Weakly Interacting Bose Gas in Random Environment

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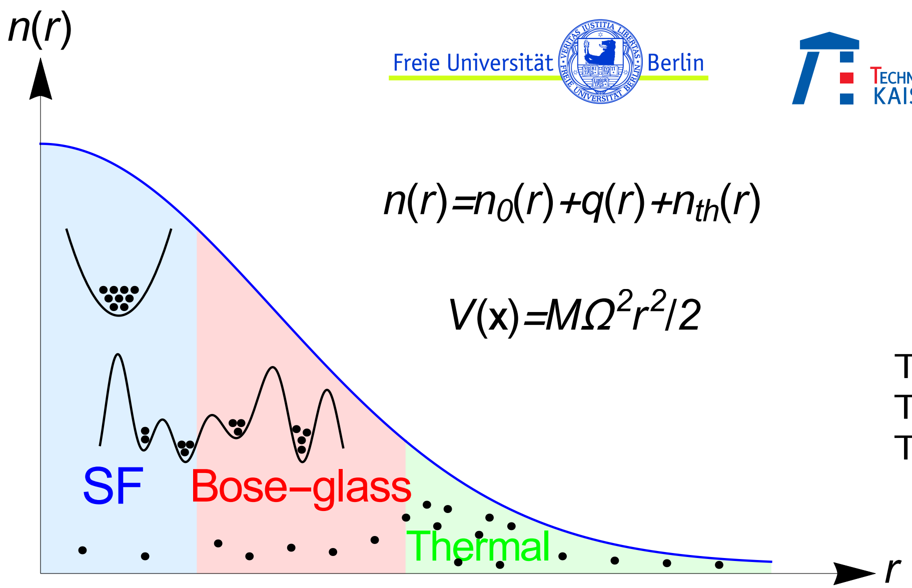
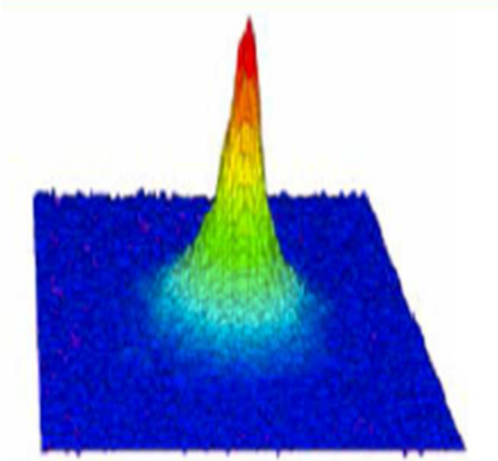
OPTIMAS



T. Khellil, A. Balaž, and A. Pelster, `arXiv:1510.04985`
T. Khellil and A. Pelster, `arXiv:1511.08882`
T. Khellil and A. Pelster, `arXiv:1512.04870`

Introduction

- Dirty BEC



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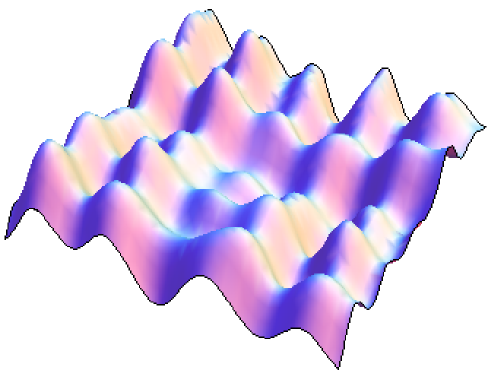
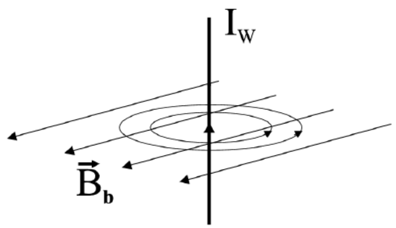
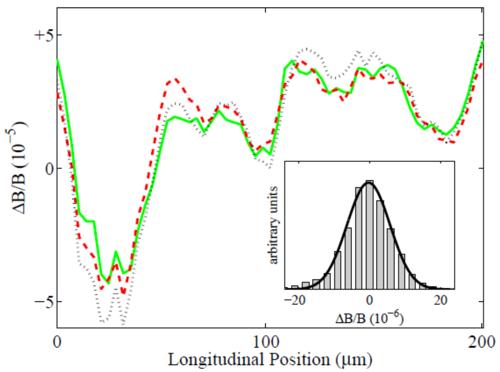


