self-Consistency Equations Densities Thomas-Fermi Radii

Self-Consistency Equations

Superfluid Region: $\tilde{n}_0(\tilde{x}, \tilde{y}, \tilde{z}) \neq 0$ and $\tilde{q}(\tilde{x}, \tilde{y}, \tilde{z}) \neq 0$

$$ilde{n}_0(ilde{x}, ilde{y}, ilde{z}) = \left[\sqrt{- ilde{\mu}+2 ilde{n}(ilde{x}, ilde{y}, ilde{z})+ ilde{x}^2+k^2 ilde{y}^2+\lambda^2 ilde{z}^2}+ ilde{d}
ight]$$

$$ilde{q}(ilde{x}, ilde{y}, ilde{z}) = rac{ ilde{d} ilde{n}_0(ilde{x}, ilde{y}, ilde{z})}{\sqrt{- ilde{\mu}+2 ilde{n}(ilde{x}, ilde{y}, ilde{z})+ ilde{x}^2+k^2 ilde{y}^2+\lambda^2 ilde{z}^2}}$$

$$\tilde{n}(\tilde{x}, \tilde{y}, \tilde{z}) = \frac{\left[\sqrt{-\tilde{\mu} + 2\tilde{n}(\tilde{x}, \tilde{y}, \tilde{z}) + \tilde{x}^2 + k^2 \tilde{y}^2 + \lambda^2 \tilde{z}^2} + \tilde{d}\right]^3}{\sqrt{-\tilde{\mu} + 2\tilde{n}(\tilde{x}, \tilde{y}, \tilde{z}) + \tilde{x}^2 + k^2 \tilde{y}^2 + \lambda^2 \tilde{z}^2}}$$

Bose-glass Region:
$$\tilde{n}_0(\tilde{x}, \tilde{y}, \tilde{z}) = 0$$
 and $\tilde{q}(\tilde{x}, \tilde{y}, \tilde{z}) = \tilde{n}(\tilde{x}, \tilde{y}, \tilde{z}) \neq 0$
 $-\tilde{\mu} + 2\tilde{n}(\tilde{x}, \tilde{y}, \tilde{z}) + \tilde{x}^2 + k^2 \tilde{y}^2 + \lambda^2 \tilde{z}^2 = 0$

Tama Khellil Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 11/28

2



• Discriminant of Cardan method in superfluid region for $\tilde{n}(\tilde{r})$

$$\begin{split} \delta &= -\frac{4\tilde{\mu}^3}{27} + \frac{4\tilde{\mu}^2\tilde{d}^2}{3} + 8\tilde{\mu}\tilde{d}^4 + 4\tilde{d}^6 + \left(\frac{4\tilde{\mu}^2}{9} - \frac{8\tilde{\mu}\tilde{d}^2}{3} - 8\tilde{d}^4\right)\left(\tilde{x}^2 + k^2\tilde{y}^2 + \lambda^2\tilde{z}^2\right) \\ &+ \left(-\frac{4\tilde{\mu}}{9} + \frac{4\tilde{d}^2}{3}\right)\left(\tilde{x}^2 + k^2\tilde{y}^2 + \lambda^2\tilde{z}^2\right)^2 + \frac{4}{27}\left(\tilde{x}^2 + k^2\tilde{y}^2 + \lambda^2\tilde{z}^2\right)^3 \end{split}$$



Tama Khellil Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 12/28

э



• Discriminant of Cardan method in superfluid region for $\tilde{n}(\tilde{r})$

$$\begin{split} \delta &= -\frac{4\tilde{\mu}^3}{27} + \frac{4\tilde{\mu}^2\tilde{d}^2}{3} + 8\tilde{\mu}\tilde{d}^4 + 4\tilde{d}^6 + \left(\frac{4\tilde{\mu}^2}{9} - \frac{8\tilde{\mu}\tilde{d}^2}{3} - 8\tilde{d}^4\right)\left(\tilde{x}^2 + k^2\tilde{y}^2 + \lambda^2\tilde{z}^2\right) \\ &+ \left(-\frac{4\tilde{\mu}}{9} + \frac{4\tilde{d}^2}{3}\right)\left(\tilde{x}^2 + k^2\tilde{y}^2 + \lambda^2\tilde{z}^2\right)^2 + \frac{4}{27}\left(\tilde{x}^2 + k^2\tilde{y}^2 + \lambda^2\tilde{z}^2\right)^3 \end{split}$$



