Bose-Einstein Condensates of Atoms and Photons – A Comprehensive Overview

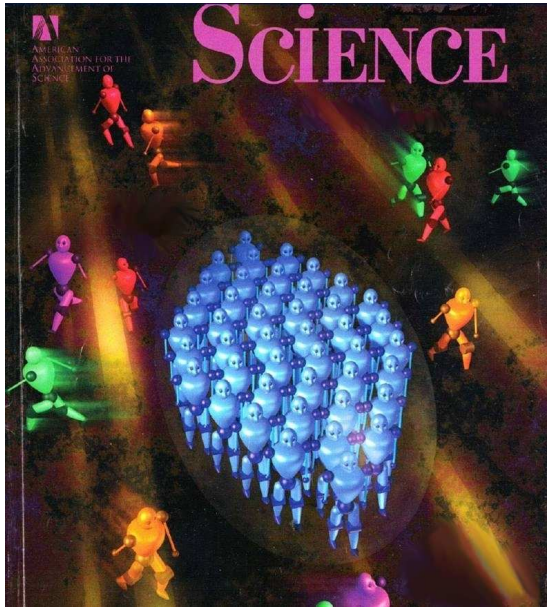
Axel Pelster

SFB/Transregio 49
Frankfurt – Kaiserslautern - Mainz
Condensed matter systems with variable many-body interactions

Technische Universität Kaiserslautern

OPTIMAS
TU Kaiserslautern

1. BEC of Atoms
2. BEC of Photons
3. Two-Mode Laser Model
4. Atoms in Optical Lattice
5. Photons in Cavity Lattice
6. Conclusion



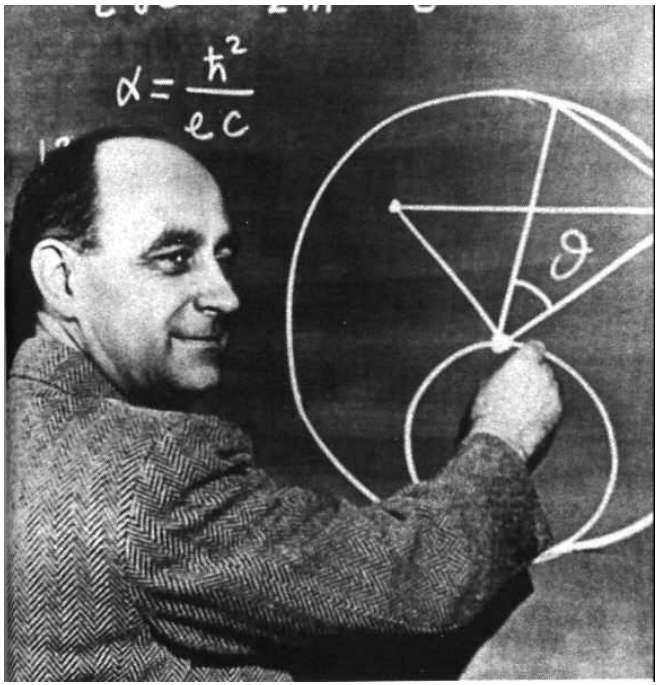
1.1 Identical Quantum Particles

Bosons:

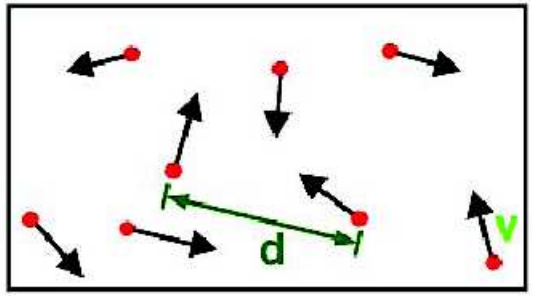
- integer spin
- symmetric wave function

Fermions:

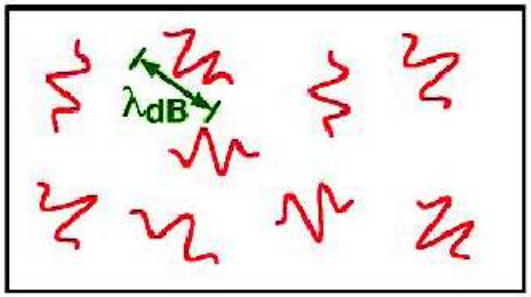
- half-integer spin
- anti-symmetric wave function



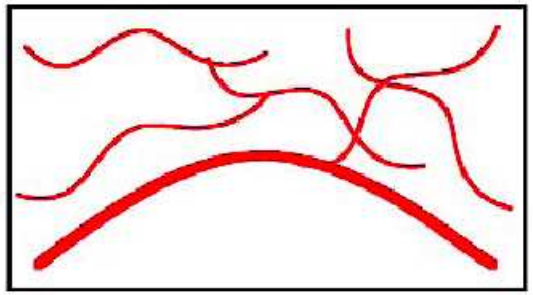
1.2 What is Bose-Einstein Condensation?



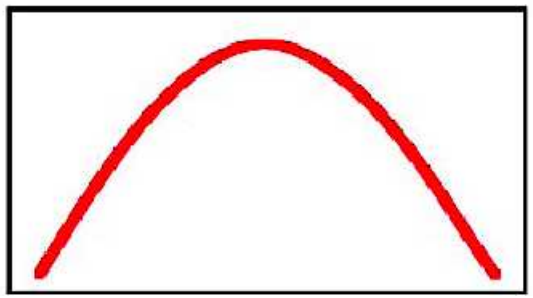
High Temperature T:
thermal velocity v
density d^{-3}
"Billiard balls"



Low Temperature T:
De Broglie wavelength
$\lambda_{dB}=h/mv \propto T^{-1/2}$
"Wave packets"



$T=T_{crit}$: Bose-Einstein Condensation
$\lambda_{dB} \approx d$
"Matter wave overlap"

