Hartree-Fock Theory of a Harmonically Trapped Dirty Bose-Einstein Condensate

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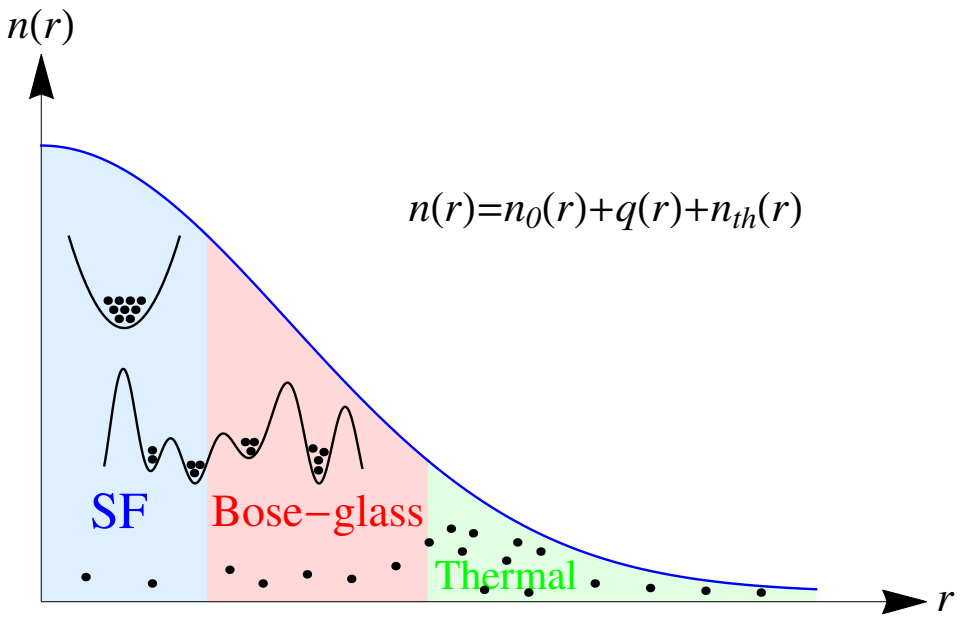
New Year Seminar

Outline



Introduction

- **Superfluid Helium in Porous Media:** (persistence of superfluidity)
  Reppy et al., PRL **51**, 666 (1983)
- **Laser Speckles:** (controlled randomness)
  Inguscio et al., PRL **95**, 070401 (2005)
  Aspect et al., PRL **95**, 170409 (2005)
- **Wire Traps:** (undesired randomness)
  Schmiedmayer et al., PRA **76**, 063621 (2007)
  Fortágh and Zimmermann, RMP **79**,235 (2007)
- **Localized Atomic Species:**
  Gavish and Castin, PRL **95**, 020401 (2005) (theoretical suggestion)
  Schneble et al., PRL **107**, 145306 (2011) (experimental realization)
- **Incommensurate Lattices:** (quasi-disorder)
  Lewenstein et al., PRL **91**, 080403 (2003)
  Ertmer et al., PRL **95**, 170411 (2005)

Model System

- Action of a Bose Gas

$$\mathcal{A} = \int_0^{\hbar\beta} d\tau \int d\mathbf{r} \left\{ \psi^* (\mathbf{r}, \tau) \left[\hbar \frac{\partial}{\partial \tau} - \frac{\hbar^2}{2M} \Delta + V(\mathbf{r}) + U(\mathbf{r}) - \mu \right] \psi (\mathbf{r}, \tau) + \frac{g}{2} |\psi (\mathbf{r}, \tau)|^4 \right\}$$

- Properties
  - trap potential $V(\mathbf{r})$
  - disorder potential $U(\mathbf{r})$
  - chemical potential μ
  - repulsive interaction $g = \frac{4\pi\hbar^2 a}{M}$
  - periodic Bose fields $\psi(\mathbf{r}, \tau + \hbar\beta) = \psi(\mathbf{r}, \tau)$

Random Potential

- Disorder Ensemble Average

$$\overline{\bullet} = \int \mathcal{D}U \bullet P[U], \qquad \int \mathcal{D}U = \prod_{\mathbf{r}} \int_{-\infty}^{+\infty} dU(\mathbf{r}), \qquad \int \mathcal{D}U P[U] = 1$$

- Assumption

$$\overline{U(\mathbf{r})} = 0, \qquad \overline{U(\mathbf{r}_1)U(\mathbf{r}_2)} = R^{(2)}(\mathbf{r}_1 - \mathbf{r}_2)$$

- Characteristic Functional

$$\overline{\exp\left\{i \int d\mathbf{r} J(\mathbf{r}) U(\mathbf{r})\right\}} = \exp\left\{\sum_{n=2}^{\infty} \frac{i^n}{n!} \int d\mathbf{r}_1 \cdots \int d\mathbf{r}_n R^{(n)}(\mathbf{r}_1, \ldots, \mathbf{r}_n) J(\mathbf{r}_1) \cdots J(\mathbf{r}_n)\right\}$$

Grand-Canonical Potential

- Aim

$$\mathcal{F} = -\frac{1}{\beta}\overline{\ln \mathcal{Z}}$$

$$\mathcal{Z} = \oint \mathcal{D}\psi^* \mathcal{D}\psi e^{-\mathcal{A}[\psi^*,\psi]/\hbar}$$

- Problem

$$\overline{\ln \mathcal{Z}} \neq \ln \overline{\mathcal{Z}}$$

- Solution: Replica Trick

$$\mathcal{F} = -\frac{1}{\beta} \lim_{\mathcal{N}\to 0} \frac{\overline{\mathcal{Z}^{\mathcal{N}}} - 1}{\mathcal{N}}$$

G. Parisi, J. Phys. France **51**, 1595 (1990)
M. Mezard and G. Parisi, J. Phys. I France **1**, 809 (1991)

Replica Trick

- Disorder Averaged Partition Function

$$\overline{\mathcal{Z}^{\mathcal{N}}} = \overline{\left\{ \prod_{\alpha'=1}^{\mathcal{N}} \oint \mathcal{D}\psi^*_{\alpha'} \mathcal{D}\psi_{\alpha'} \right\} e^{-\sum_{\alpha=1}^{\mathcal{N}} \mathcal{A}[\psi^*_\alpha, \psi_\alpha]/\hbar}} = \oint \left\{ \prod_{\alpha=1}^{\mathcal{N}} \mathcal{D}\psi^*_\alpha \mathcal{D}\psi_\alpha \right\} e^{-\mathcal{A}^{(\mathcal{N})}/\hbar}$$