Tama Khellil Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 13/28

Densities



Tama Khellil

Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 14/28

Self-Consistency Equations Densities Thomas-Fermi Radii

Thomas-Fermi Radii

 $\bullet~^{87}\mathrm{Rb},~N=10^6,~\omega_{\mathrm{x}}=320\pi~\mathrm{Hz},~k=\sqrt{2}$, $\lambda=2$ and $a=5.29~\mathrm{nm}$



Condensate Radii

$$\begin{split} \tilde{x}_{\mathrm{TF}_{1}} &= \sqrt{\tilde{\mu} - 3\tilde{d}^{2} - 6\sqrt{3}\tilde{d}^{2}\cos\left(\frac{\pi}{18}\right)}, \qquad \tilde{y}_{\mathrm{TF}_{1}} = \tilde{x}_{\mathrm{TF}_{1}}/k, \qquad \tilde{z}_{\mathrm{TF}_{1}} = \tilde{x}_{\mathrm{TF}_{1}}/\lambda \\ \bullet \text{ Cloud Radii} \end{split}$$

$$ilde{\mathbf{x}}_{\mathrm{TF}_2} = \sqrt{ ilde{\mu}}, \qquad ilde{\mathbf{y}}_{\mathrm{TF}_2} = ilde{\mathbf{x}}_{\mathrm{TF}_2}/k, \qquad ilde{\mathbf{z}}_{\mathrm{TF}_2} = ilde{\mathbf{x}}_{\mathrm{TF}_2}/\lambda$$

Tama Khellil Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 15/28

Self-Consistency Equations Densities Thomas-Fermi Radii

Thomas-Fermi Radii

 $\bullet~^{87}\mathrm{Rb},~N=10^6,~\omega_x=320\pi~\mathrm{Hz},~k=\sqrt{2}$, $\lambda=2$ and $a=5.29~\mathrm{nm}$



Condensate Radii

$$\begin{split} \tilde{x}_{\mathrm{TF}_{1}} &= \sqrt{\tilde{\mu} - 3\tilde{d}^{2} - 6\sqrt{3}\tilde{d}^{2}\cos\left(\frac{\pi}{18}\right)}, \qquad \tilde{y}_{\mathrm{TF}_{1}} = \tilde{x}_{\mathrm{TF}_{1}}/k, \qquad \tilde{z}_{\mathrm{TF}_{1}} = \tilde{x}_{\mathrm{TF}_{1}}/\lambda \\ \bullet \text{ Cloud Radii} \end{split}$$

$$ilde{\mathbf{x}}_{\mathrm{TF}_2} = \sqrt{ ilde{\mu}}, \qquad ilde{\mathbf{y}}_{\mathrm{TF}_2} = ilde{\mathbf{x}}_{\mathrm{TF}_2}/k, \qquad ilde{\mathbf{z}}_{\mathrm{TF}_2} = ilde{\mathbf{x}}_{\mathrm{TF}_2}/\lambda$$

Tama Khellil Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 16/28

Self-Consistency Equations Densities Thomas-Fermi Radii

Assumptions

- $V(\mathbf{r}) = \frac{1}{2}M\Omega^2 r^2$
- Thomas-Fermi approximation

• Length scale
$$I = \sqrt{\frac{\hbar}{M\Omega}}$$

• Energy scale
$$\mu_0 = \frac{15^{2/5}}{2} \left(\frac{aN}{l}\right)^{2/5} \hbar \Omega$$

• Dimensionless quantities

$$ilde{r}=\sqrt{rac{M\Omega^2}{2\mu_0}}r, \qquad ilde{n}(ilde{r})=rac{gn(\mathbf{r})}{\mu_0}, \qquad ilde{\mu}=rac{\mu-d^2}{\mu_0}, \qquad ilde{d}=rac{d}{\sqrt{\mu_0}}$$