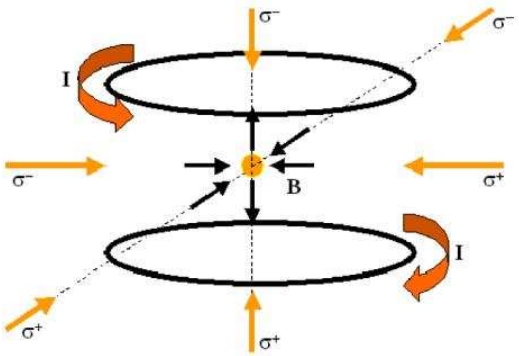


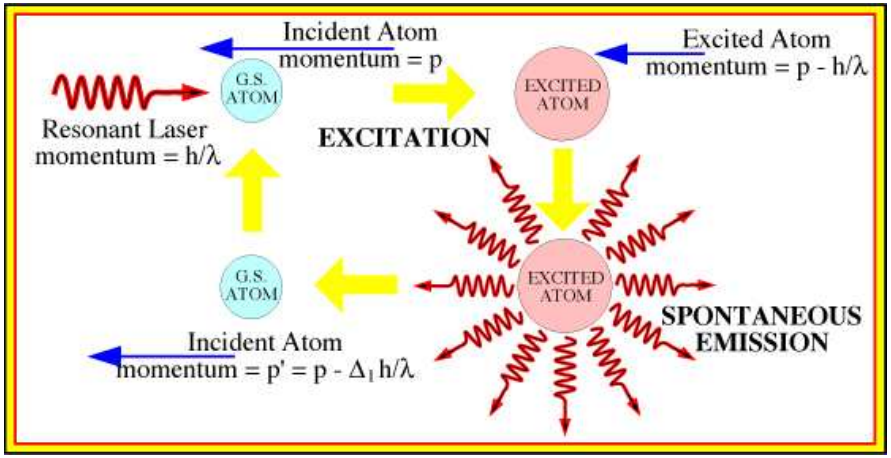
T=0: Pure Bose condensate
"Giant matter wave"

- $\lambda_{\mathrm{dB}} = \dfrac{\hbar}{\sqrt{2Mk_BT}}$
- $n = \dfrac{1}{d^3}$
- $\dfrac{\lambda_{\mathrm{dB}}}{d} \approx 1$
- $T_c \approx \dfrac{\hbar^2 n^{3/2}}{2Mk_B}$

