Mean-Field Phase Diagram of Disordered Bosons in a Lattice at Non-Zero Temperature

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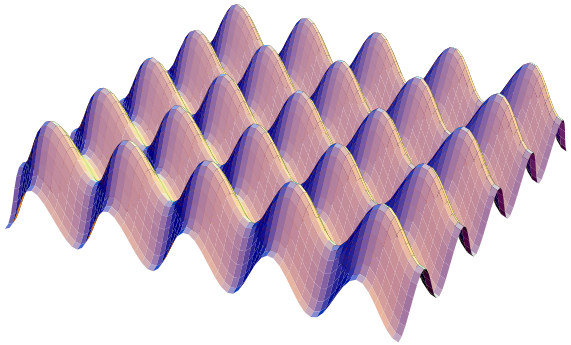
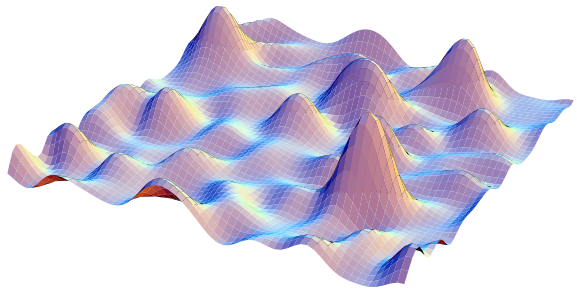
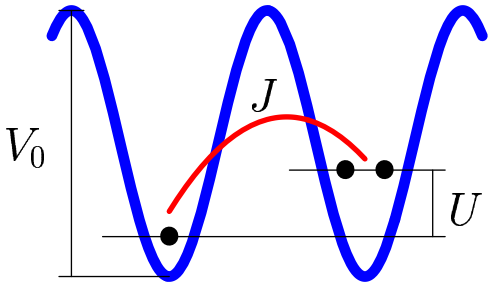
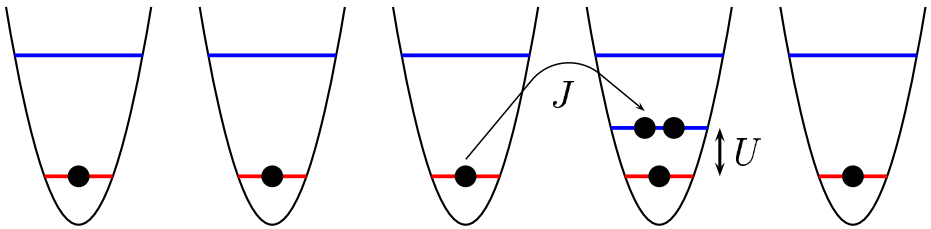






Suprafluid versus Mott-Isolator

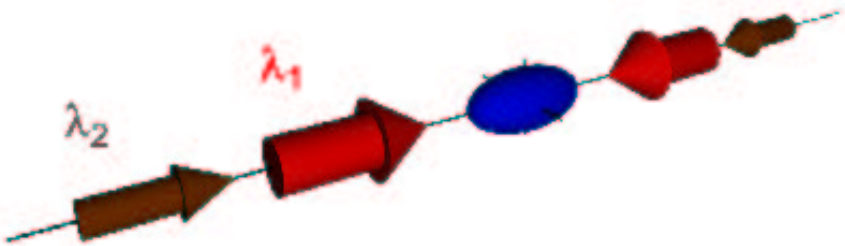


$$\hat{H}_{BH} = \underbrace{-J\sum_i \left(\hat{a}_i^\dagger \hat{a}_{i+1} + \hat{a}_{i+1}^\dagger \hat{a}_i\right)}_{\text{tunneling (hopping)}} + \underbrace{\frac{U}{2}\sum_i \hat{n}_i\left(\hat{n}_i - 1\right)}_{\text{onsite interaction}} - \underbrace{\mu \sum_i \hat{n}_i}_{\text{part. cons.}} \quad , \quad \hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$$