- **Uncontrolled disorder:**

Wire Traps

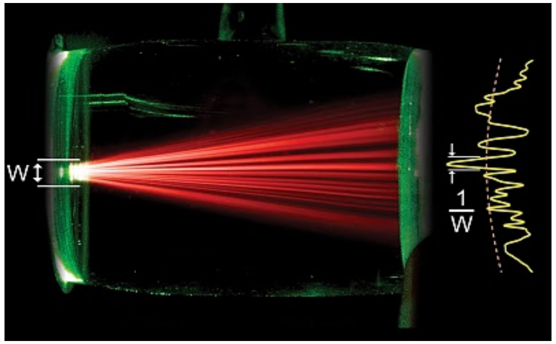


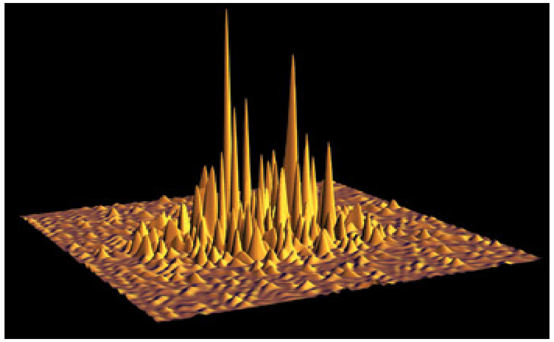


Schmiedmayer group:
PRA **76**, 063621 (2007)

- **Controlled disorder:**

Laser Speckles





Inguscio group: PRL **95**, 070401 (2005)
Aspect group: PRL **95**, 170409 (2005)

Definitions

2-point function $$\lim_{|\mathbf{x}-\mathbf{x}'|\to\infty} \overline{\langle \psi(\mathbf{x},\tau)\psi^*(\mathbf{x}',\tau)\rangle} = \sqrt{n_0(\mathbf{x})n_0(\mathbf{x}')}$$

4-point function $$\lim_{|\mathbf{x}-\mathbf{x}'|\to\infty} \overline{|\langle \psi(\mathbf{x},\tau)\psi^*(\mathbf{x}',\tau)\rangle|^2} = [n_0(\mathbf{x}) + q(\mathbf{x})]\,[n_0(\mathbf{x}') + q(\mathbf{x}')]$$

Superfluid (SF)	Bose-Glass (BG)	Thermal
$n(\mathbf{x}) = n_0(\mathbf{x}) + q(\mathbf{x}) + n_{\mathrm{th}}(\mathbf{x})$	$n_0(\mathbf{x}) = 0$ $n(\mathbf{x}) = q(\mathbf{x}) + n_{\mathrm{th}}(\mathbf{x})$	$q(\mathbf{x}) = n_0(\mathbf{x}) = 0$ $n(\mathbf{x}) = n_{\mathrm{th}}(\mathbf{x})$

Region: with respect to spatial coordinates
Phase: with respect to disorder strength or temperature