1.3 Cooling Techniques

magneto-optical trap

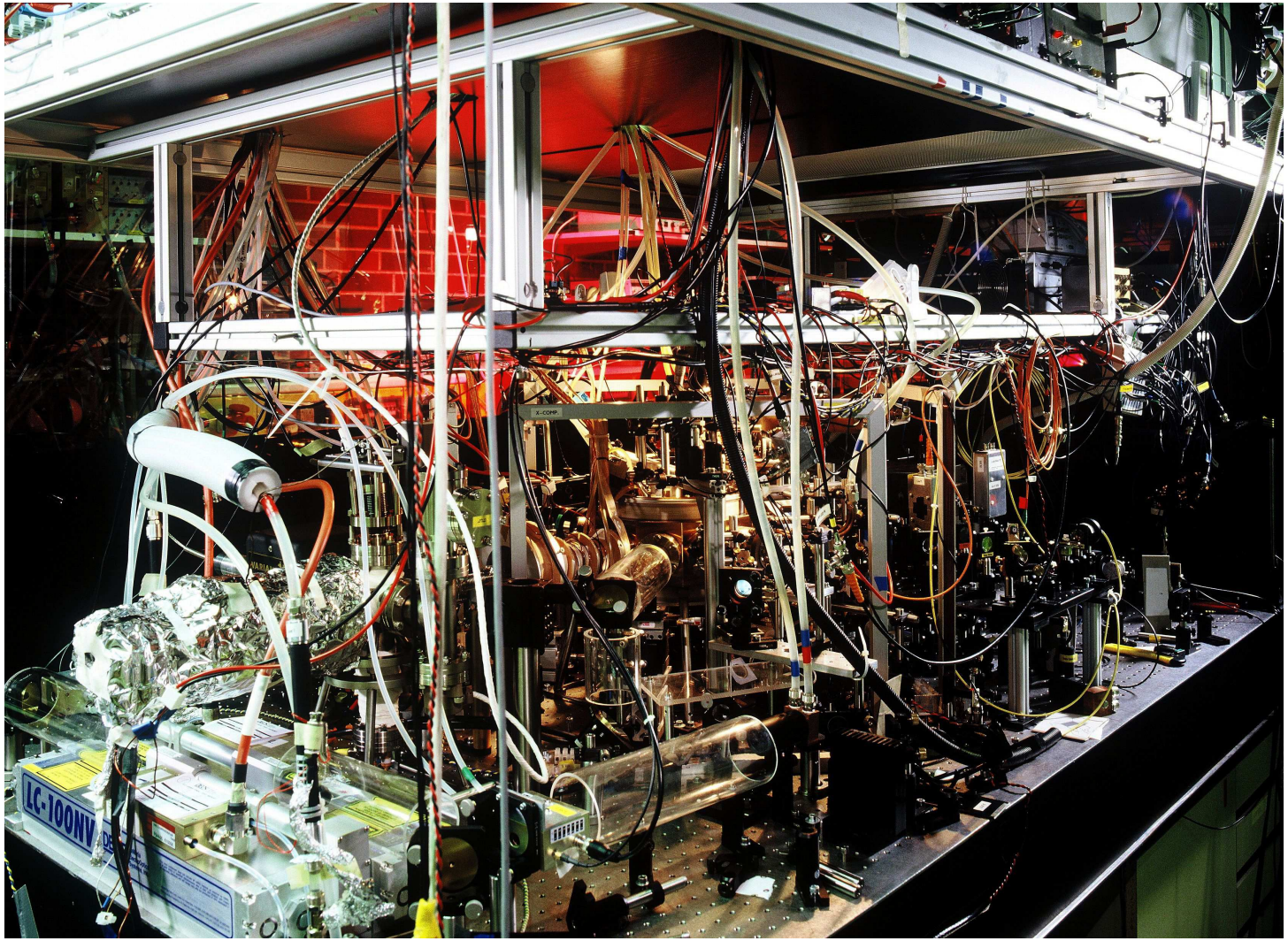




laser cooling

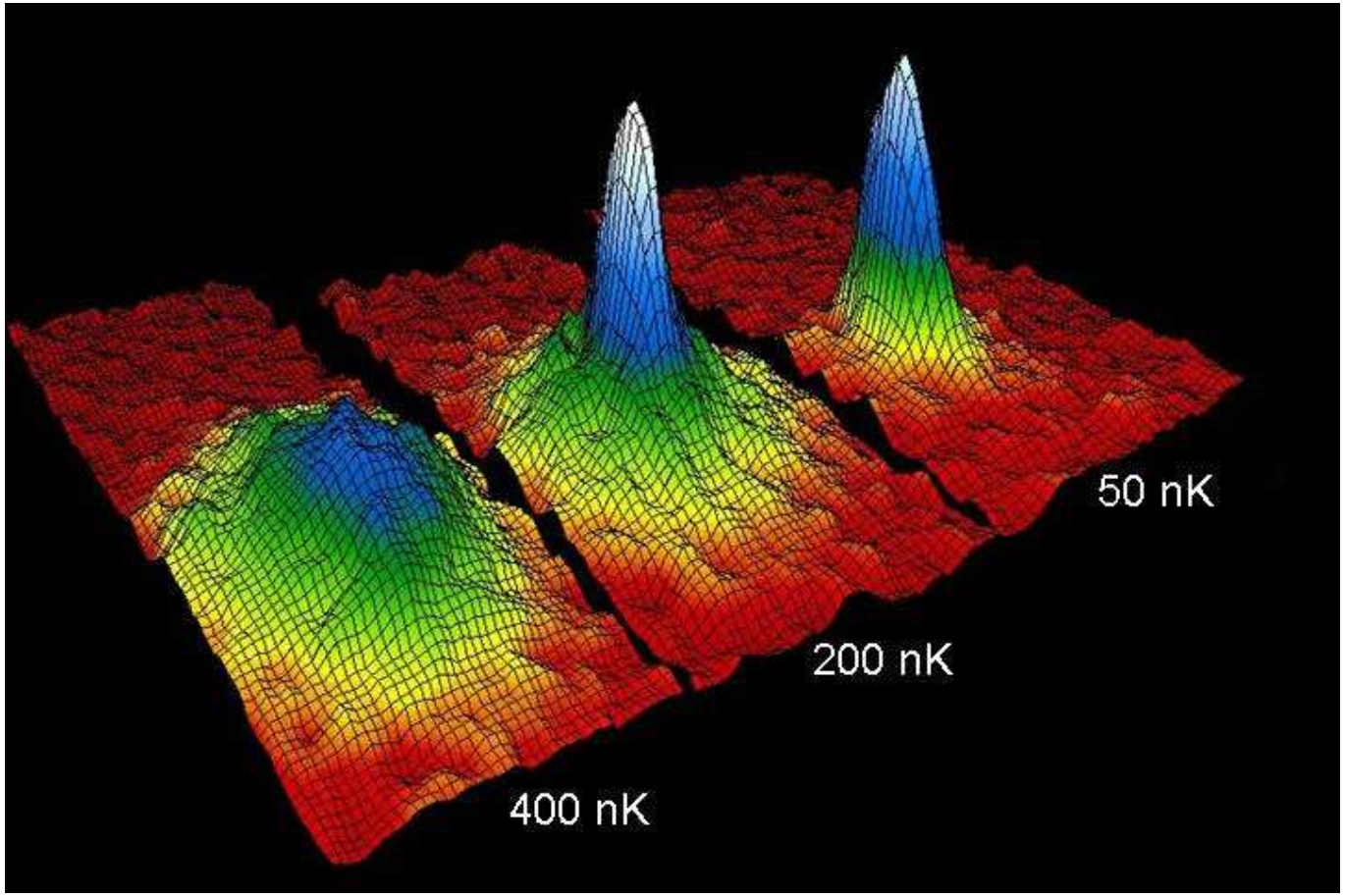
evaporative cooling

1.4 Experimental Apparatus



Costs about 1.000.000 EUR

1.5 Time-of-Flight Absorption Pictures



JILA (1995): $^{87}_{37}$Rb , N=20 000 , $\omega_1 = \omega_2 = \omega_3/\sqrt{8} = 2\pi\times$ 120 Hz