- $J << U$: fixed particle number n at each site (MI)
- $J >> U$: superposition of states with different particle numbers (SF)

$$\lim_{|i-j|\to\infty} \langle a_i^\dagger a_j \rangle \neq 0 \qquad \text{(ODLRO)}$$

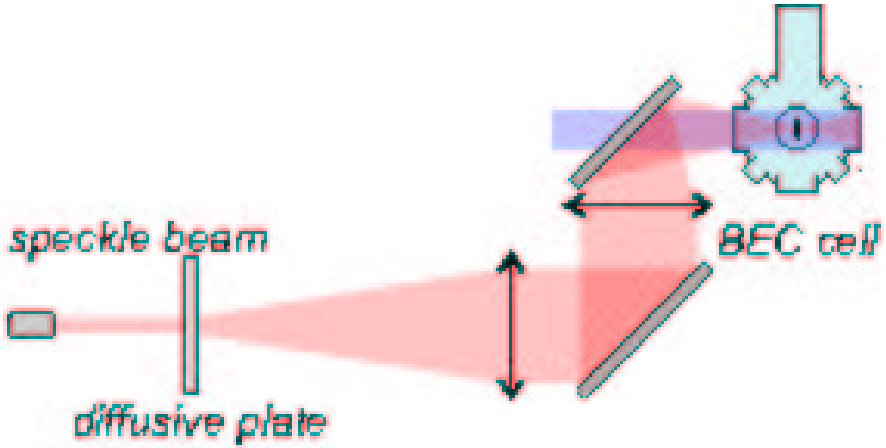
Incommensurable Lattice (Quasi-Disorder)

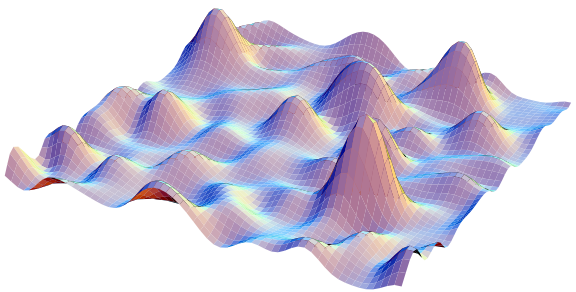


$$V_L(\mathbf{r}) = V_1 \cos^2 k_1 x + V_2 \cos^2 k_2 x$$

$$\frac{k_2}{k_1} = \frac{\lambda_1}{\lambda_2} = \frac{\sqrt{5}-1}{2} \qquad \text{(golden cut)}$$