Tama Khellil Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 17/28

イロト イヨト イヨト

Self-Consistency Equations Densities Thomas-Fermi Radii

Self-Consistency Equations

 \tilde{n}_{t}

Superfluid Region: $\tilde{n}_0(\tilde{r}) \neq 0$, $\tilde{q}(\tilde{r}) \neq 0$ and $\tilde{n}_{\rm th}(\tilde{r}) \neq 0$

$$\tilde{n}(\tilde{r}) = \tilde{n}_0(\tilde{r}) + \tilde{q}(\tilde{r}) + \tilde{n}_{\rm th}(\tilde{r})$$

$$(\tilde{r}) = \left[\sqrt{2}\tilde{r} + \tilde{r} + \tilde{r}\right]$$

$$\tilde{m}_0\left(\tilde{r}\right) = \left\lfloor \sqrt{-\tilde{\mu} + 2\tilde{n}(\tilde{r}) + \tilde{r}^2} + \tilde{d} \right\rfloor^2$$

$$egin{aligned} & ilde{q}\left(ilde{r}
ight)=rac{ ilde{d} ilde{n}_{0}\left(ilde{r}
ight)}{\sqrt{- ilde{\mu}+2 ilde{n}(ilde{r})+ ilde{r}^{2}}} \ _{
m h}\left(ilde{r}
ight)=rac{ extbf{g}}{ ilde{\mu}}\left(rac{ extbf{M}}{2\pieta\hbar^{2}}
ight)^{3/2}arsigma_{3/2}\left(e^{eta ilde{\mu}}\left[ilde{\mu}-2 ilde{n}(ilde{r})- ilde{r}^{2}
ight]}
ight), \end{aligned}$$

(日)

э

Self-Consistency Equations Densities Thomas-Fermi Radii

Self-Consistency Equations

• Decoupled self-consistency equations in superfluid region

$$\tilde{n}\left(\tilde{r}\right) = \frac{\left[\sqrt{-\tilde{\mu} + 2\tilde{n}(\tilde{r}) + \tilde{r}^{2}} + \tilde{d}\right]^{3}}{\sqrt{-\tilde{\mu} + 2\tilde{n}(\tilde{r}) + \tilde{r}^{2}}} + \frac{g}{\tilde{\mu}} \left(\frac{M}{2\pi\beta\hbar^{2}}\right)^{3/2} \varsigma_{3/2} \left(e^{\beta\tilde{\mu} \left[\tilde{\mu} - 2\tilde{n}(\tilde{r}) - \tilde{r}^{2}\right]}\right)$$

$$\left[\sqrt{\tilde{n}_{0}(\tilde{r})}-\tilde{d}\right]^{2}+\tilde{\mu}-\tilde{r}^{2}-\frac{2\tilde{n}_{0}^{\frac{3}{2}}(\tilde{r})}{\sqrt{\tilde{n}_{0}(\tilde{r})}-\tilde{d}}-\frac{2g}{\tilde{\mu}}\left(\frac{M}{2\pi\beta\hbar^{2}}\right)^{3/2}\varsigma_{3/2}\left(e^{-\beta\tilde{\mu}}\left[\sqrt{\tilde{n}_{0}(\tilde{r})}-\tilde{d}\right]^{2}\right)=0$$

$$\begin{aligned} & \left[\frac{\tilde{a}^2}{2\tilde{q}^2(\tilde{r})}\left[\frac{\tilde{a}(\tilde{r})}{2\tilde{a}} + \frac{1}{2}\sqrt{\frac{\tilde{q}^2(\tilde{r})}{\tilde{a}^2} - 4\tilde{q}(\tilde{r})}\right]^4 - \frac{\tilde{q}(\tilde{r})}{\tilde{a}}\left[\frac{\tilde{q}(\tilde{r})}{2\tilde{a}} + \frac{1}{2}\sqrt{\frac{\tilde{q}^2(\tilde{r})}{\tilde{a}^2} - 4\tilde{q}(\tilde{r})}\right] \\ & + \frac{\tilde{\mu} - \tilde{r}^2}{2} - \frac{g}{\tilde{\mu}}\left(\frac{M}{2\pi\beta\hbar^2}\right)^{3/2}\varsigma_{3/2}\left(e^{-\beta\tilde{\mu}\cdot\frac{\tilde{a}^2}{q^2(\tilde{r})}\left[\frac{\tilde{q}(\tilde{r})}{2\tilde{a}} + \frac{1}{2}\sqrt{\frac{\tilde{q}^2(\tilde{r})}{\tilde{a}^2} - 4\tilde{q}(\tilde{r})}\right]^4\right) = 0\end{aligned}$$