Self-Consistency Equations

$$n(\mathbf{x}) = n_0(\mathbf{x}) + q(\mathbf{x}) + \underbrace{\lim_{\eta \downarrow 0} \sum_{m=-\infty}^{\infty} e^{i\omega_m \eta} \frac{Q_m(\mathbf{x})}{\hbar\beta}}_{n_{\mathrm{th}}(\mathbf{x})}$$

$$\left[-\mu + 2gn(\mathbf{x}) + V(\mathbf{x}) - gn_0(\mathbf{x}) - \frac{D}{\hbar} Q_0(\mathbf{x}) - \frac{\hbar^2}{2M}\Delta\right] \sqrt{n_0(\mathbf{x})} = 0$$

$$q(\mathbf{x}) = D\Gamma\left(2 - \frac{n}{2}\right)\left(\frac{M}{2\pi\hbar^2}\right)^{n/2} \frac{[q(\mathbf{x}) + n_0(\mathbf{x})]}{\left[-\mu + 2gn(\mathbf{x}) + V(\mathbf{x}) - \frac{D}{\hbar} Q_0(\mathbf{x})\right]^{2-\frac{n}{2}}}$$

$$Q_m(\mathbf{x}) = \Gamma\left(1 - \frac{n}{2}\right)\hbar\left(\frac{M}{2\pi\hbar^2}\right)^{n/2}\left[-i\hbar\omega_m - \mu + 2gn(\mathbf{x}) + V(\mathbf{x}) - \frac{D}{\hbar} Q_m(\mathbf{x})\right]^{\frac{n}{2}-1}$$

Matsubara frequencies $\omega_m = 2\pi m / (\hbar\beta)$

T. Khellil and A. Pelster, `arXiv:1511.08882`

Thomas-Fermi Approximation (TF)

$$\left[-\mu + 2gn(\mathbf{x}) + \frac{1}{2}M\Omega^2 x^2 - gn_0(\mathbf{x}) - \frac{D}{\hbar}Q_0(\mathbf{x}) - \frac{\hbar^2}{2M}\Delta\right]\sqrt{n_0(\mathbf{x})} = 0, \qquad g = 2a\hbar\omega_r$$

- Energy scale $\bar{\mu} = \hbar\omega_r\left(\frac{3}{2\sqrt{2}}N\frac{a}{l}\sqrt{\frac{\Omega}{\omega_r}}\right)^{2/3}$, $l = \sqrt{\frac{\hbar}{M\Omega}}$, coordinate scale $R_{\mathrm{TF}} = l\sqrt{\frac{2\bar{\mu}}{\hbar\Omega}}$, density scale $\bar{n} = \frac{\bar{\mu}}{g}$, dimensionless disorder strength $\tilde{D} = \frac{\xi^3}{\mathcal{L}^3}$, $\xi = \frac{l^2}{R_{\mathrm{TF}}}$, and $\mathcal{L} = \left(\frac{\hbar^4}{M^2 D}\right)^{1/3}$

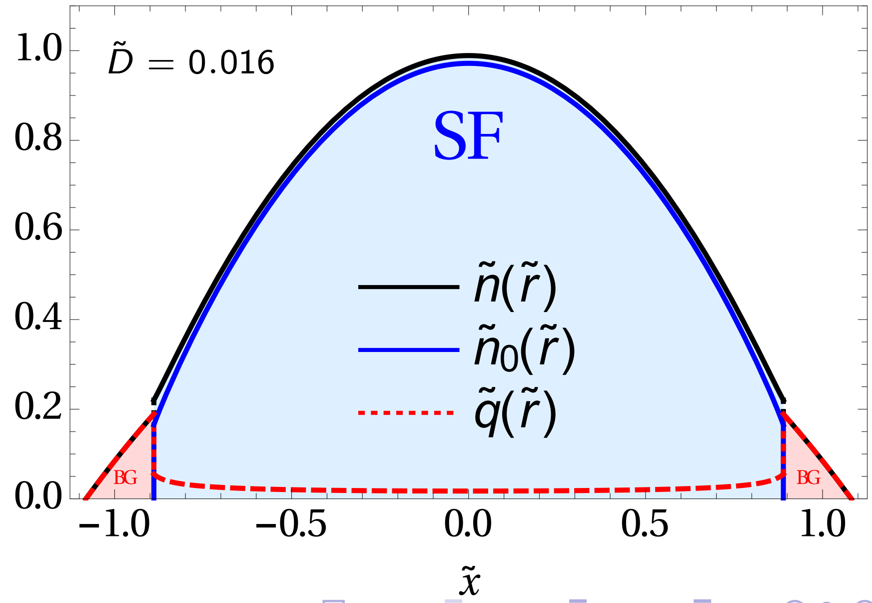
- $^{87}\mathrm{Rb}$, $N = 10^6$, $\Omega = 2\pi \times 50\,\mathrm{Hz}$,