Laser-Speckle

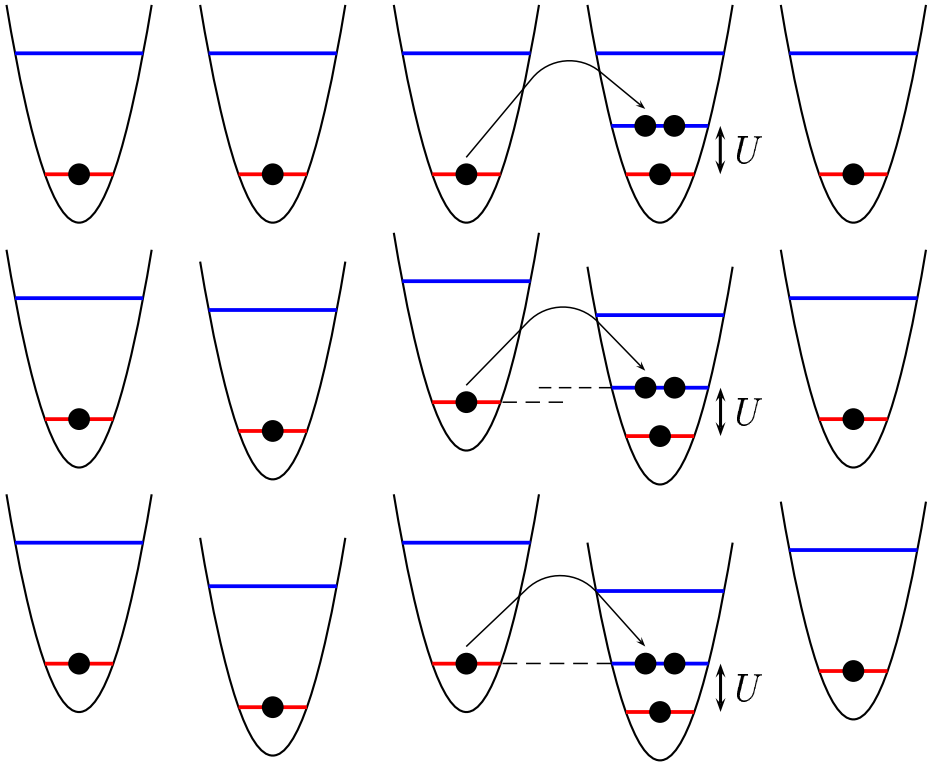




Quantum Phase Transitions of "Dirty" Bosons

potential wells shifted by $\epsilon_i \in \left[-\frac{D}{2}, \frac{D}{2}\right]$

$$\hat{H}_{BH} = -J \sum_i \left(\hat{a}_i^\dagger \hat{a}_{i+1} + \hat{a}_{i+1}^\dagger \hat{a}_i \right) + \frac{U}{2} \sum_i \hat{n}_i \left(\hat{n}_i - 1 \right) - \sum_i \left(\mu + \epsilon_i \right) \hat{n}_i$$



$D = 0$
(no disorder)

$D < U$
(weak disorder)

$D > U$
(strong disorder)

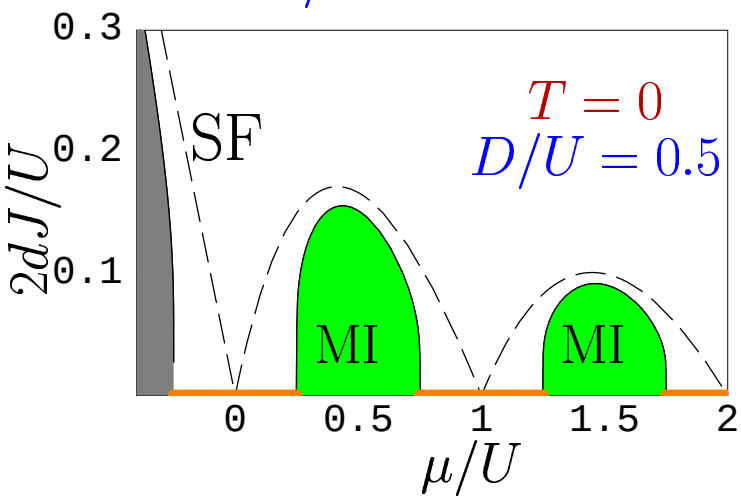
Homogeneous Disorder Distribution

all values $\epsilon_i \in \left[-\frac{D}{2}, \frac{D}{2}\right]$ have equal probability and are independent

K.V.Krutitsky, A.Pelster, and R.Graham, New J. Phys. **8**, 187 (2006)

