Dissipative Two-Mode Tavis-Cummings Model with Time-Delayed Feedback Control

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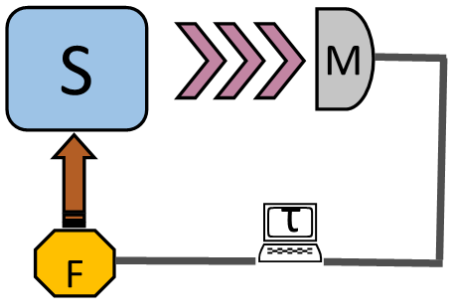
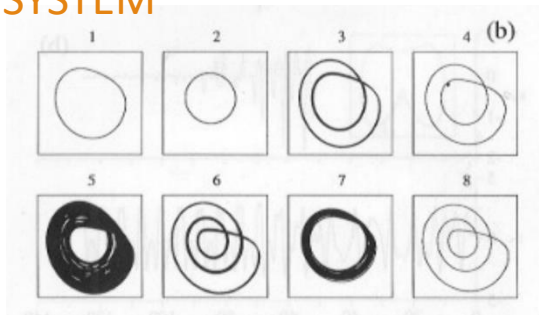
Why Feedback?

W. Just, A. Pelster et al. , Phil. Trans. R. Soc. A **368** (2010)



PYRAGAS CHAOS CONTROL IN A ROSSLER SYSTEM

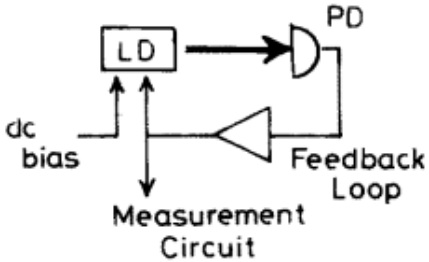


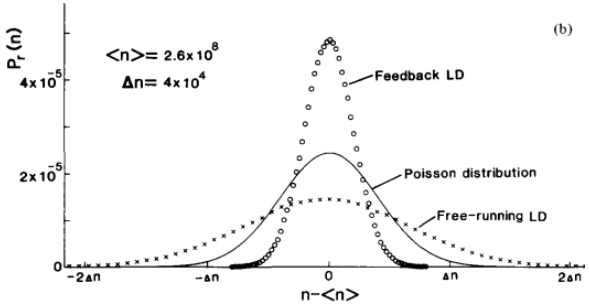
Control ~ $\left(X(t-\tau)-X(t)\right)$

Non-invasive

Stabilizes fixed points, unstable limit cycles

SUB-POISSONIAN STATISTICS IN A FEEDBACK SEMICONDUCTOR LASER



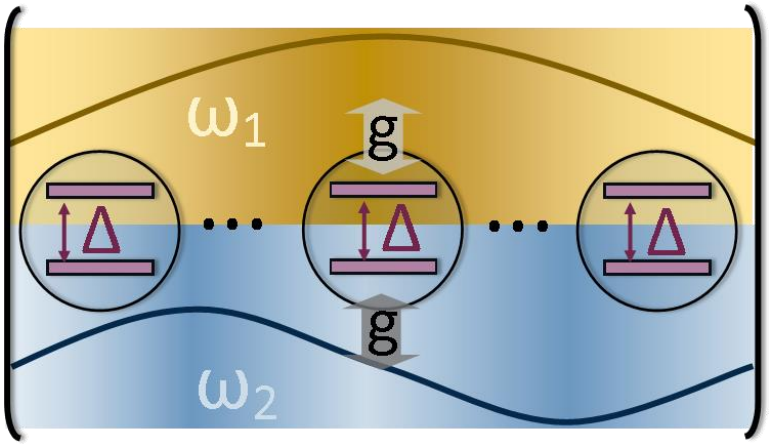


S. Machida et al., Opt. Comm. **57** (1986)

K. Pyragas, Phys. Let. A **170** (1992)

From Tavis-Cummings to a Two-mode-Laser

$$H = \sum_{m=1}^{2} \omega_m a_m^\dagger a_m + \frac{\Delta}{2} J^z + g \sum_{m=1}^{2} (a_m J^+ + a_m^\dagger J^-)$$