  $\omega_r = 2\pi \times 179\,\mathrm{Hz}$, and $a = 5.29\,\mathrm{nm}$

- Length scales: $l_r = \sqrt{\frac{\hbar}{M\omega_r}} = 806.04\,\mathrm{nm}$,

  $l = 1.52\,\mu m$, $\xi = 45.6\,\mathrm{nm}$, and $R_{\mathrm{TF}} = 50.9\,\mu\mathrm{m}$

  Quasi one-dimensional regime: $a \ll l_r < l$
  TF approximation: $\xi \ll R_{\mathrm{TF}}$



Methods

- Numerical method
  - Gross-Pitaevskii equation for the ground state
    $$\left[-\frac{\hbar^2\Delta}{2m} - \mu + U(x) + V(x) + \frac{g}{2}\psi^*(x)\psi(x)\right]\psi(x) = 0$$
    Gaussian correlation function $D(x) = \frac{D}{\sqrt{2\pi}\lambda}\exp\left\{-\frac{x^2}{2\lambda^2}\right\}$
  - Generating random potential $U(x) = \frac{1}{\sqrt{\mathrm{N}}}\sum_{n=0}^{\mathrm{N}-1}\left[A_n \cos(k_n x) + B_n \sin(k_n x)\right]$

    J. Majda and P. Kramer, Phys. Rep. **314,** 237 (1999)
  - C program for solving time-(in)dependent Gross-Pitaevskii equation in 1D

    D. Vudragovic et al., Comput. Phys. Commun. **183**, 2021 (2012)
  - Particle density $n(x) = \overline{\psi(x)^2}$, condensate density $n_0(x) = \overline{\psi(x)}^2$, deviation $q(x) = n(x) - n_0(x)$

Problem: TF-approximated and numerical results are not compatible

Solution: variational method

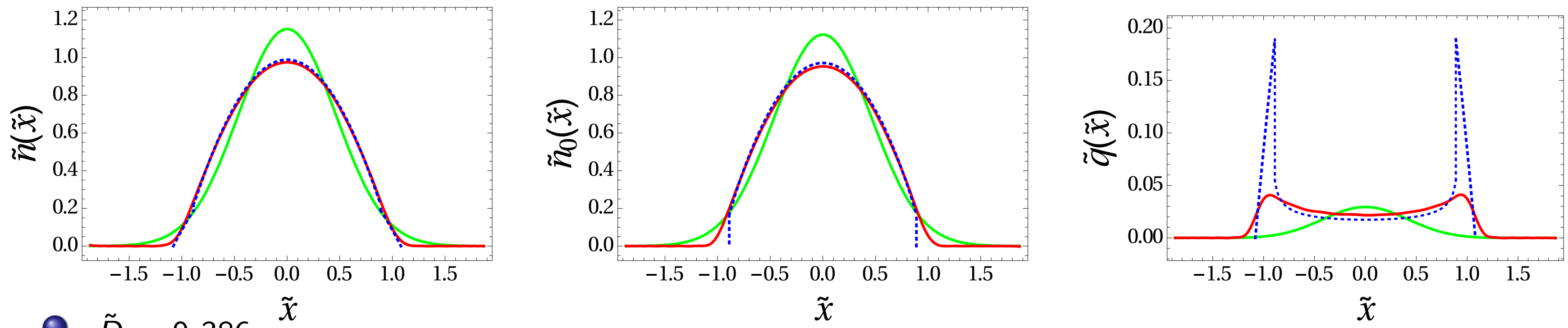
- Variational method

$$\tilde{n}_0\left(\tilde{x}\right) = \alpha e^{-\sigma\tilde{x}^2}, \quad \tilde{n}_0\left(\tilde{x}\right) + \tilde{q}\left(\tilde{x}\right) = \gamma e^{-\theta\tilde{x}^2}, \quad \tilde{Q}_0(\tilde{x}) = \frac{\tilde{q}\left(\tilde{x}\right) + \tilde{n}_0\left(\tilde{x}\right)}{\tilde{D}} - \left(\zeta + \eta\tilde{x}^2\right)$$