1.6 Periodic Table of Chemical Elements

Periode | ☐ = Hauptgruppen | ☐ = Nebengruppen | ☐ = Edelgase | Schale

Periode	I	II	IIIa	IVa	Va	VIa	VIIa	VIIIa			Ia	Ib	III	IV	V	VI	VII	VIII	Schale
1	1,008 H 1 Wasserstoff																	4,003 He 2 Helium	K
2	6,941 Li 3 Litium	9,012 Be 4 Beryllium											10,811 B 5 Bor	12,011 C 6 Kohlenstoff	14,007 N 7 Stickstoff	15,999 O 8 Sauerstoff	18,998 F 9 Fluor	20,180 Ne 10 Neon	L
3	22,990 Na 11 Natrium	24,305 Mg 12 Magnesium											26,982 Al 13 Aluminium	28,086 Si 14 Silicium	30,974 P 15 Phosphor	32,066 S 16 Schwefel	35,453 Cl 17 Chlor	39,948 Ar 18 Argon	M
4	39,098 K 19 Kalium	40,078 Ca 20 Calcium	44,956 Sc 21 Scandium	47,88 Ti 22 Titan	50,942 V 23 Vanadin	51,996 Cr 24 Chrom	54,938 Mn 25 Mangan	55,847 Fe 26 Eisen	58,933 Co 27 Kobalt	58,69 Ni 28 Nickel	63,546 Cu 29 Kupfer	65,39 Zn 30 Zink	69,723 Ga 31 Gallium	72,61 Ge 32 Germanium	74,922 As 33 Arsen	78,96 Se 34 Selen	79,904 Br 35 Brom	83,8 Kr 36 Krypton	N
5	85,468 Rb 37 Rubidium	87,62 Sr 38 Strontium	88,906 Y 39 Yttrium	91,224 Zr 40 Zirconium	92,906 Nb 41 Niob	95,94 Mo 42 Molybdän	98,906 Tc 43 * Technetium	101,07 Ru 44 Ruthenium	102,906 Rh 45 Rhodium	106,42 Pd 46 Palladium	107,868 Ag 47 Silber	112,411 Cd 48 Cadmium	114,82 In 49 Indium	118,71 Sn 50 Zinn	121,75 Sb 51 Antimon	127,6 Te 52 Tellur	126,904 I 53 Iod	131,29 Xe 54 Xenon	O
6	132,905 Cs 55 Cäsium	137,327 Ba 56 Barium	138,906 La 57 Lanthan	178,49 Hf 72 Hafnium	180,948 Ta 73 Tantal	183,85 W 74 Wolfram	186,207 Re 75 Rhenium	190,2 Os 76 Osmium	192,22 Ir 77 Iridium	195,08 Pt 78 Platin	196,967 Au 79 Gold	200,59 Hg 80 Quecksilber	204,383 Tl 81 Thallium	207,2 Pb 82 Blei	208,98 Bi 83 Bismut	208,982 Po 84 * Polonium	209,987 At 85 * Astat	222,018 Rn 86 * Radon	P
7	223,02 Fr 87 * Francium	226,025 Ra 88 * Radium	227,028 Ac 89 * Actinium	261,109 Rf 104 * Rutherfordium	262,114 Ha 105 * Hahnium	263,118 Sg 106 * Seaborgium	262,123 Ns 107 * Nielsbohrium	ca. 265 Hs 108 * Hassium	ca. 268 Mt 109 * Meitnerium	ca. 269 Ds 110 * Darmstadtium	ca. 272 Rg 111 * Roentgenium	ca. 277 ? 112 *		ca. 289 ? 114 *		ca. 289 ? 116 *		ca. 293 ? 118 *	Q

Lanthanide

6	140,12 Ce 58 Cer	140,91 Pr 59 Praseodym	144,24 Nd 60 Neodym	145 Pm 61 * Promethium	150,35 Sm 62 Samarium	151,96 Eu 63 Europium	157,25 Gd 64 Gadolinium	158,92 Tb 65 Terbium	162,50 Dy 66 Dysprosium	164,93 Ho 67 Holmium	167,26 Er 68 Erbium	168,93 Tm 69 Thulium	173,04 Yb 70 Ytterbium	174,97 Lu 71 Lutetium

Actinide

7	232,04 Th 90 * Thorium	231 Pa 91 * Protactinium	238,03 U 92 * Uran	237 Np 93 * Neptunium	244 Pu 94 * Plutonium	243 Am 95 * Americium	247 Cm 96 * Curium	247 Bk 97 * Berkelium	251 Cf 98 * Californium	254 Es 99 * Einsteinium	257 Fm 100 * Fermium	258 Md 101 * Mendelevium	259 No 102 * Nobelium	260 Lr 103 * Lawrencium

Aggregatzustand unter Normalbedingungen:

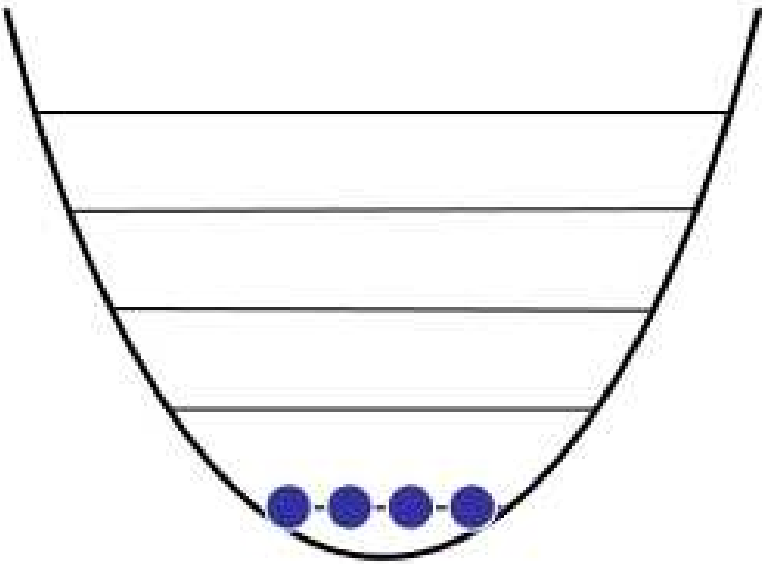
- Fe fest
- Hg flüssig
- He gasförmig

* = radioaktives Element

1.7 Ground-State of Bosons

atoms
non-vanishing mass

photons
vanishing mass



Bose-Einstein condensate

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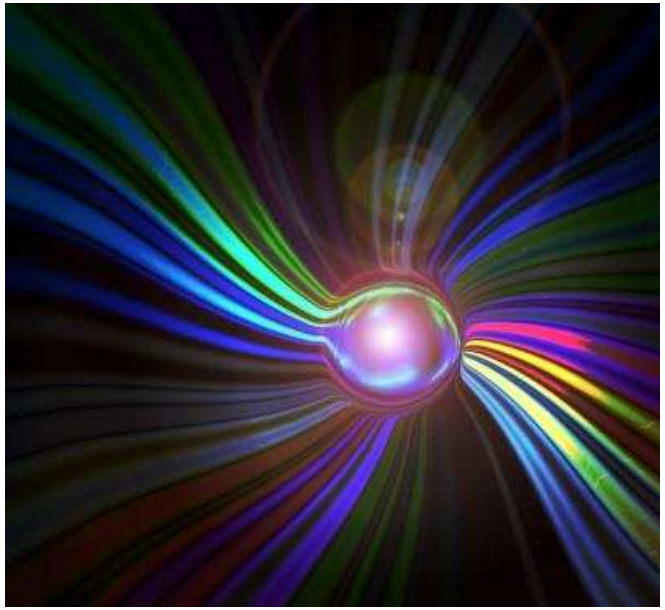
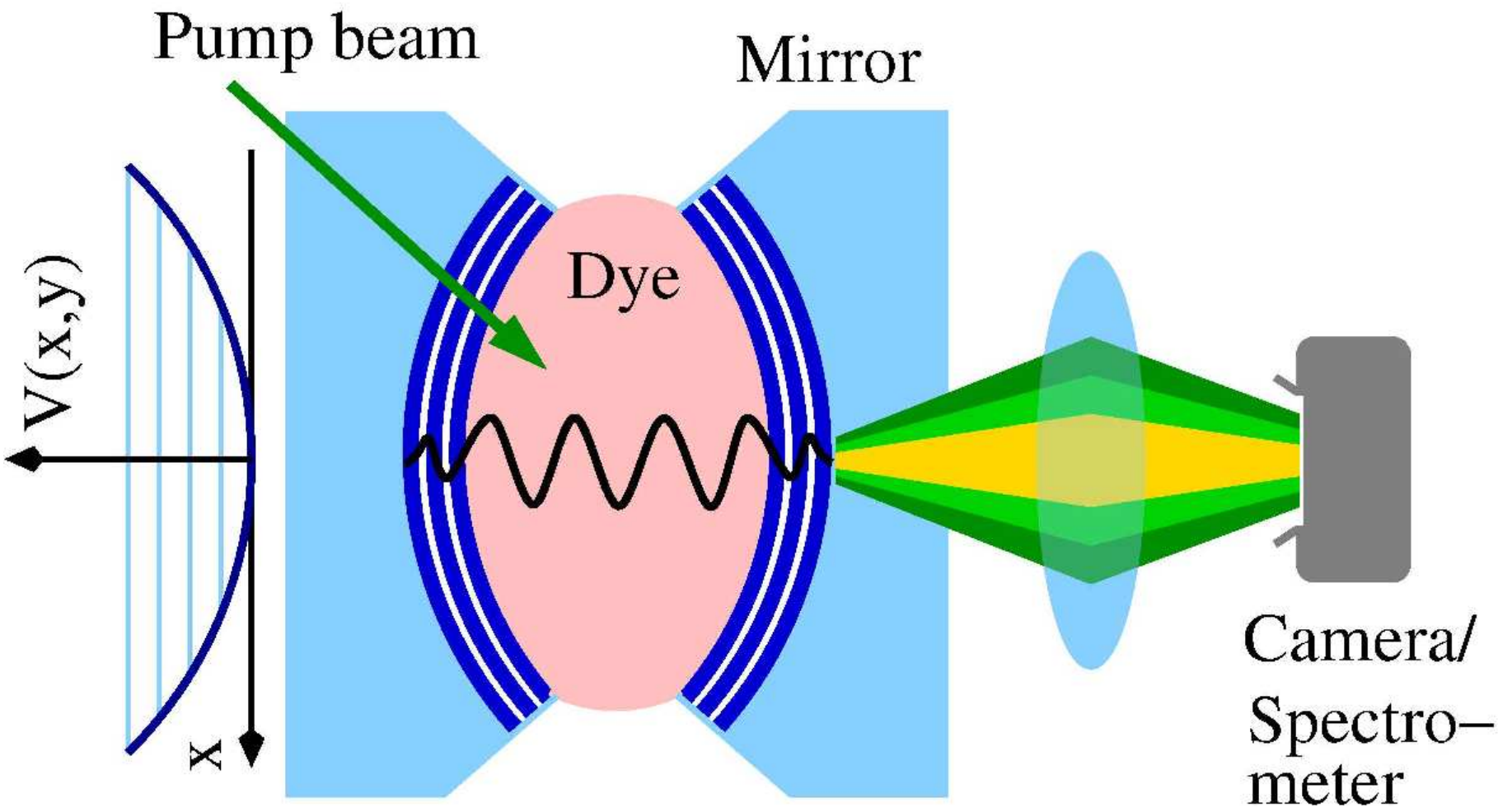


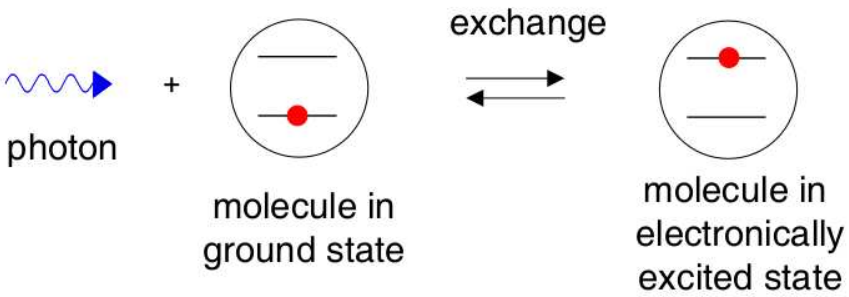
2.1 Set-Up of Bonn Experiment

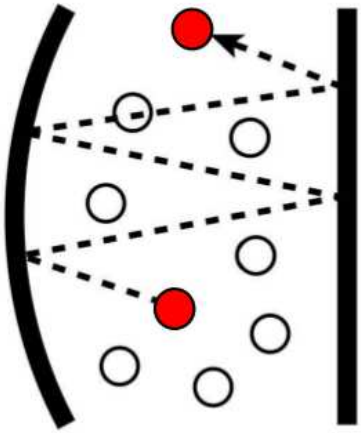


Klärs, Vewinger, and Weitz, Nature Phys. **6**, 512 (2010)