ω_i: Frequencies of the cavity modes

Δ: Atomic Frequency

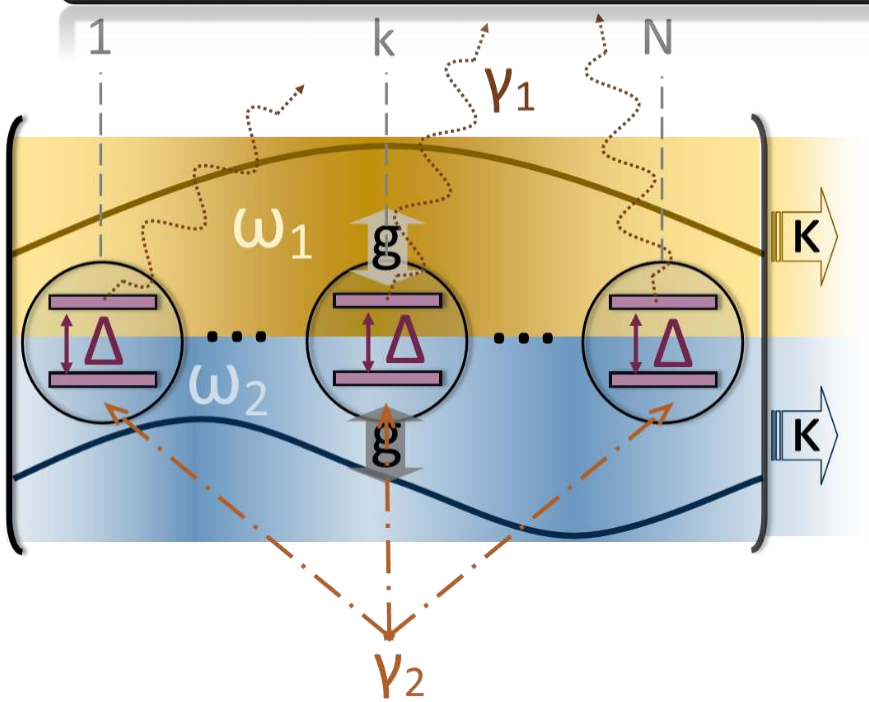
g: Coupling

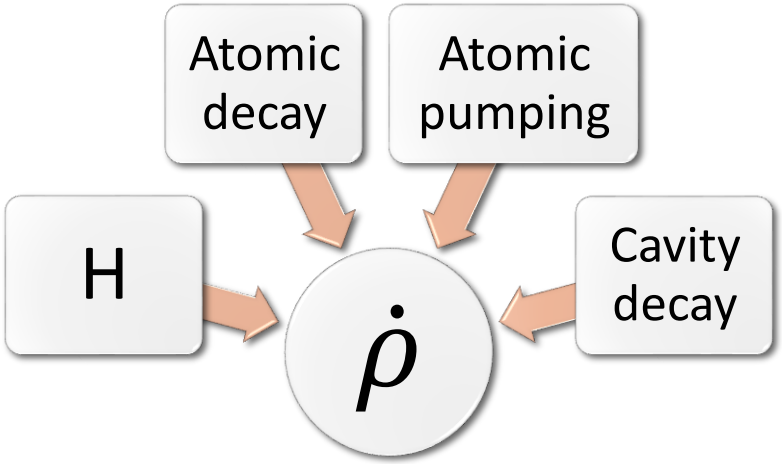
$J^\pm = \sum_i \sigma_i^\pm$ collective spin