- Replicated Action

$$\begin{aligned}
\mathcal{A}^{(\mathcal{N})} = & \int_0^{\hbar\beta} d\tau \int d\mathbf{r} \sum_{\alpha=1}^{\mathcal{N}} \left\{ \psi^*_\alpha(\mathbf{r},\tau) \left[\hbar \frac{\partial}{\partial \tau} - \frac{\hbar^2}{2M} \Delta + V(\mathbf{r}) - \mu \right] \psi_\alpha(\mathbf{r},\tau) \right. \\
& \left. + \frac{g}{2} |\psi_\alpha(\mathbf{r},\tau)|^4 \right\} + \sum_{n=2}^{\infty} \frac{1}{n!} \left(\frac{-1}{\hbar} \right)^{n-1} \int_0^{\hbar\beta} d\tau_1 \cdots \int_0^{\hbar\beta} d\tau_n \int d\mathbf{r}_1 \cdots \int d\mathbf{r}_n \\
& \times \sum_{\alpha_1=1}^{\mathcal{N}} \cdots \sum_{\alpha_n=1}^{\mathcal{N}} R^{(n)}(\mathbf{r}_1, \ldots, \mathbf{r}_n) |\psi_{\alpha_1}(\mathbf{r}_1,\tau_1)|^2 \cdots |\psi_{\alpha_n}(\mathbf{r}_n,\tau_n)|^2
\end{aligned}$$

Assumptions

- Bogoliubov background method $\psi_\alpha(\mathbf{r},\tau) = \Psi_\alpha(\mathbf{r},\tau) + \delta\psi_\alpha(\mathbf{r},\tau)$
- Hartree-Fock theory

- In the replica limit $\mathcal{N} \to 0$ higher-order disorder cumulants are negligible: only $R^{(2)}(\mathbf{r})$ contributes
- Disorder amounts to attractive interaction for $n = 2$

- Semiclassical approximations due to $V(\mathbf{r})$
- Replica symmetry

$$\Rightarrow \begin{cases} \Psi_\alpha(\mathbf{r},\tau) = \sqrt{n_0(\mathbf{r})} \\ \langle \delta\psi_\alpha(\mathbf{r},\tau)\,\delta\psi_{\alpha'}(\mathbf{r}',\tau')\rangle = \mathcal{Q}\left(\mathbf{r}-\mathbf{r}', \frac{\mathbf{r}+\mathbf{r}'}{2}, \tau-\tau'\right)\delta_{\alpha\alpha'} + q\left(\frac{\mathbf{r}+\mathbf{r}'}{2}, \tau-\tau'\right) \\ n(\mathbf{r}) = \Psi_\alpha(\mathbf{r},\tau)\Psi^*_\alpha(\mathbf{r},\tau) + \langle \delta\psi_\alpha(\mathbf{r},\tau)\,\delta\psi_\alpha(\mathbf{r},\tau)\rangle \end{cases}$$

Remark: Homogeneous case worked out in Ref.

R. Graham and A. Pelster, Int. J. Bif. Chaos **19**, 2745 (2009)

Self-Consistency Equations

$$n(\mathbf{r}) = n_0(\mathbf{r}) + q(\mathbf{r}) + n_{\mathrm{th}}(\mathbf{r})$$

$$\left\{ -gn_0(\mathbf{r}) + \left[\sqrt{-\mu + d^2 + 2gn(\mathbf{r}) + V(\mathbf{r})} + d \right]^2 - \frac{\hbar^2}{2M}\Delta \right\} \sqrt{n_0(\mathbf{r})} = 0$$

$$q(\mathbf{r}) = \frac{d n_0(\mathbf{r})}{\sqrt{-\mu + d^2 + 2gn(\mathbf{r}) + V(\mathbf{r})}}$$

$$n_{\mathrm{th}}(\mathbf{r}) = \left(\frac{M}{2\pi\beta\hbar^2} \right)^{3/2} \varsigma_{3/2}\left(e^{\beta\,[\mu - d^2 - 2gn(\mathbf{r}) - V(\mathbf{r})]} \right)$$

$$N = \int n(\mathbf{r}) d\mathbf{r}$$

where $\varsigma_\nu(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^\nu}$, $R^{(2)}(\mathbf{r}_1 - \mathbf{r}_2) = D\delta(\mathbf{r}_1 - \mathbf{r}_2)$, $d = \sqrt{\pi} D \left(\frac{M}{2\pi\hbar^2} \right)^{3/2}$

Assumptions

- $V(\mathbf{r}) = M\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right)/2$
- Thomas-Fermi approximation
- $T = 0$
- Length scales $l_i = \sqrt{\frac{\hbar}{M\omega_i}}, \; i = x, y, z$
- Energy scale $\bar{\mu} = \frac{15^{2/5}}{2}\left(\frac{aN}{(l_x l_y l_z)^{1/3}}\right)^{2/5} \hbar\left(\omega_x \omega_y \omega_z\right)^{1/3}$
- Trap aspect ratios $k = \omega_y/\omega_x$, $\lambda = \omega_z/\omega_x$

- Dimensionless quantities

$$\tilde{x} = \sqrt{\frac{M\omega_x^2}{2\bar{\mu}}}x, \qquad \tilde{n}(\tilde{x}, \tilde{y}, \tilde{z}) = \frac{gn(x,y,z)}{\bar{\mu}}, \qquad \tilde{\mu} = \frac{\mu - d^2}{\bar{\mu}}, \qquad \tilde{d} = \frac{d}{\sqrt{\bar{\mu}}}$$

Self-Consistency Equations

Superfluid Region: $\tilde{n}_0(\tilde{x}, \tilde{y}, \tilde{z}) \neq 0$ and $\tilde{q}(\tilde{x}, \tilde{y}, \tilde{z}) \neq 0$

$$\tilde{n}_0(\tilde{x}, \tilde{y}, \tilde{z}) = \left[\sqrt{-\tilde{\mu} + 2\tilde{n}(\tilde{x}, \tilde{y}, \tilde{z}) + \tilde{x}^2 + k^2\tilde{y}^2 + \lambda^2\tilde{z}^2} + \tilde{d}\right]^2$$

$$\tilde{q}(\tilde{x}, \tilde{y}, \tilde{z}) = \frac{\tilde{d}\tilde{n}_0(\tilde{x}, \tilde{y}, \tilde{z})}{\sqrt{-\tilde{\mu} + 2\tilde{n}(\tilde{x}, \tilde{y}, \tilde{z}) + \tilde{x}^2 + k^2\tilde{y}^2 + \lambda^2\tilde{z}^2}}$$