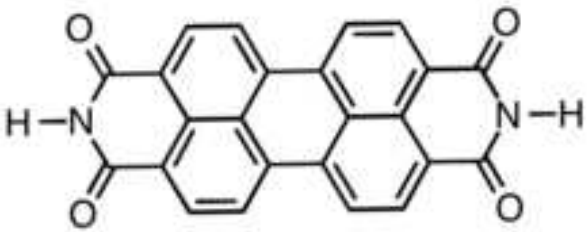
Klärs, Schmitt, Vewinger, and Weitz, Nature **468**, 545 (2010)

2.2 Thermalization of Photons



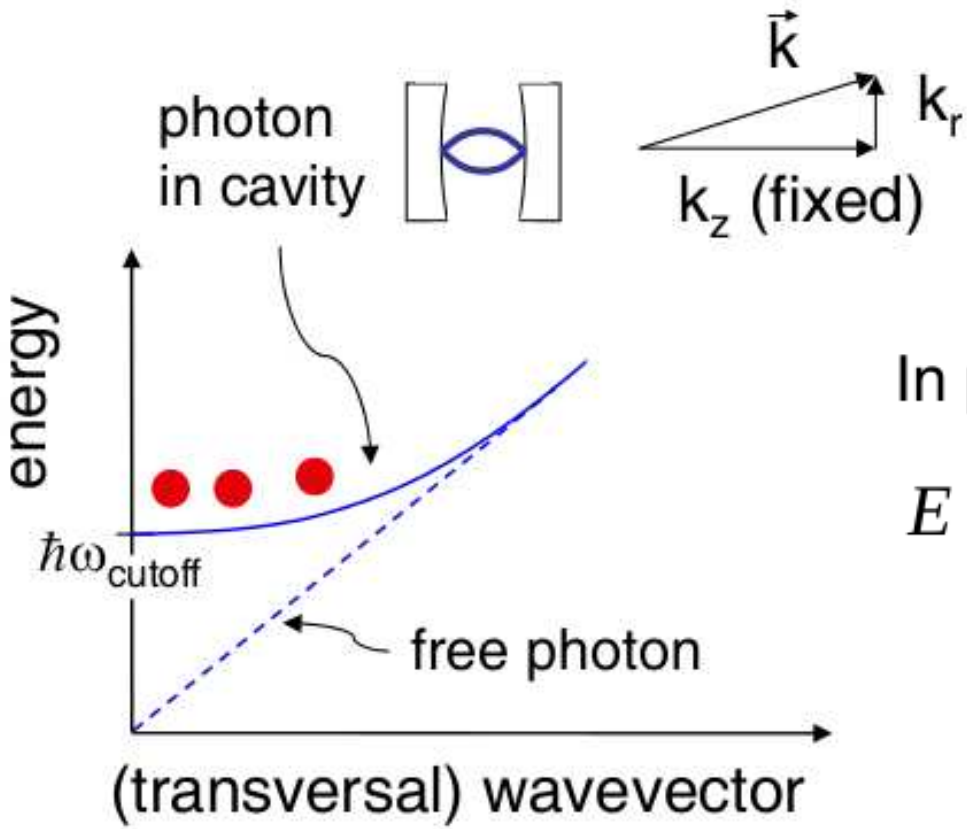


photons are multiply absorbed and emitted by dye molecules in resonator



Perylene-diimide (PDI)

2.3 Quadratic Photon Dispersion



In paraxial approximation (k_z>>k_r):

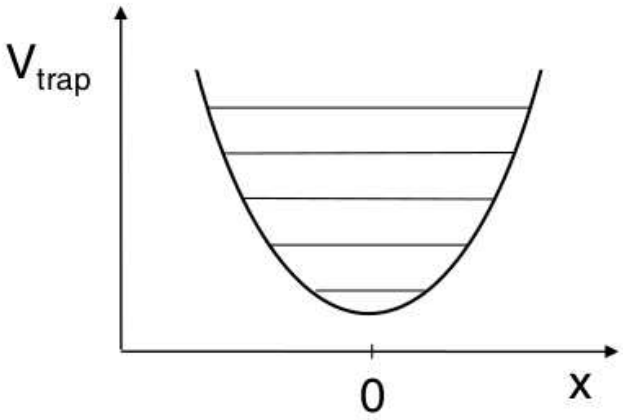
$$E = \hbar c\sqrt{k_z^2 + k_r^2} \cong \hbar c\left(k_z + \frac{k_r^2}{2k_z}\right)$$

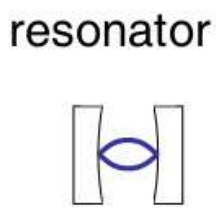
$$= m_{eff}c^2 + \frac{(\hbar k_r)^2}{2m_{eff}}$$