$J^z = \frac{1}{2} \sum_i \sigma_i^z$

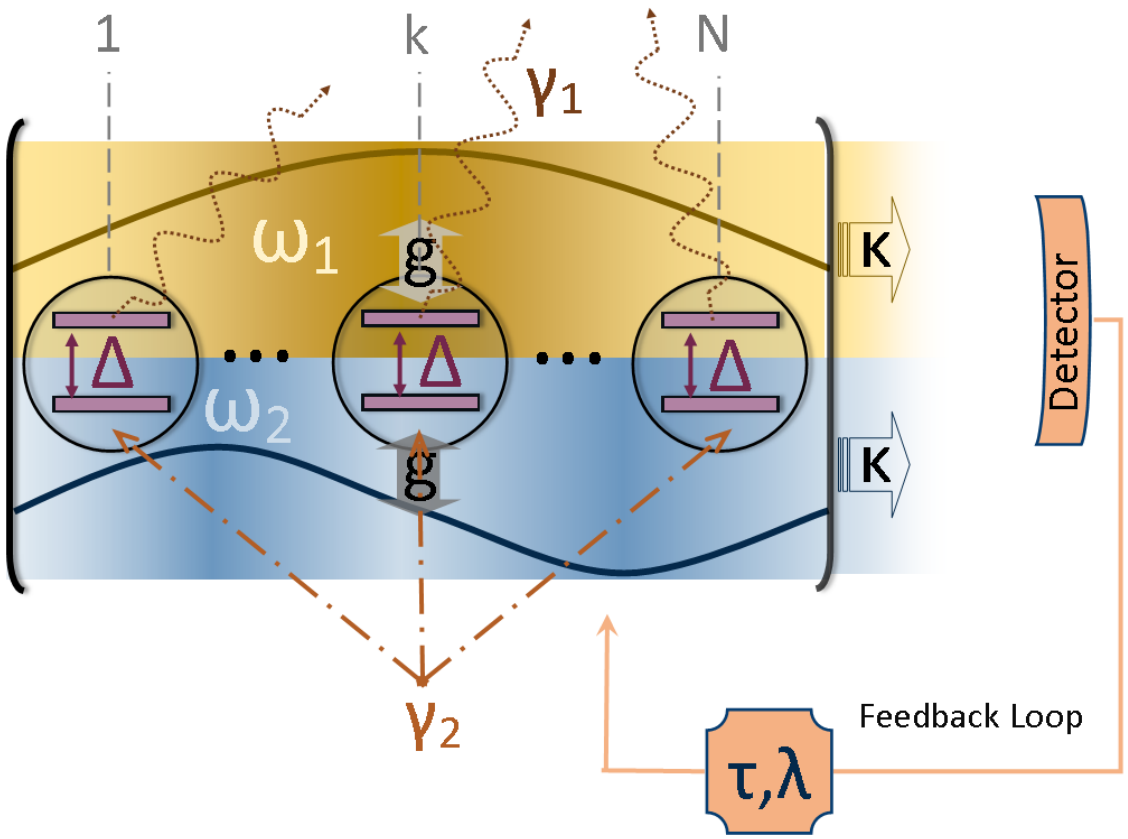
From Tavis-Cummings to a Two-mode-Laser

$$\frac{d\hat{\rho}(t)}{dt} = -i[\hat{H},\hat{\rho}] - \kappa\mathcal{L}[\hat{a}_1]\hat{\rho} - \kappa\mathcal{L}[\hat{a}_2]\hat{\rho} - \frac{\gamma_2}{2}\sum_{k=1}^{N}\mathcal{L}[\hat{\sigma}_k^+]\hat{\rho} - \frac{\gamma_1}{2}\sum_{k=1}^{N}\mathcal{L}[\hat{\sigma}_k^-]\hat{\rho},$$





Two-mode-Laser with Feedback



Math Tools

Lindblad Master Eq.

- $\dot{\hat{\rho}} = -i[\hat{H}, \hat{\rho}] - \frac{\kappa}{2} D[\hat{O}_k]\hat{\rho}$
- Lindbladian $D[\hat{O}_k]\hat{\rho} = \hat{O}_k^\dagger \hat{O}_k \hat{\rho} + \hat{\rho}\hat{O}_k^\dagger \hat{O}_k - 2\hat{O}_k \hat{\rho} \hat{O}_k^\dagger$
- $\hat{O}_i$ - System operator

Mean-Field and Equations of Motion

- $\partial_t \langle \hat{O} \rangle = Tr(\hat{O}\dot{\hat{\rho}})$
- Factorized operator $\langle \hat{O}_1 \hat{O}_2 \rangle \approx \langle \hat{O}_1 \rangle \langle \hat{O}_2 \rangle$, valid in thermodynamic limit
- Semiclassical equations: $\partial_t \vec{v}(t) = \vec{f}(\vec{v})$,
  $\vec{v} \quad = (\langle \hat{O}_1 \rangle, \langle \hat{O}_2 \rangle \dots)$