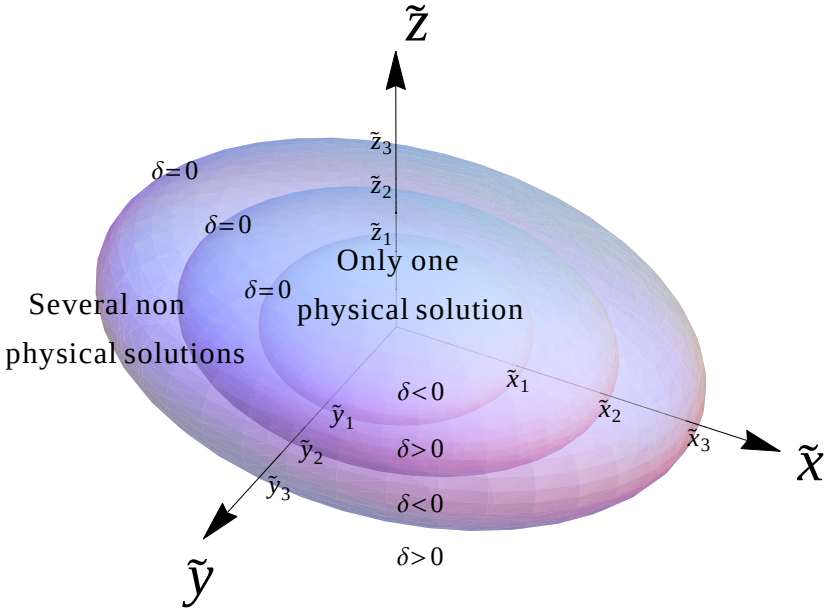
$$\tilde{n}(\tilde{x}, \tilde{y}, \tilde{z}) = \frac{\left[\sqrt{-\tilde{\mu} + 2\tilde{n}(\tilde{x}, \tilde{y}, \tilde{z}) + \tilde{x}^2 + k^2\tilde{y}^2 + \lambda^2\tilde{z}^2} + \tilde{d}\right]^3}{\sqrt{-\tilde{\mu} + 2\tilde{n}(\tilde{x}, \tilde{y}, \tilde{z}) + \tilde{x}^2 + k^2\tilde{y}^2 + \lambda^2\tilde{z}^2}}$$

Bose-glass Region: $\tilde{n}_0(\tilde{x}, \tilde{y}, \tilde{z}) = 0$ and $\tilde{q}(\tilde{x}, \tilde{y}, \tilde{z}) = \tilde{n}(\tilde{x}, \tilde{y}, \tilde{z}) \neq 0$

$$-\tilde{\mu} + 2\tilde{n}(\tilde{x}, \tilde{y}, \tilde{z}) + \tilde{x}^2 + k^2\tilde{y}^2 + \lambda^2\tilde{z}^2 = 0$$

- Discriminant of Cardan method in superfluid region for $\tilde{n}(\tilde{r})$

$$\delta = -\frac{4\tilde{\mu}^3}{27} + \frac{4\tilde{\mu}^2\tilde{d}^2}{3} + 8\tilde{\mu}\tilde{d}^4 + 4\tilde{d}^6 + \left(\frac{4\tilde{\mu}^2}{9} - \frac{8\tilde{\mu}\tilde{d}^2}{3} - 8\tilde{d}^4\right)\left(\tilde{x}^2 + k^2\tilde{y}^2 + \lambda^2\tilde{z}^2\right)$$
$$+ \left(-\frac{4\tilde{\mu}}{9} + \frac{4\tilde{d}^2}{3}\right)\left(\tilde{x}^2 + k^2\tilde{y}^2 + \lambda^2\tilde{z}^2\right)^2 + \frac{4}{27}\left(\tilde{x}^2 + k^2\tilde{y}^2 + \lambda^2\tilde{z}^2\right)^3$$



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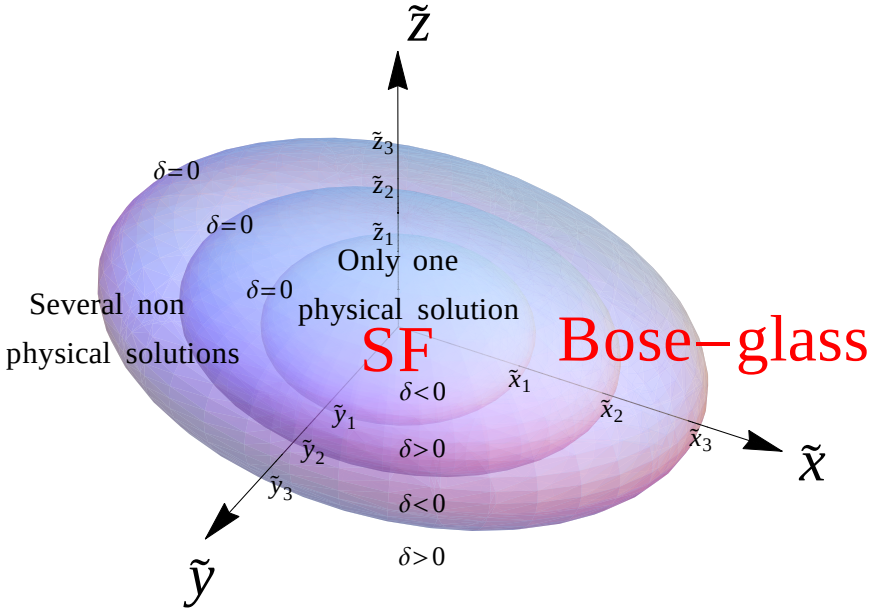
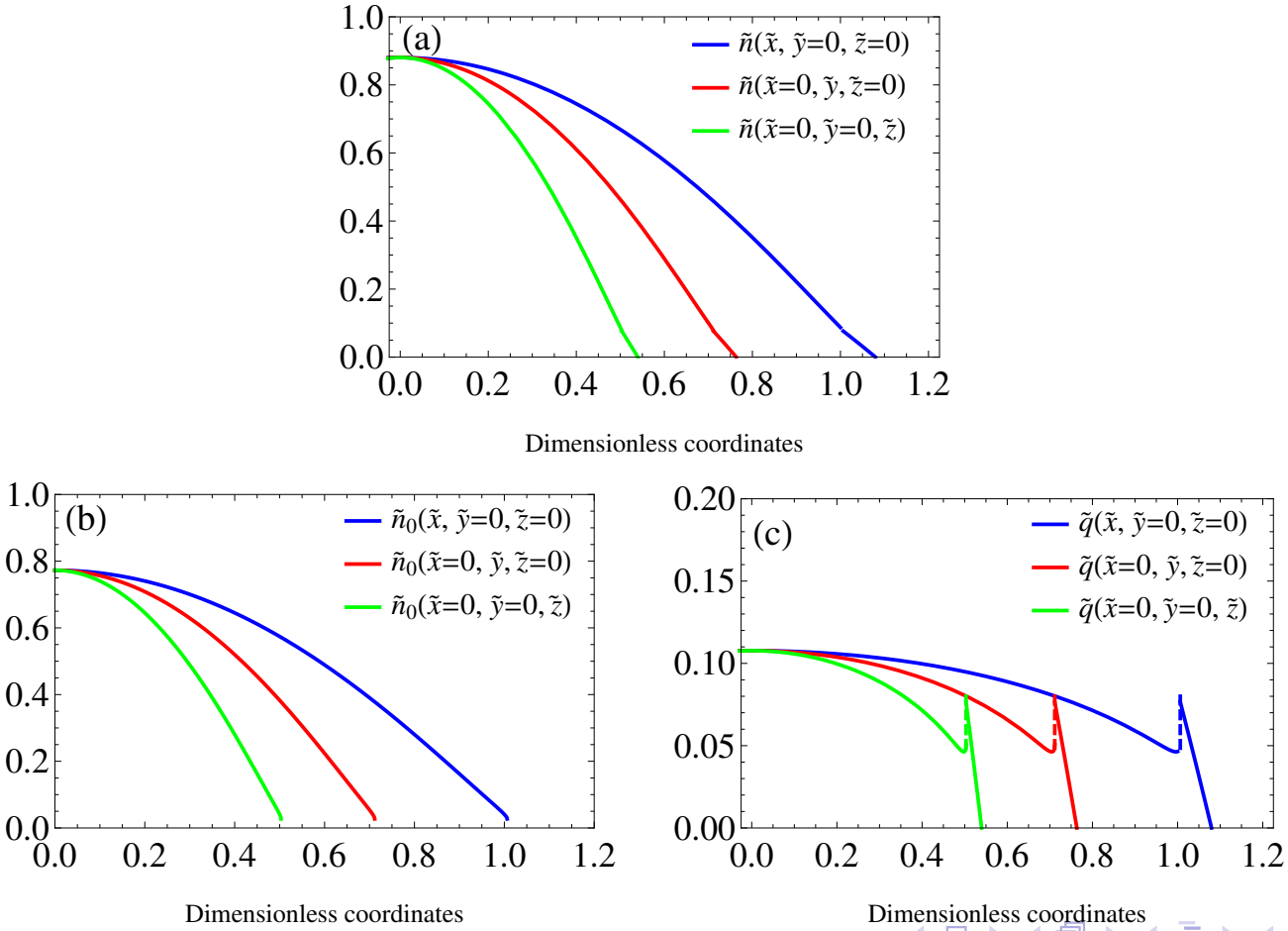
$$+ \left(-\frac{4\tilde{\mu}}{9} + \frac{4\tilde{d}^2}{3}\right)\left(\tilde{x}^2 + k^2\tilde{y}^2 + \lambda^2\tilde{z}^2\right)^2 + \frac{4}{27}\left(\tilde{x}^2 + k^2\tilde{y}^2 + \lambda^2\tilde{z}^2\right)^3$$