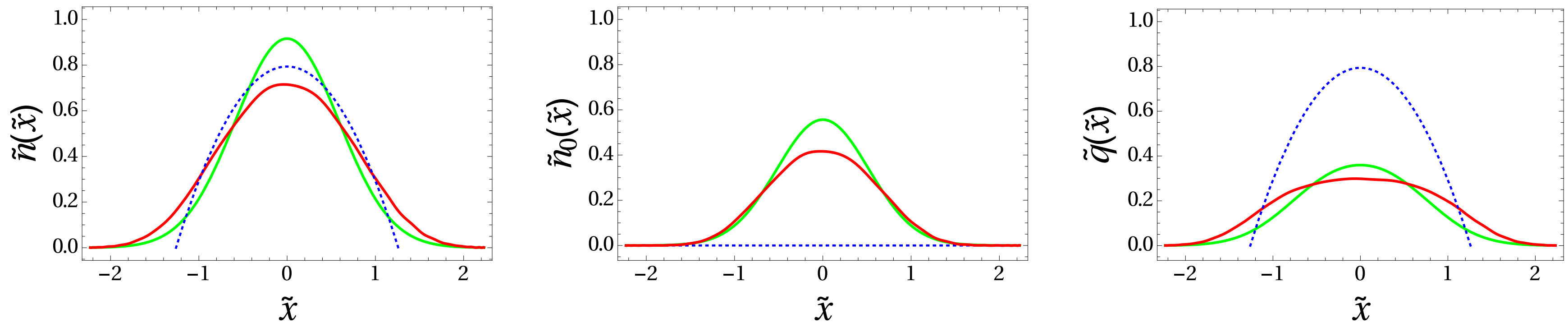
Extremise $\tilde{\mathcal{F}}$ with respect to α, σ, γ, θ, ζ, and η