Math Tools

Fixed Points

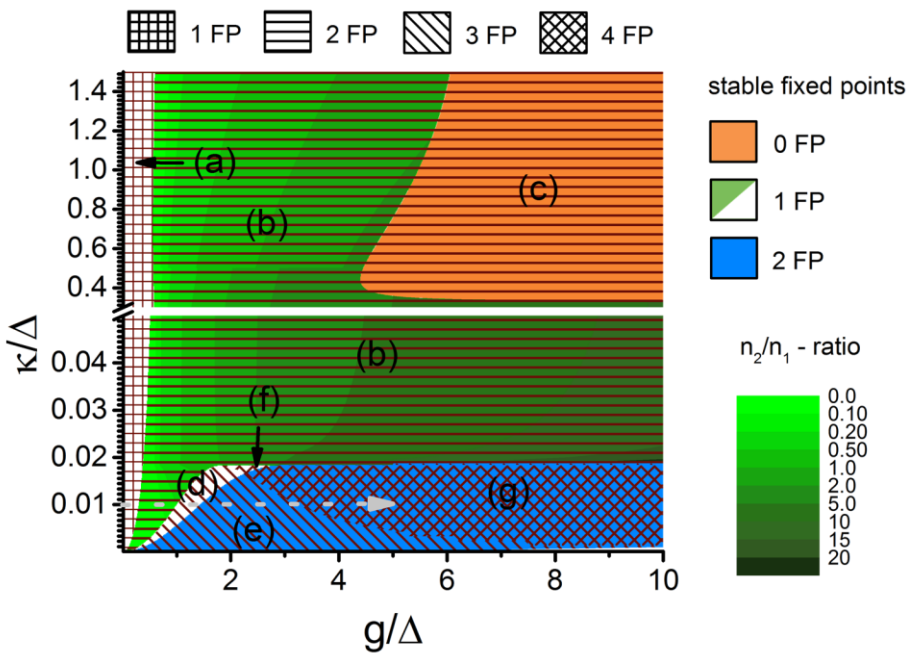
- $\vec{f}(\vec{v}^0) = 0 \implies \vec{v}^0 = (\langle \hat{O}_1^0 \rangle, \langle \hat{O}_2^0 \rangle, \dots)$

Introduce Pyragas Feedback into the Equations

Stability and Feedback

- Linear stability analysis: $\vec{v} = \vec{v}_0 + \delta\vec{v}$
- $\det\left(\Lambda \cdot \mathbf{1} - \boldsymbol{B} - (\boldsymbol{B} - \boldsymbol{A}) \cdot e^{-\Lambda\,\tau}\right) = 0$
- Numeric: Beyond linear stability

Stability Diagram



- Meanfield Eqs.

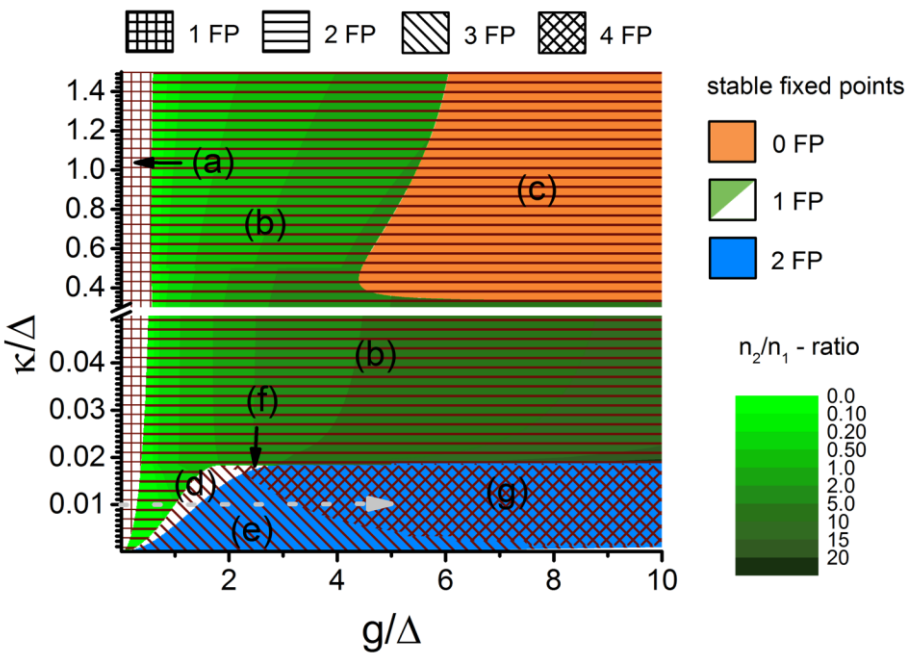
$$\partial_t \vec{v}(t) = \vec{f}(\vec{v})$$