with $m_{eff} = \hbar k_z / c \equiv \hbar\omega_{cutoff} / c^2$

2.4 Photon Trapping

- **Harmonic potential from mirror curvature:**





- **2D gas of massive photons:**

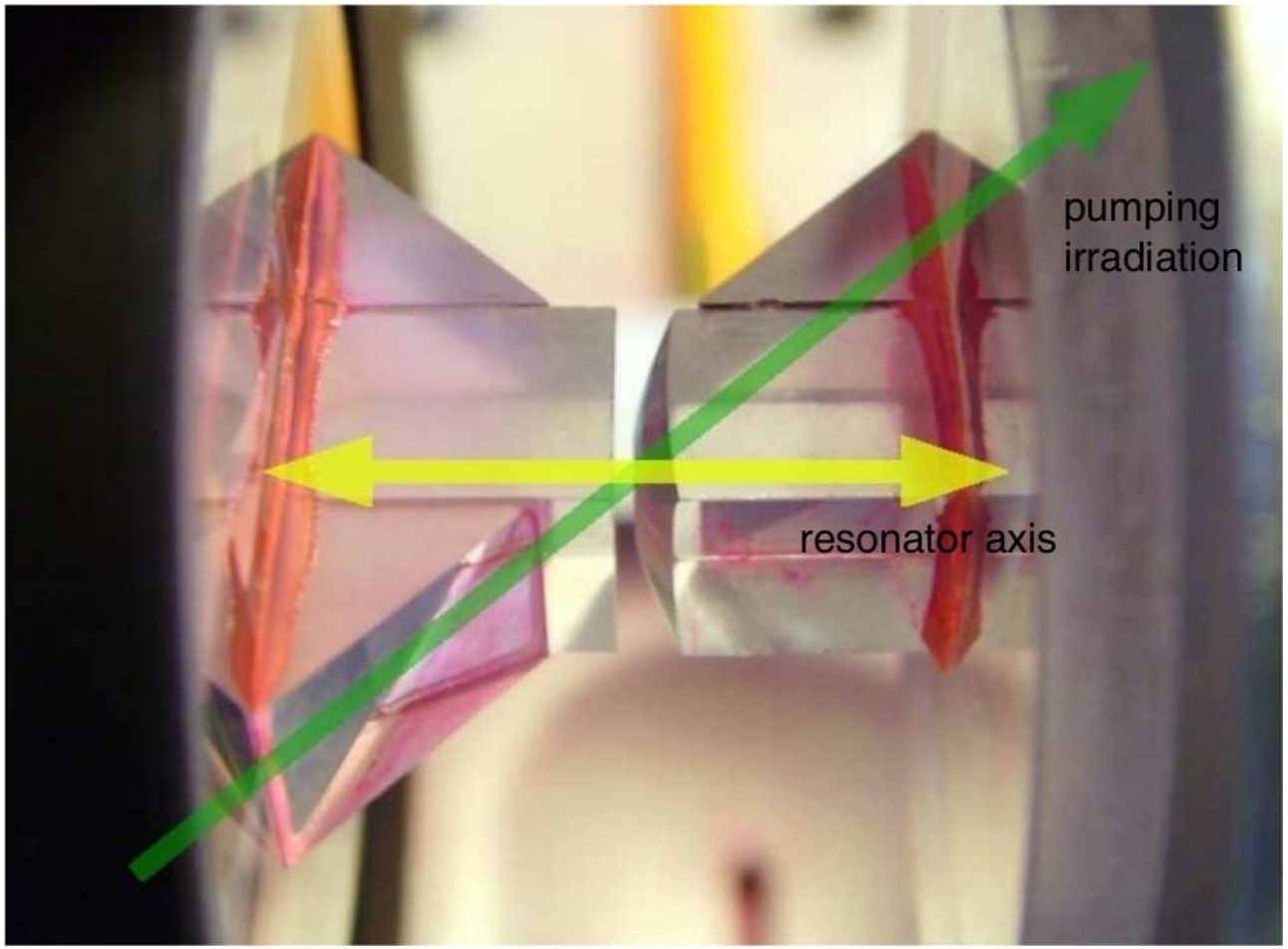
$$E = m_{eff}c^2 + \frac{\hbar^2 k_r^2}{2m_{eff}} + \frac{1}{2}m_{eff}\Omega^2 r^2\,, \qquad m_{eff} = \frac{\hbar\omega_{cutoff}}{c^2}$$

- **BEC transition:**

$$N > N_{\rm c} = \frac{\pi^2}{3}\left(\frac{k_{\rm B}T}{\hbar\Omega}\right)^2 \approx 77\,000\,, \qquad N_{\rm c}^{\rm exp} = (6.3 \pm 2.4)\cdot 10^4$$

$$T = 300\ \text{K}, \quad \Omega = 2\pi\cdot 4\cdot 10^{10}\ \text{Hz}, \quad m_{eff} = 6.7\cdot 10^{-36}\ \text{kg} \approx 10^{-10}\, m_{\rm Rb}$$