Comparison of Analytical, Numerical, and Variational Results

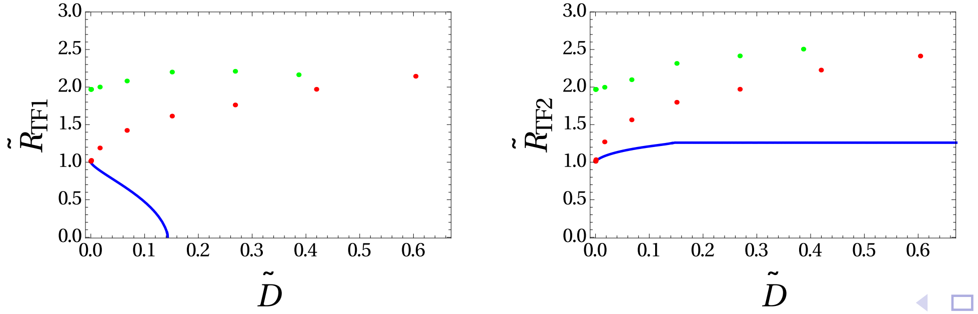
- $\tilde{D} = 0.016$



- $\tilde{D} = 0.386$



- TF Radii

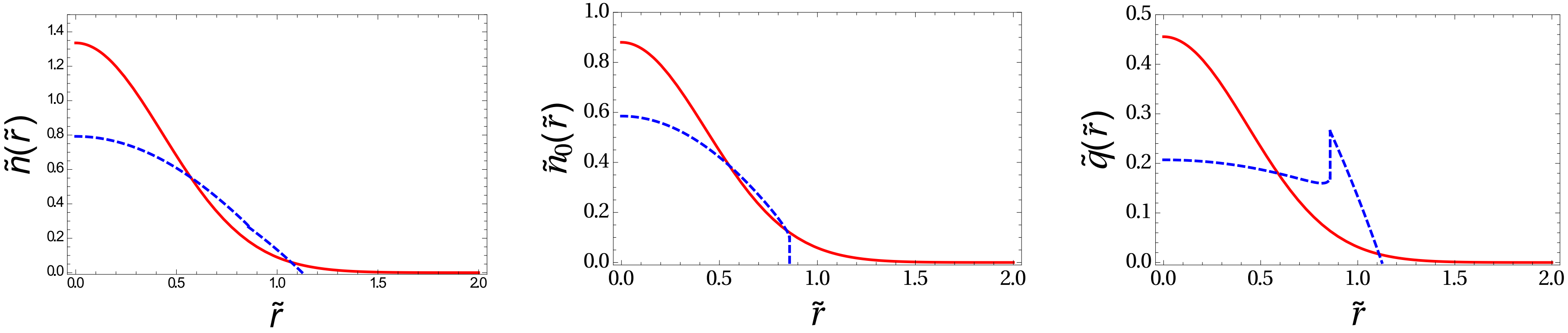


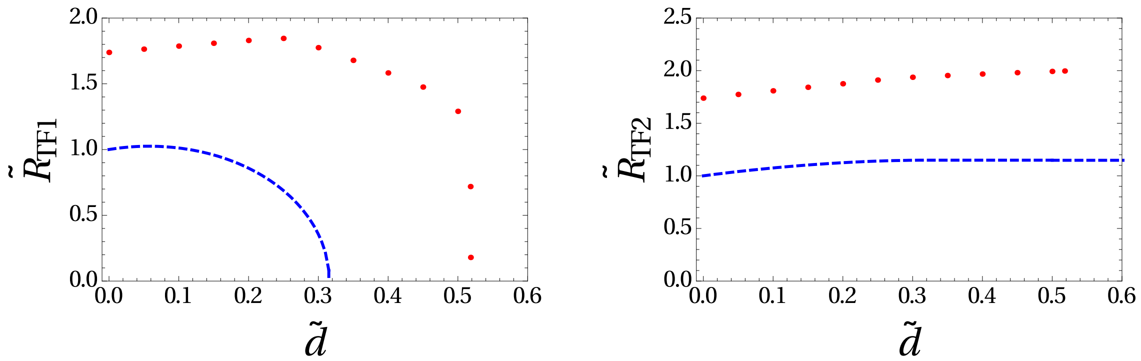
Green $\longrightarrow$ Variational

Red $\longrightarrow$ Numerical

Blue $\longrightarrow$ Analytical

Isotropic 3D $T = 0$ for $\tilde{d} = 0.2$





Red $\longrightarrow$ Variational

Blue $\longrightarrow$ Analytical

Quantum phase transition from superfluid to Bose-glass in agreement with

T. Nattermann and V. L. Pokrovsky, Phys. Rev. Lett. **100**, 060402 (2008) $\tilde{d}_{\rm c} = 0.115$

Comparison of 1D and 3D

1D	3D
QPT in homogeneous case	QPT in homogeneous case
qualitative agreement with Huang-Meng theory	qualitative agreement with Huang-Meng theory
Factor of $2\sqrt{2}$ discrepancy with Huang-Meng	Factor of $\sqrt{2}$ discrepancy with Huang-Meng
μ decreases with disorder strength	μ increases with disorder strength
Tiny TF approximation disorder validity range	Larger TF approximation disorder validity range
TF and variational results disagree totally	TF and variational results agree qualitatively
Weak disorder: BG region at the trap border Strong disorder: BG region at the trap center	BG region at the trap center
No QPT in weak and intermediate disorder regime	**QPT in intermediate disorder regime**