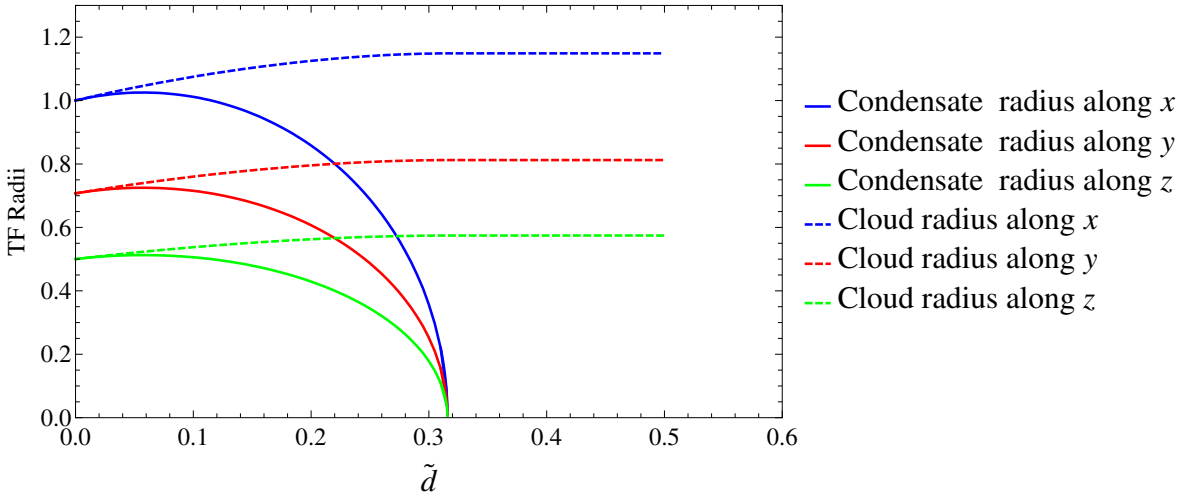
Densities

- $^{87}\mathrm{Rb}$, $N = 10^6$, $\tilde{d} = 0.107$, $\tilde{\mu} = 1.165$, $\omega_x = 320\pi$ Hz, $k = \sqrt{2}$, $\lambda = 2$ and $a = 5.29$ nm



Thomas-Fermi Radii

- $^{87}\mathrm{Rb}$, $N = 10^6$, $\omega_x = 320\pi$ Hz, $k = \sqrt{2}$, $\lambda = 2$ and $a = 5.29$ nm



- Condensate Radii

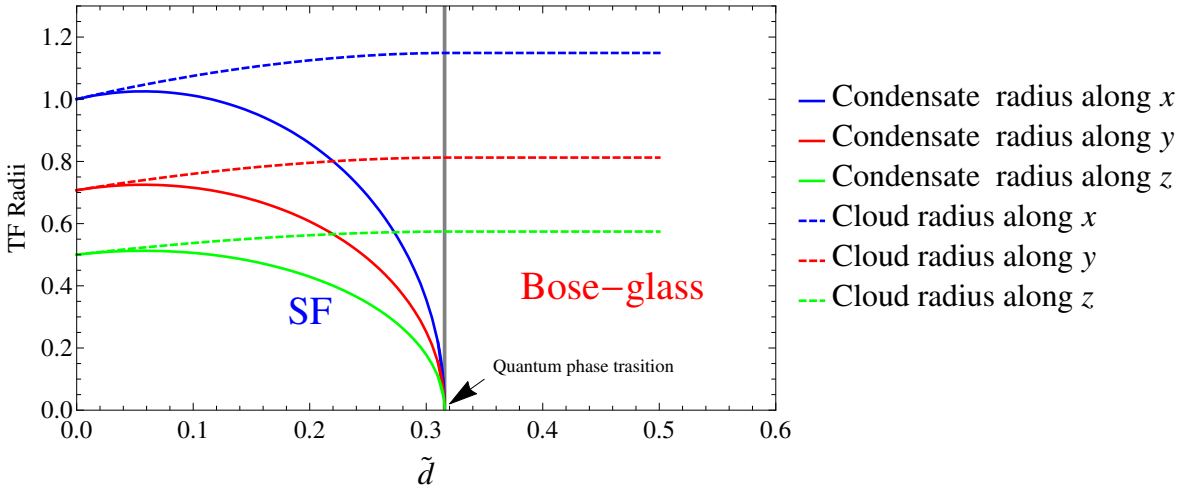
$$\tilde{x}_{\mathrm{TF}_1} = \sqrt{\tilde{\mu} - 3\tilde{d}^2 - 6\sqrt{3}\tilde{d}^2 \cos\left(\frac{\pi}{18}\right)}, \qquad \tilde{y}_{\mathrm{TF}_1} = \tilde{x}_{\mathrm{TF}_1}/k, \qquad \tilde{z}_{\mathrm{TF}_1} = \tilde{x}_{\mathrm{TF}_1}/\lambda$$