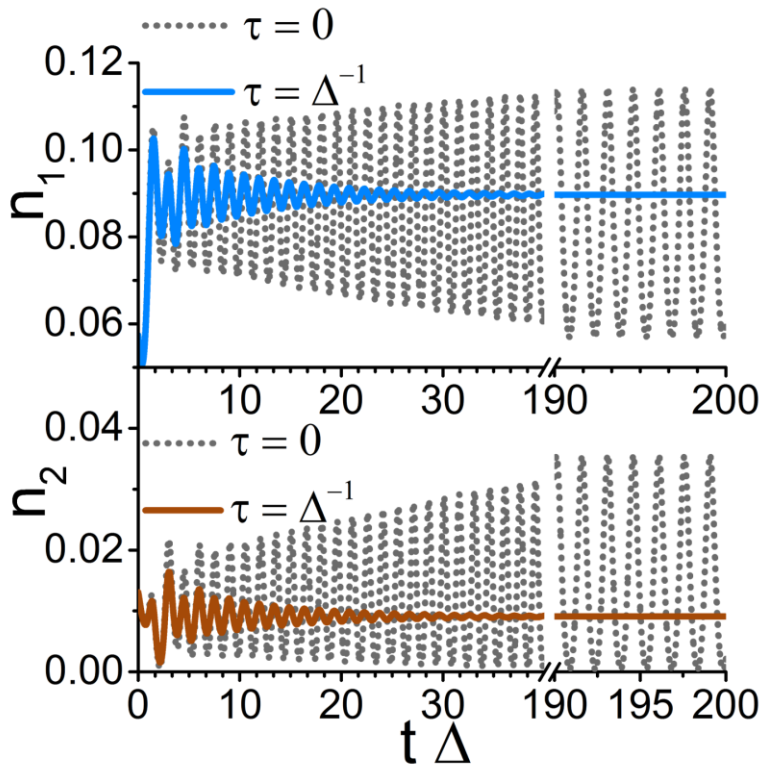
$$\vec{v} = \left(\langle \hat{a}_1 \rangle, \langle \hat{a}_2 \rangle, \langle \hat{J}_+ \rangle, \langle \hat{J}_- \rangle, \langle \hat{J}_z \rangle\right)$$

- Steady state: $\partial_t \vec{v} = 0$ → exact solution

Area	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Fixed Points	1	2	2	3	3	4	4
Stable	1	1	0	1	2	1	2

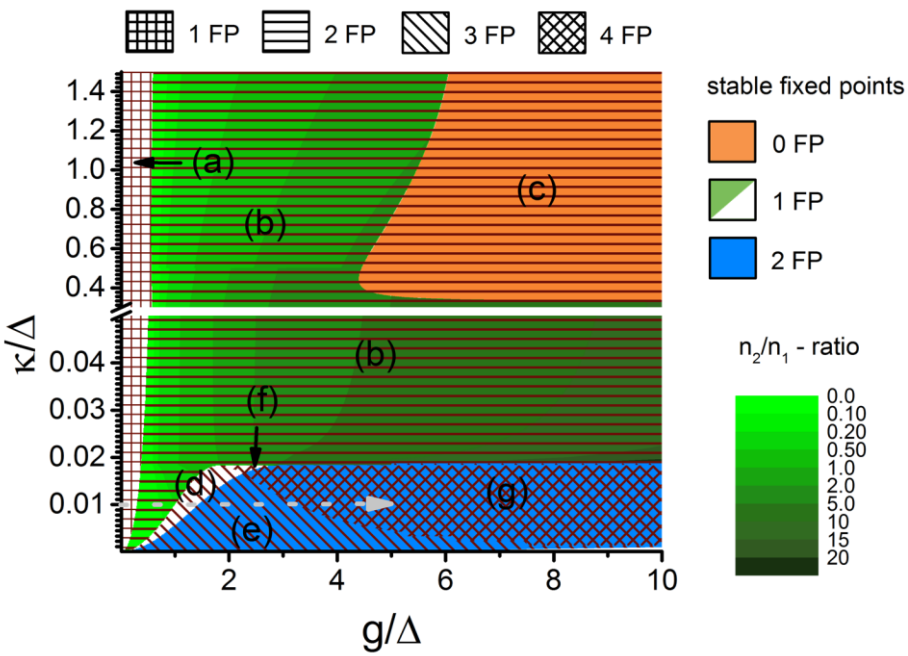
Pyragas Feedback: Stabilization (Area (c))