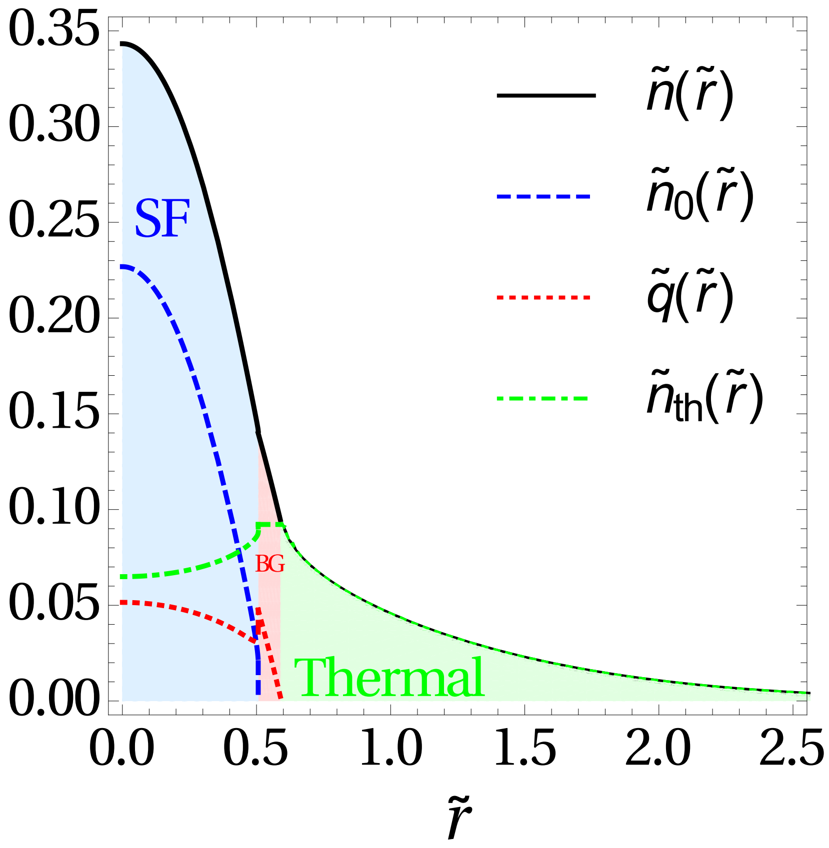
T. Khellil, A. Balaž, and A. Pelster, `arXiv:1510.04985`

T. Khellil and A. Pelster, `arXiv:1512.04870`

Isotropic 3D Finite Temperature (TF)

$^{87}\mathrm{Rb}$, $N = 10^6$, $\Omega = 100\,\mathrm{Hz}$, $T = 60\ \mathrm{nK}$, $\tilde{d} = 0.088$, and $a = 5.29\,\mathrm{nm}$

with Robinson expansion $\varsigma_{\nu}(e^x) = \Gamma(1-\nu)(-x)^{\nu-1} + \sum_{k=0}^{\infty} \frac{x^k}{k!}\varsigma(\nu-k)\,,\ x < 0$



Announcement

616th Wilhelm and Else Heraeus Seminar

Ultracold Quantum Gases -

Current Trends and Future Perspectives

organized by Carlos Sá de Melo and Axel Pelster

Bad Honnef (Germany); May 9 – 13, 2016

Invited Speakers: Eugene Demler (USA), Rembert Duine (Netherthelands), Tilman Esslinger (Switzerland), Michael Fleischhauer (Germany), Thierry Giamarchi (Switzerland), Rudi Grimm (Austria), Johannes Hecker-Denschlag (Germany), Andreas Hemmerich (Germany), Jason Ho (USA), Walter Hofstetter (Germany), Randy Hulet (USA), Massimo Inguscio (Italy), Corinna Kollath (Germany), Stefan Kuhr (UK), Kazimierz Rzazewski (Poland), Anna Sanpera (Spain), Luis Santos (Germany), Jörg Schmiedmayer (Austria), Dan Stamper-Kurn (USA), Sandro Stringari (Italy), Leticia Tarruell (Spain), Jacques Tempere (Belgium), Päivi Törmä (Finland), Matthias Weidemüller (Germany), Eugene Zaremba (Canada), Peter Zoller (Austria)

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