- Cloud Radii

$$\tilde{x}_{\mathrm{TF}_2} = \sqrt{\tilde{\mu}}, \qquad \tilde{y}_{\mathrm{TF}_2} = \tilde{x}_{\mathrm{TF}_2}/k, \qquad \tilde{z}_{\mathrm{TF}_2} = \tilde{x}_{\mathrm{TF}_2}/\lambda$$

Thomas-Fermi Radii

- $^{87}\mathrm{Rb}$, $N = 10^6$, $\omega_x = 320\pi$ Hz, $k = \sqrt{2}$, $\lambda = 2$ and $a = 5.29$ nm



- Condensate Radii

$$\tilde{x}_{\mathrm{TF}_1} = \sqrt{\tilde{\mu} - 3\tilde{d}^2 - 6\sqrt{3}\tilde{d}^2 \cos\left(\frac{\pi}{18}\right)}, \qquad \tilde{y}_{\mathrm{TF}_1} = \tilde{x}_{\mathrm{TF}_1}/k, \qquad \tilde{z}_{\mathrm{TF}_1} = \tilde{x}_{\mathrm{TF}_1}/\lambda$$

- Cloud Radii

$$\tilde{x}_{\mathrm{TF}_2} = \sqrt{\tilde{\mu}}, \qquad \tilde{y}_{\mathrm{TF}_2} = \tilde{x}_{\mathrm{TF}_2}/k, \qquad \tilde{z}_{\mathrm{TF}_2} = \tilde{x}_{\mathrm{TF}_2}/\lambda$$

Assumptions

- $V(\mathbf{r}) = \frac{1}{2} M\Omega^2 r^2$
- Thomas-Fermi approximation
- Length scale $l = \sqrt{\frac{\hbar}{M\Omega}}$
- Energy scale $\mu_0 = \frac{15^{2/5}}{2}\left(\frac{aN}{l}\right)^{2/5}\hbar\Omega$

- Dimensionless quantities

$$\tilde{r} = \sqrt{\frac{M\Omega^2}{2\mu_0}}\, r, \qquad \tilde{n}(\tilde{r}) = \frac{gn(\mathbf{r})}{\mu_0}, \qquad \tilde{\mu} = \frac{\mu - d^2}{\mu_0}, \qquad \tilde{d} = \frac{d}{\sqrt{\mu_0}}$$

Self-Consistency Equations

Superfluid Region: $\tilde{n}_0(\tilde{r}) \neq 0$, $\tilde{q}(\tilde{r}) \neq 0$ and $\tilde{n}_{\mathrm{th}}(\tilde{r}) \neq 0$

$$\tilde{n}(\tilde{r}) = \tilde{n}_0(\tilde{r}) + \tilde{q}(\tilde{r}) + \tilde{n}_{\mathrm{th}}(\tilde{r})$$

$$\tilde{n}_0(\tilde{r}) = \left[\sqrt{-\tilde{\mu} + 2\tilde{n}(\tilde{r}) + \tilde{r}^2} + \tilde{d}\right]^2$$

$$\tilde{q}(\tilde{r}) = \frac{\tilde{d}\tilde{n}_0(\tilde{r})}{\sqrt{-\tilde{\mu} + 2\tilde{n}(\tilde{r}) + \tilde{r}^2}}$$

$$\tilde{n}_{\mathrm{th}}(\tilde{r}) = \frac{g}{\bar{\mu}}\left(\frac{M}{2\pi\beta\hbar^2}\right)^{3/2} \varsigma_{3/2}\left(e^{\beta\bar{\mu}\,[\tilde{\mu}-2\tilde{n}(\tilde{r})-\tilde{r}^2]}\right)$$

Self-Consistency Equations

- Decoupled self-consistency equations in superfluid region

$$\tilde{n}(\tilde{r}) = \frac{\left[\sqrt{-\tilde{\mu} + 2\tilde{n}(\tilde{r}) + \tilde{r}^2} + \tilde{d}\right]^3}{\sqrt{-\tilde{\mu} + 2\tilde{n}(\tilde{r}) + \tilde{r}^2}} + \frac{g}{\bar{\mu}} \left(\frac{M}{2\pi\beta\hbar^2}\right)^{3/2} \varsigma_{3/2}\left(e^{\beta\bar{\mu}\,[\tilde{\mu} - 2\tilde{n}(\tilde{r}) - \tilde{r}^2]}\right)$$

$$\left[\sqrt{\tilde{n}_0(\tilde{r})} - \tilde{d}\right]^2 + \tilde{\mu} - \tilde{r}^2 - \frac{2\tilde{n}_0^{\frac{3}{2}}(\tilde{r})}{\sqrt{\tilde{n}_0(\tilde{r})} - \tilde{d}} - \frac{2g}{\bar{\mu}} \left(\frac{M}{2\pi\beta\hbar^2}\right)^{3/2} \varsigma_{3/2}\left(e^{-\beta\bar{\mu}\,\left[\sqrt{\tilde{n}_0(\tilde{r})} - \tilde{d}\right]^2}\right) = 0$$