$T = 0$

$D/U < 1$



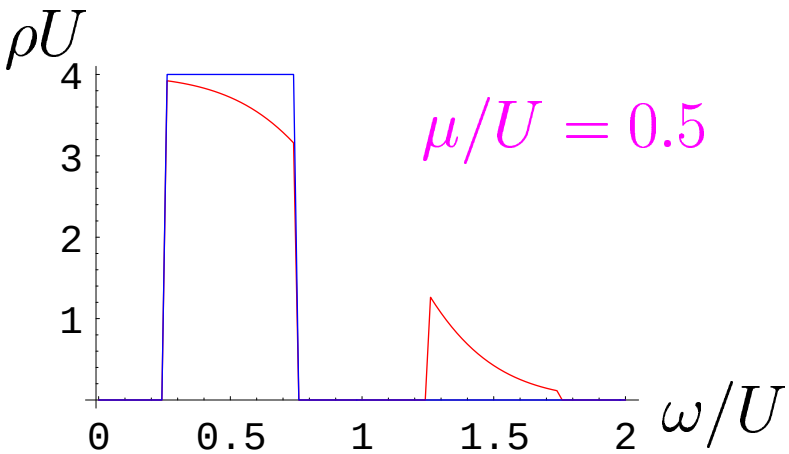
SF: $\lim_{|i-j|\to\infty} \langle a_i^\dagger a_j \rangle \neq 0$

	MI	BG
compressibility κ	0	$\neq 0$
density of states $\rho(0)$	0	$\neq 0$

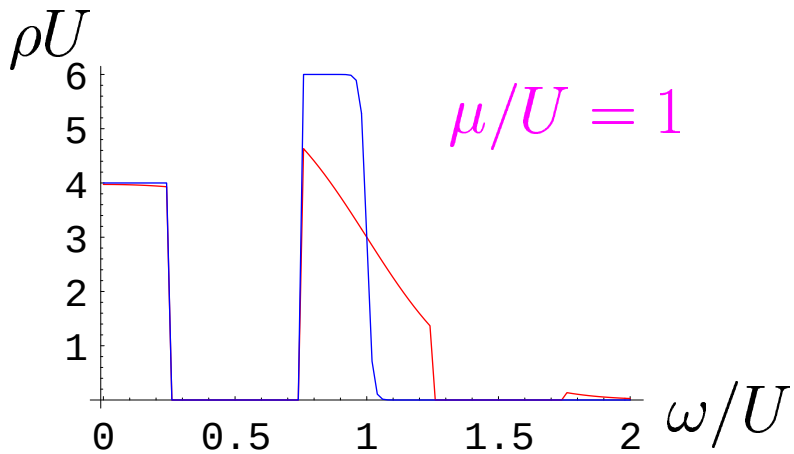
Non-Vanishing Temperature

$$0 < \frac{kT}{U} \ll 1 : \qquad \kappa(T) \approx \kappa(0) + \alpha\, e^{-\frac{\Delta E}{kT}} \neq 0 \ (!)$$

Density of States



$\rho(0) = 0$ (MI)



$\rho(0) \neq 0$ (BG)