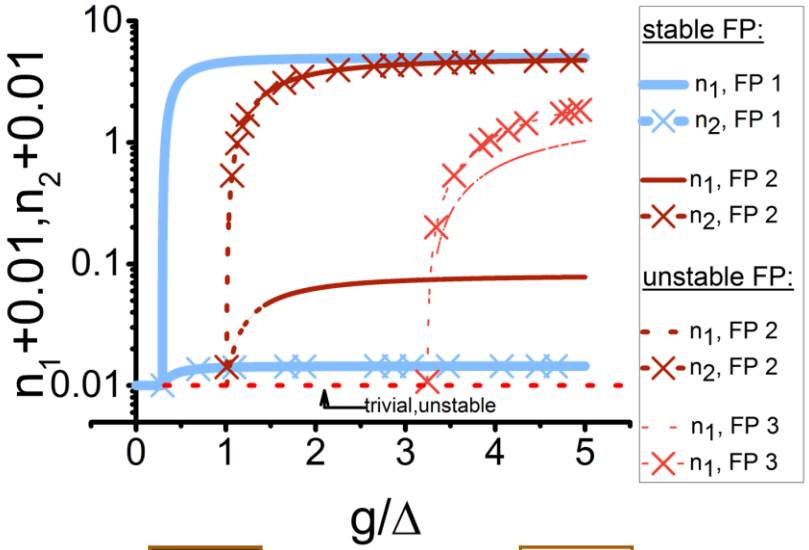


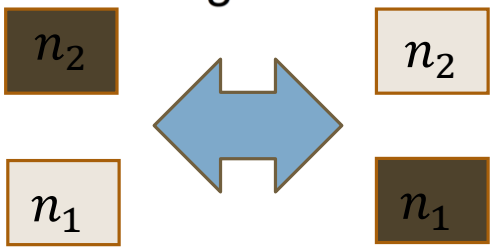


$$\dot{J}_z = \cdots + \lambda(J_z[t-\tau] - J_z[t])$$

Pyragas Feedback: Selecting the Lasing Mode (Area (e))



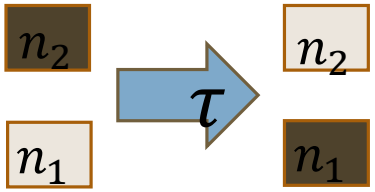




Pyragas Feedback: Selecting the Lasing Mode (Area (e))

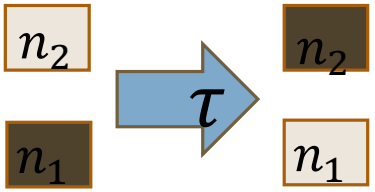
SWITCHING TO GROUND STATE OCCUPATION

$$\omega_1 \rightarrow (\omega_1 + \lambda(n_2(t-\tau) - n_2(t)))$$



SWITCHING TO EXCITED STATE OCCUPATION

$$\dot{a}_1 = \cdots + \lambda(a_1(t-\tau) - a_1(t))$$



Summary&Outlook

TC-Laser

Various Pyragas Schemas

- Stabilization
- Choice of the lasing mode

Outlook

- Connection to Thermalization [*]
- Control of Thermalization
- Beyond Meanfield Pyragas

W. Kopylov et al. PRA **92** (2015)

* P. Kirton and J. Keeling, PRL **111** (2013)