$$\frac{\tilde{d}^2}{2\tilde{q}^2(\tilde{r})}\left[\frac{\tilde{q}(\tilde{r})}{2\tilde{d}} + \frac{1}{2}\sqrt{\frac{\tilde{q}^2(\tilde{r})}{\tilde{d}^2} - 4\tilde{q}(\tilde{r})}\right]^4 - \frac{\tilde{q}(\tilde{r})}{\tilde{d}}\left[\frac{\tilde{q}(\tilde{r})}{2\tilde{d}} + \frac{1}{2}\sqrt{\frac{\tilde{q}^2(\tilde{r})}{\tilde{d}^2} - 4\tilde{q}(\tilde{r})}\right]$$
$$+ \frac{\tilde{\mu} - \tilde{r}^2}{2} - \frac{g}{\bar{\mu}}\left(\frac{M}{2\pi\beta\hbar^2}\right)^{3/2} \varsigma_{3/2}\left(e^{-\beta\bar{\mu}\,\frac{\tilde{d}^2}{\tilde{q}^2(\tilde{r})}\left[\frac{\tilde{q}(\tilde{r})}{2\tilde{d}} + \frac{1}{2}\sqrt{\frac{\tilde{q}^2(\tilde{r})}{\tilde{d}^2} - 4\tilde{q}(\tilde{r})}\right]^4}\right) = 0$$

$$\tilde{n}_{\mathrm{th}}(\tilde{r}) = \frac{g}{\bar{\mu}}\left(\frac{M}{2\pi\beta\hbar^2}\right)^{3/2} \varsigma_{3/2}\left(e^{-\beta\bar{\mu}\,\left[\sqrt[3]{\frac{6\tilde{d}^2 + \tilde{\mu} - 2\tilde{n}_{\mathrm{th}}(\tilde{r}) - \tilde{r}^2 + i\sqrt{-\alpha(\tilde{r})}}{2}} + \sqrt[3]{\frac{6\tilde{d}^2 + \tilde{\mu} - 2\tilde{n}_{\mathrm{th}}(\tilde{r}) - \tilde{r}^2 - i\sqrt{-\alpha(\tilde{r})}}{2}} - 2\tilde{d}\right]^2}\right)$$

where $\alpha(\tilde{r}) = \frac{4}{27}\left\{27\tilde{d}^6 + 54\tilde{d}^4\left[\tilde{\mu} - 2\tilde{n}_{\mathrm{th}}(\tilde{r}) - \tilde{r}^2\right] + 9\tilde{d}^2\left[-\tilde{\mu} + 2\tilde{n}_{\mathrm{th}}(\tilde{r}) + \tilde{r}^2\right]^2 - \left[\tilde{\mu} - 2\tilde{n}_{\mathrm{th}}(\tilde{r}) - \tilde{r}^2\right]^3\right\}$

Self-Consistency Equations

Bose-glass Region: $\tilde{n}_0(\tilde{r}) = 0$, $\tilde{q}(\tilde{r}) \neq 0$ and $\tilde{n}_{\rm th}(\tilde{r}) \neq 0$

$$\tilde{q}(\tilde{r}) = \frac{\tilde{\mu} - \tilde{r}^2}{2} - \frac{g}{\bar{\mu}} \left(\frac{M}{2\pi\beta\hbar^2} \right)^{3/2} \varsigma\left(\frac{3}{2}\right)$$

$$\tilde{n}_{\rm th}(\tilde{r}) = \frac{g}{\bar{\mu}} \left(\frac{M}{2\pi\beta\hbar^2} \right)^{3/2} \varsigma\left(\frac{3}{2}\right)$$

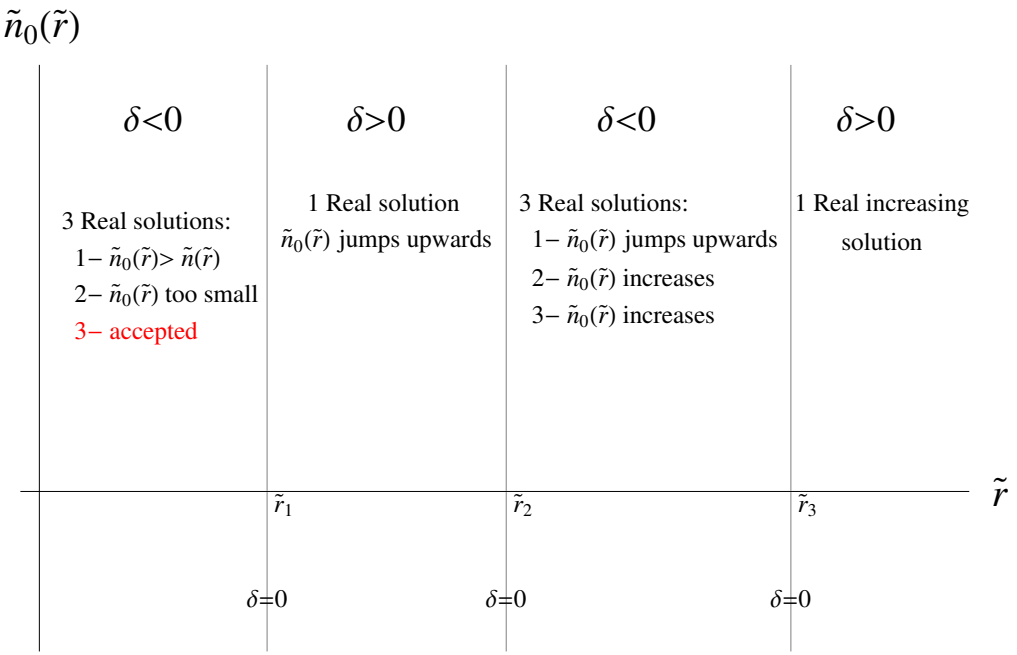
$$\tilde{n}(\tilde{r}) = \frac{\tilde{\mu} - \tilde{r}^2}{2}$$

Thermal Region: $\tilde{n}_0(\tilde{r}) = \tilde{q}(\tilde{r}) = 0$ and $\tilde{n}_{\rm th}(\tilde{r}) = \tilde{n}(\tilde{r}) \neq 0$

$$\tilde{n}_{\rm th}(\tilde{r}) = \frac{g}{\bar{\mu}} \left(\frac{M}{2\pi\beta\hbar^2} \right)^{3/2} \varsigma_{3/2}\left(e^{\beta\bar{\mu}\,[\tilde{\mu} - 2\tilde{n}_{\rm th}(\tilde{r}) - \tilde{r}^2]} \right)$$

- Robinson Formula $\varsigma_{\nu}(e^{x}) = \Gamma(1-\nu)(-x)^{\nu-1} + \sum_{k=0}^{\infty} \frac{x^k}{k!} \varsigma(\nu - k), \quad x < 0$
- Discriminant of Cardan method in superfluid region for $\tilde{n}_0(\tilde{r})$

$$\delta = -\frac{4\tilde{\mu}^3}{27} + \frac{4\tilde{\mu}^2\tilde{d}^2}{3} + 8\tilde{\mu}\tilde{d}^4 + 4\tilde{d}^6 + \left(\frac{4\tilde{\mu}^2}{9} - \frac{8\tilde{\mu}\tilde{d}^2}{3} - 8\tilde{d}^4\right)\tilde{r}^2 + \left(-\frac{4\tilde{\mu}}{9} + \frac{4\tilde{d}^2}{3}\right)\tilde{r}^4 + \frac{4}{27}\tilde{r}^6$$