$$\tilde{n}_{\rm th}(\tilde{r}) = \frac{g}{\tilde{\mu}} \left(\frac{M}{2\pi\beta\hbar^2}\right)^{3/2} \varsigma_{3/2} \left(e^{-\beta\tilde{\mu}} \left[\sqrt[3]{\frac{6\tilde{d}^2 + \tilde{\mu} - 2\tilde{n}_{\rm th}(\tilde{r}) - \tilde{r}^2 + i\sqrt{-\alpha(\tilde{r})}}{2} + \sqrt[3]{\frac{6\tilde{d}^2 + \tilde{\mu} - 2\tilde{n}_{\rm th}(\tilde{r}) - \tilde{r}^2 - i\sqrt{-\alpha(\tilde{r})}}{2} - 2\tilde{d}} \right]^2 \right)$$
where $\alpha(\tilde{r}) = \frac{4}{27} \left\{ 27\tilde{d}^6 + 54\tilde{d}^4 \left[\tilde{\mu} - 2\tilde{n}_{\rm th}(\tilde{r}) - \tilde{r}^2 \right] + 9\tilde{d}^2 \left[-\tilde{\mu} + 2\tilde{n}_{\rm th}(\tilde{r}) + \tilde{r}^2 \right]^2 - \left[\tilde{\mu} - 2\tilde{n}_{\rm th}(\tilde{r}) - \tilde{r}^2 \right]^3 \right\} = 0$

Self-Consistency Equations Densities Thomas-Fermi Radii

Self-Consistency Equations

Bose-glass Region: $\tilde{n}_0(\tilde{r}) = 0$, $\tilde{q}(\tilde{r}) \neq 0$ and $\tilde{n}_{th}(\tilde{r}) \neq 0$

$$\tilde{q}\left(\tilde{r}\right) = \frac{\tilde{\mu} - \tilde{r}^2}{2} - \frac{g}{\tilde{\mu}} \left(\frac{M}{2\pi\beta\hbar^2}\right)^{3/2} \varsigma\left(\frac{3}{2}\right)$$
$$\tilde{n}_{\rm th}\left(\tilde{r}\right) = \frac{g}{\tilde{\mu}} \left(\frac{M}{2\pi\beta\hbar^2}\right)^{3/2} \varsigma\left(\frac{3}{2}\right)$$
$$\tilde{n}(\tilde{r}) = \frac{\tilde{\mu} - \tilde{r}^2}{2}$$

Thermal Region: $\tilde{n}_0(\tilde{r}) = \tilde{q}(\tilde{r}) = 0$ and $\tilde{n}_{\rm th}(\tilde{r}) = \tilde{n}(\tilde{r}) \neq 0$

$$ilde{n}_{
m th}\left(ilde{r}
ight)=rac{g}{ar{\mu}}\left(rac{M}{2\pieta\hbar^2}
ight)^{3/2}arsigma_{3/2}\left(e^{etaar{\mu}}\left[ilde{\mu}-2 ilde{n}_{
m th}(ilde{r})- ilde{r}^2
ight]
ight)$$