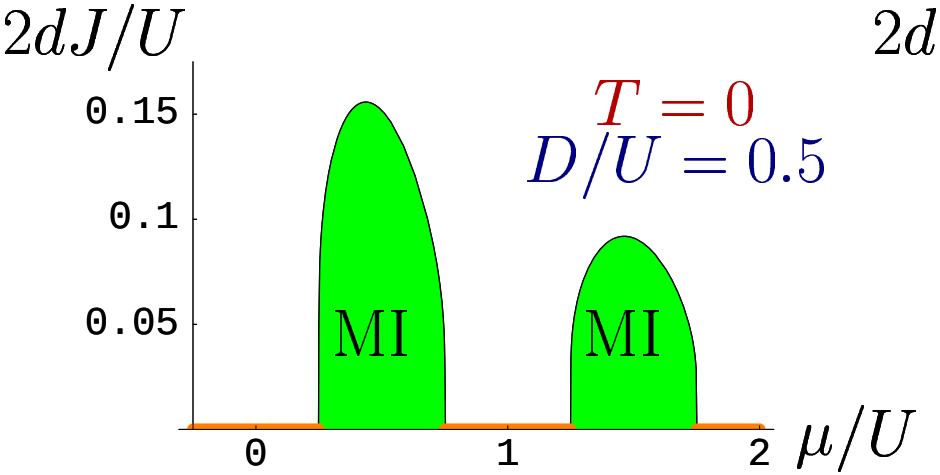
$D/U = 0.5$

$kT/U = 0.01$

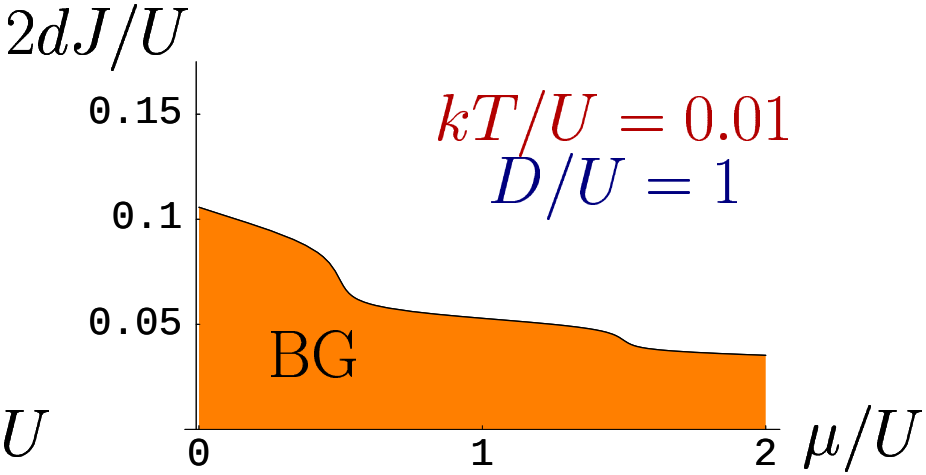
$kT/U = 0.2$

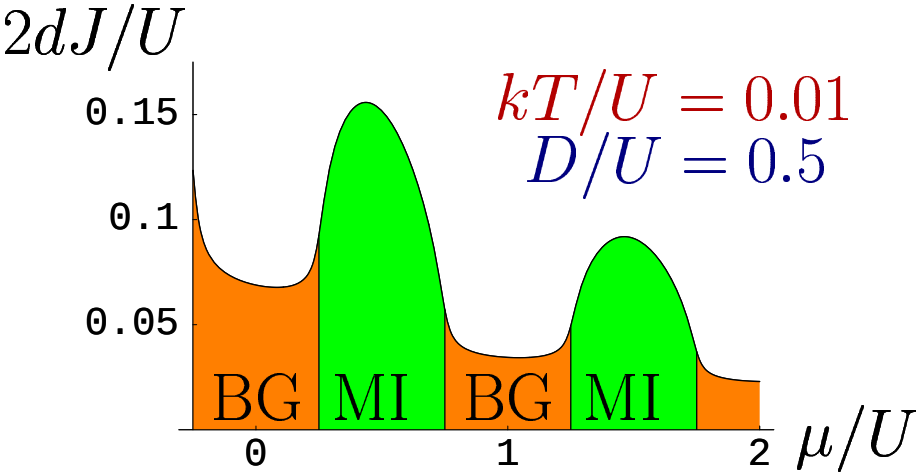
Phase Diagram

weak disorder



strong disorder





Experimental Observation of BG Phase

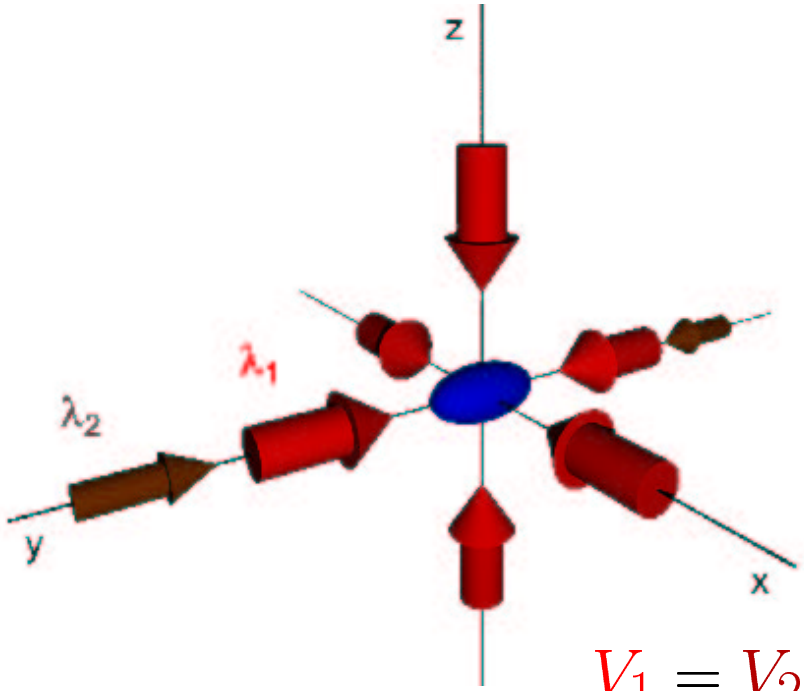
L.Fallani, J.E.Lye, V.Guarrera, C.Fort, and M.Inguscio, `cond-mat/0603655`

10^5 ^{87}Rb

$T \sim 100$ nK

$\lambda_1 = 830$ nm

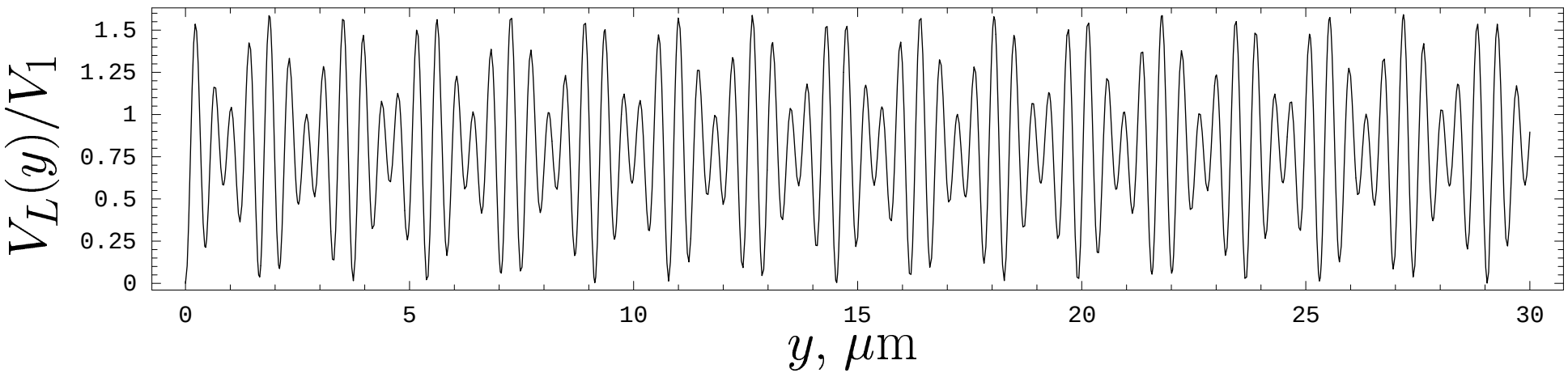
$\lambda_2 = 1076$ nm



harmonic trap +

$$V_L(\mathbf{r}) = V_1 \left(\sin^2 k_1 x + \sin^2 k_1 z\right) + V_1 \sin^2 k_1 y + V_2 \sin^2 k_2 y$$

$V_1 = V_2$



Summary and Outlook

- "clean" bosons: SF, MI
- "dirty" bosons: SF, MI, BG

  MI and BG can always be distinguished via density of states of excitations at small energies
- detecting BG phase via visibility?
- other versions of mean-field theory:

  dynamical MF, MF with locally fluctuating order parameter $\psi_i = \langle \hat{a}_i \rangle$
- beyond mean-field approaches for the phase boundary