- Robinson Formula $\varsigma_{\nu}(e^{x}) = \Gamma(1-\nu)(-x)^{\nu-1} + \sum_{k=0}^{\infty} \frac{x^k}{k!} \varsigma(\nu - k)$
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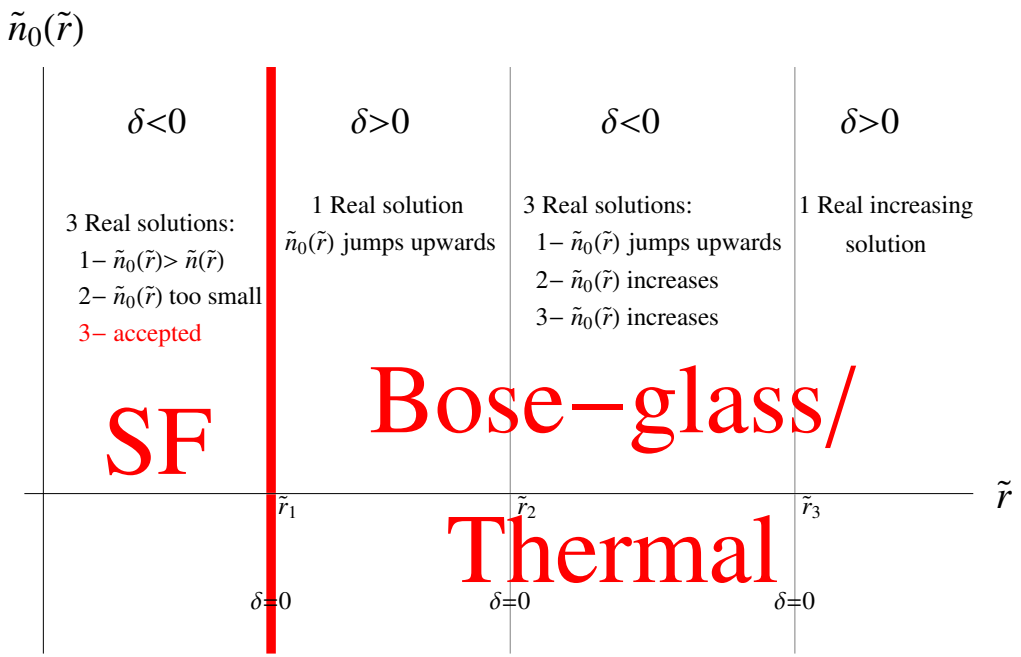
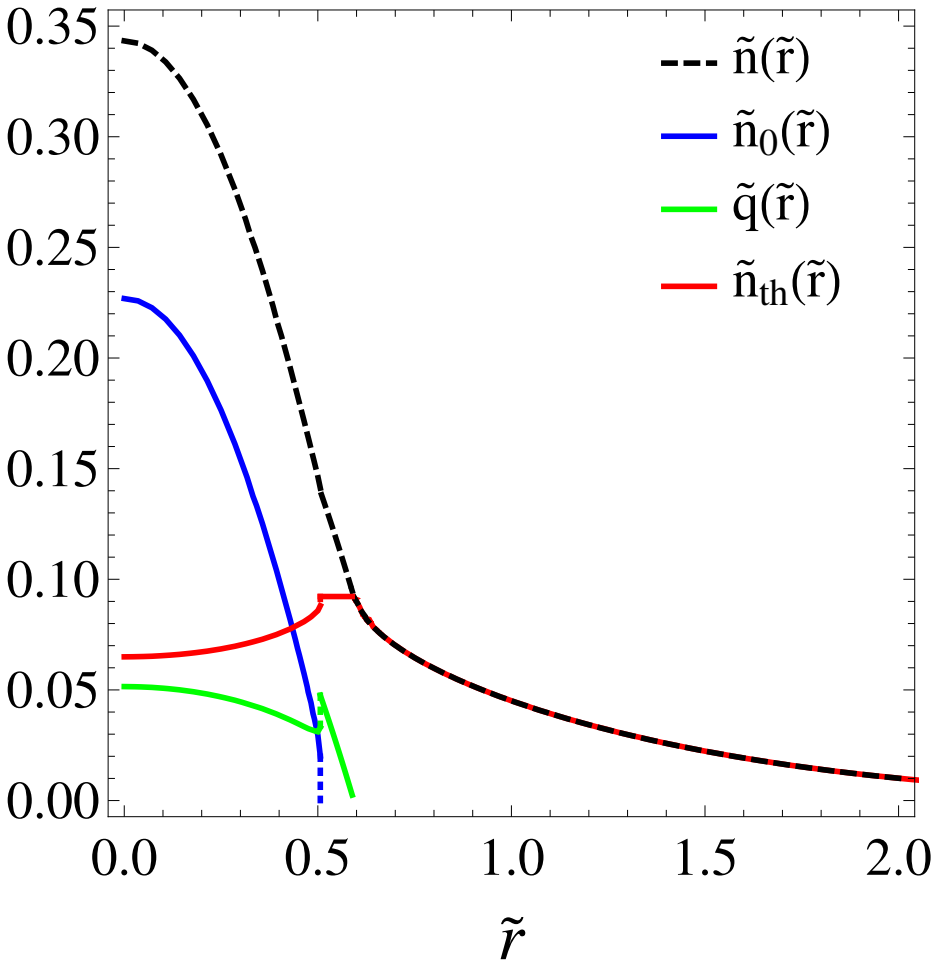


- $^{87}\mathrm{Rb}$, $N = 10^6$, $\tilde{d} = 0.088$, $\tilde{\mu} = 0.535$, $\Omega = 100$ Hz, $T = 60$ nK and $a = 5.29$ nm