Tama Khellil Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 20/28

Self-Consistency Equations Densities Thomas-Fermi Radii

- Robinson Formula $\varsigma_{\nu}(e^{x}) = \Gamma(1-\nu)(-x)^{\nu-1} + \sum_{k=0}^{\infty} \frac{x^{k}}{k!} \varsigma(\nu-k), \quad x < 0$
- Discriminant of Cardan method in superfluid region for $\tilde{n}_0(\tilde{r})$

$$\delta = -\frac{4\tilde{\mu}^3}{27} + \frac{4\tilde{\mu}^2\tilde{d}^2}{3} + 8\tilde{\mu}\tilde{d}^4 + 4\tilde{d}^6 + \left(\frac{4\tilde{\mu}^2}{9} - \frac{8\tilde{\mu}\tilde{d}^2}{3} - 8\tilde{d}^4\right)\tilde{r}^2 + \left(-\frac{4\tilde{\mu}}{9} + \frac{4\tilde{d}^2}{3}\right)\tilde{r}^4 + \frac{4}{27}\tilde{r}^6$$





Tama Khellil Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 21/28

Self-Consistency Equations Densities Thomas-Fermi Radii

- Robinson Formula $\varsigma_{\nu}(e^{x}) = \Gamma(1-\nu)(-x)^{\nu-1} + \sum_{k=0}^{\infty} \frac{x^{k}}{k!} \varsigma(\nu-k)$
- Discriminant of Cardan method in superfluid region for $\tilde{n}_0(\tilde{r})$

$$\delta = -\frac{4\tilde{\mu}^3}{27} + \frac{4\tilde{\mu}^2\tilde{d}^2}{3} + 8\tilde{\mu}\tilde{d}^4 + 4\tilde{d}^6 + \left(\frac{4\tilde{\mu}^2}{9} - \frac{8\tilde{\mu}\tilde{d}^2}{3} - 8\tilde{d}^4\right)\tilde{r}^2 + \left(-\frac{4\tilde{\mu}}{9} + \frac{4\tilde{d}^2}{3}\right)\tilde{r}^4 + \frac{4}{27}\tilde{r}^6$$



Tama Khellil Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 22/28



• ⁸⁷Rb, $N = 10^6$, $\tilde{d} = 0.088$, $\tilde{\mu} = 0.535$, $\Omega = 100$ Hz, T = 60 nK and a = 5.29 nm



Tama Khellil Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 23/28

3

Self-Consistency Equations Densities Thomas-Fermi Radii

Thomas-Fermi Radii

• $^{87}\mathrm{Rb}$, $N = 10^{6}$, $\Omega = 100~\mathrm{Hz}$, $T = 60~\mathrm{nK}$ and $a = 5.29~\mathrm{nm}$



Tama Khellil Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 24/28

▲ □ ▶ ▲ □ ▶ ▲ □ ▶

Self-Consistency Equations Densities Thomas-Fermi Radii

Thomas-Fermi Radii

• $^{87}\mathrm{Rb}$, $N = 10^{6}$, $\Omega = 100~\mathrm{Hz}$, $T = 60~\mathrm{nK}$ and $a = 5.29~\mathrm{nm}$



Tama Khellil Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 25/28

・ 同 ト ・ ヨ ト ・ ヨ ト

э

Self-Consistency Equations Densities Thomas-Fermi Radii

Thomas-Fermi Radii

• $^{87}\mathrm{Rb}$, $N = 10^{6}$, $\Omega = 100$ Hz, T = 60 nK and a = 5.29 nm



Tama Khellil Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 26/28

< ロ > < 同 > < 三 > < 三 > 、

э

 $\begin{array}{l} \mbox{Introduction} \\ \mbox{Theory} \\ \mbox{Anisotropic Trap} \ T = 0 \\ \mbox{Isotropic Trap} \ T > 0 \\ \mbox{Outlook} \end{array}$

Outlook

- Beyond Thomas-Fermi approximation
- Comparison with numerics
- General correlation function
- General interaction potential
- Time dependence of densities and Thomas-Fermi radii
- Replica Symmetry Breaking?

▲ □ ▶ ▲ □ ▶ ▲ □ ▶

 $\begin{array}{l} \mbox{Introduction} \\ \mbox{Theory} \\ \mbox{Anisotropic Trap} \ \mathcal{T} = 0 \\ \mbox{Isotropic Trap} \ \mathcal{T} > 0 \\ \mbox{Outlook} \end{array}$

Thank You For Your Attention

Tama Khellil Hartree-Fock Theory of a Harmonically Trapped Dirty BEC 28/28

▲ □ ▶ ▲ □ ▶ ▲ □ ▶