Thomas-Fermi Radii

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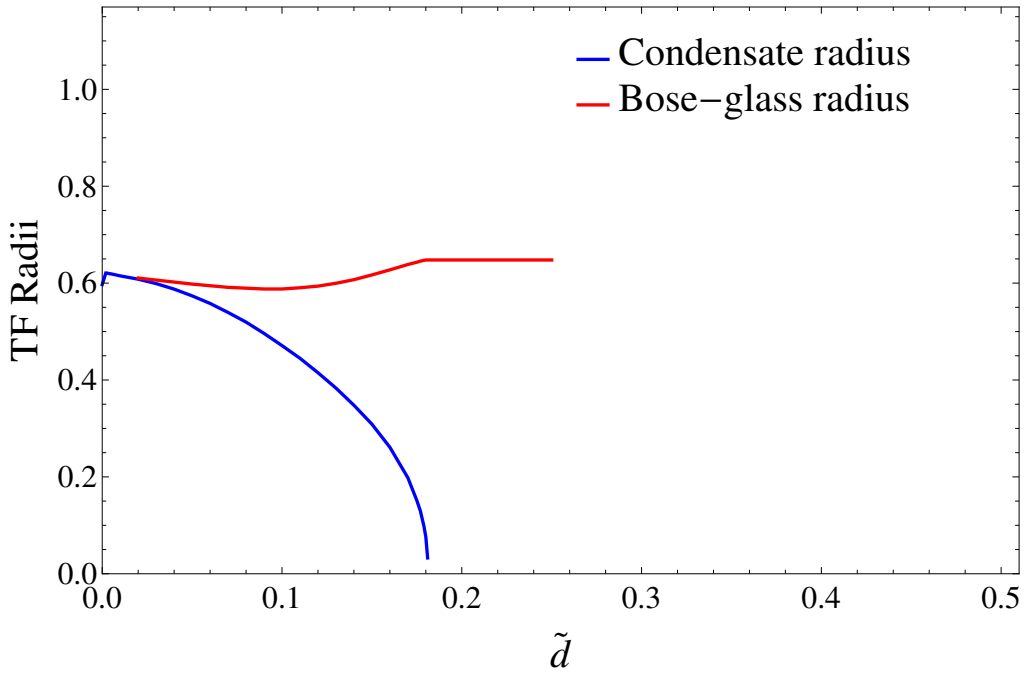

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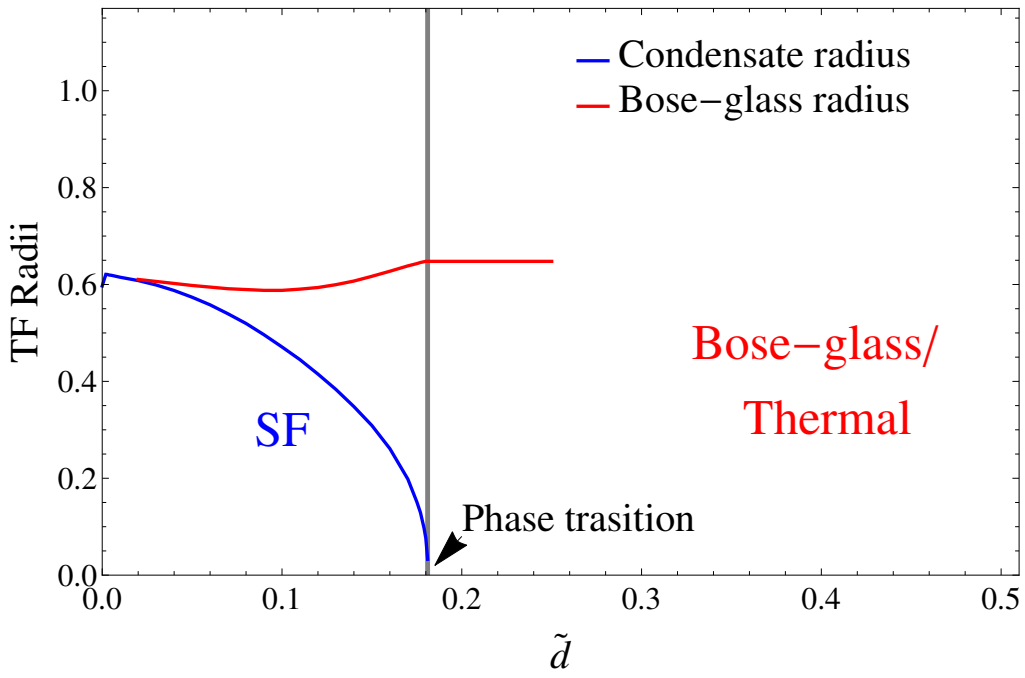
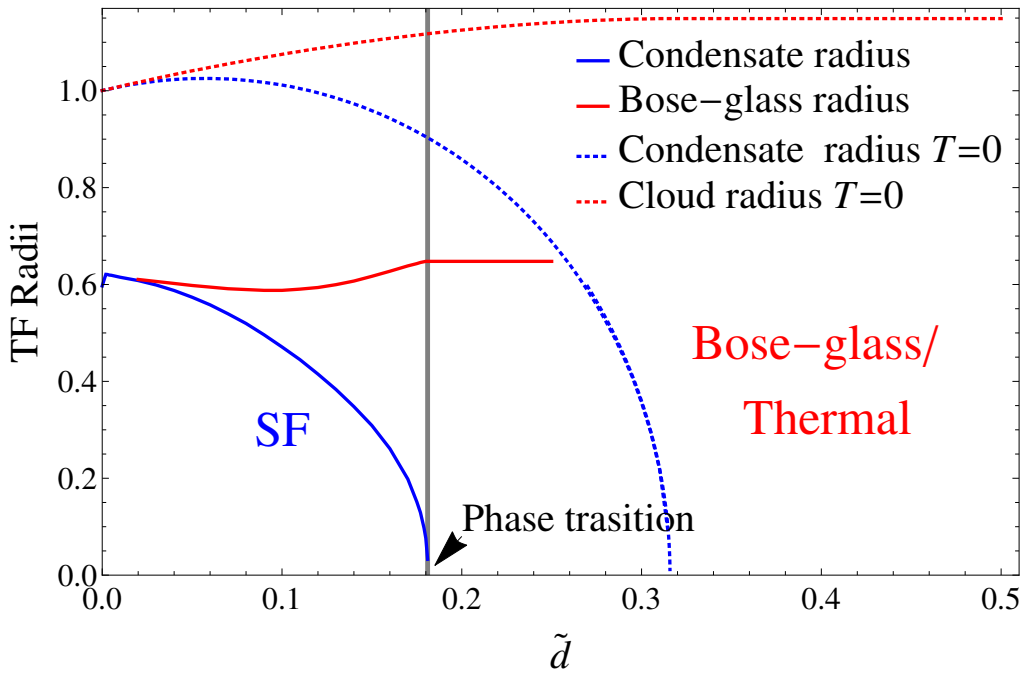


Thomas-Fermi Radii

- $^{87}\mathrm{Rb}$, $N = 10^6$, $\Omega = 100$ Hz, $T = 60$ nK and $a = 5.29$ nm



Outlook

- Beyond Thomas-Fermi approximation
- Comparison with numerics
- General correlation function
- General interaction potential
- Time dependence of densities and Thomas-Fermi radii
- Replica Symmetry Breaking?

Thank You For Your Attention