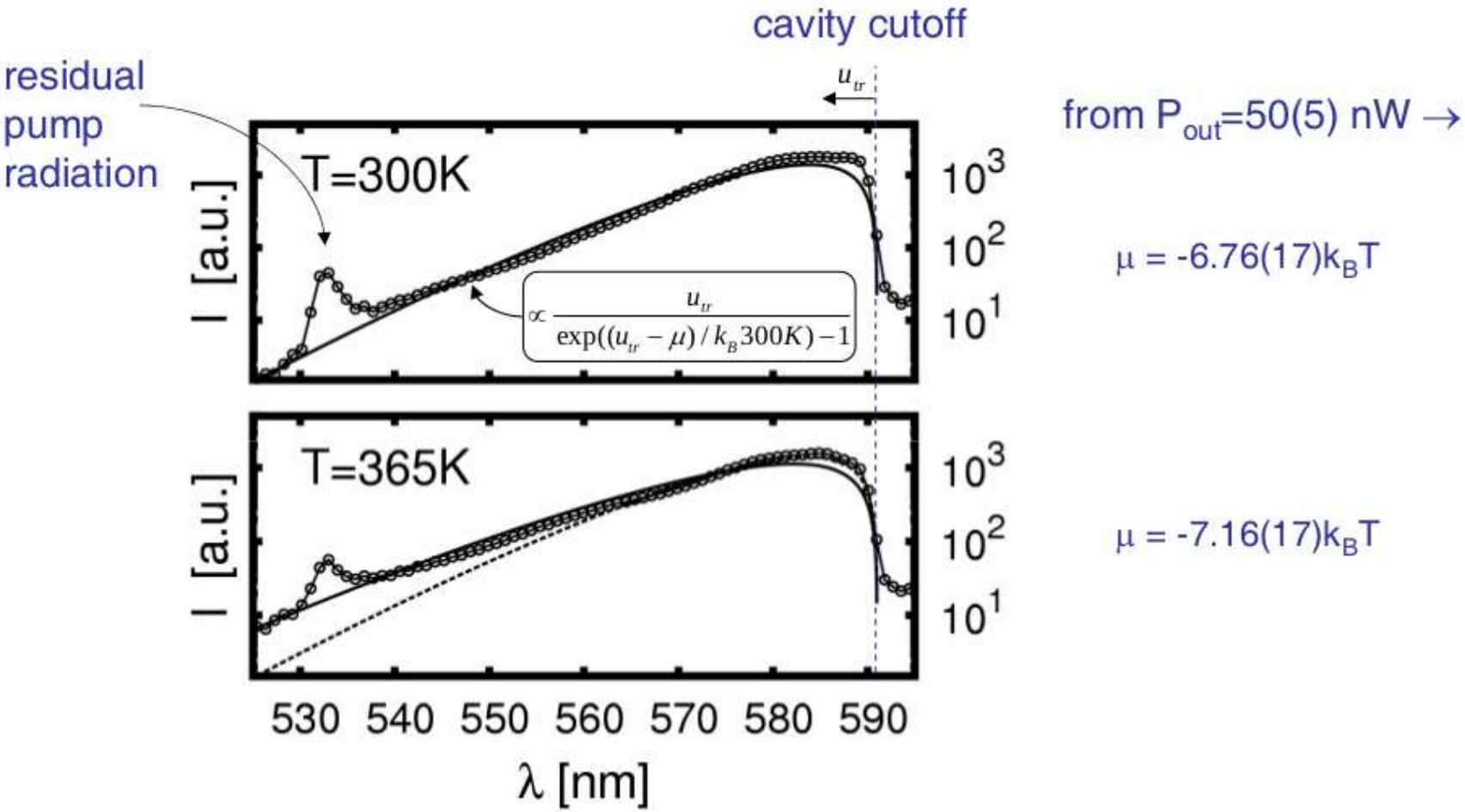
2.5 Experiment



Bonn: Klärs, Schmitt, Vewinger, and Weitz, Nature **468**, 545 (2010)

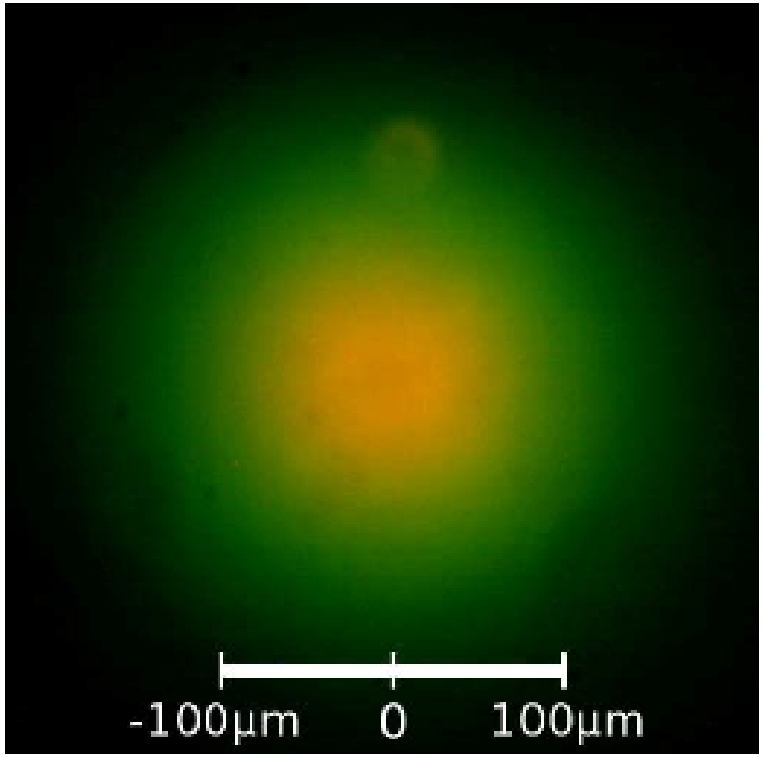
[see also **London:** Marelic and Nyman, PRA **91**, 033813 (2015)]

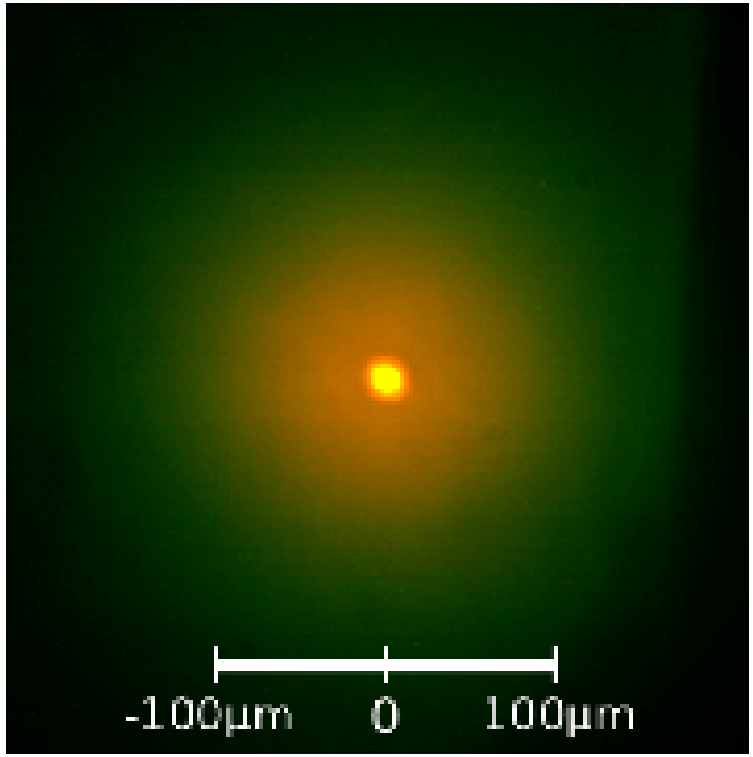
2.6 Spectrum of Thermal Photon Gas in Cavity



Klärs, Vewinger, and Weitz, Nature Phys. **6**, 512 (2010)

2.7 Photon Gas at Criticality

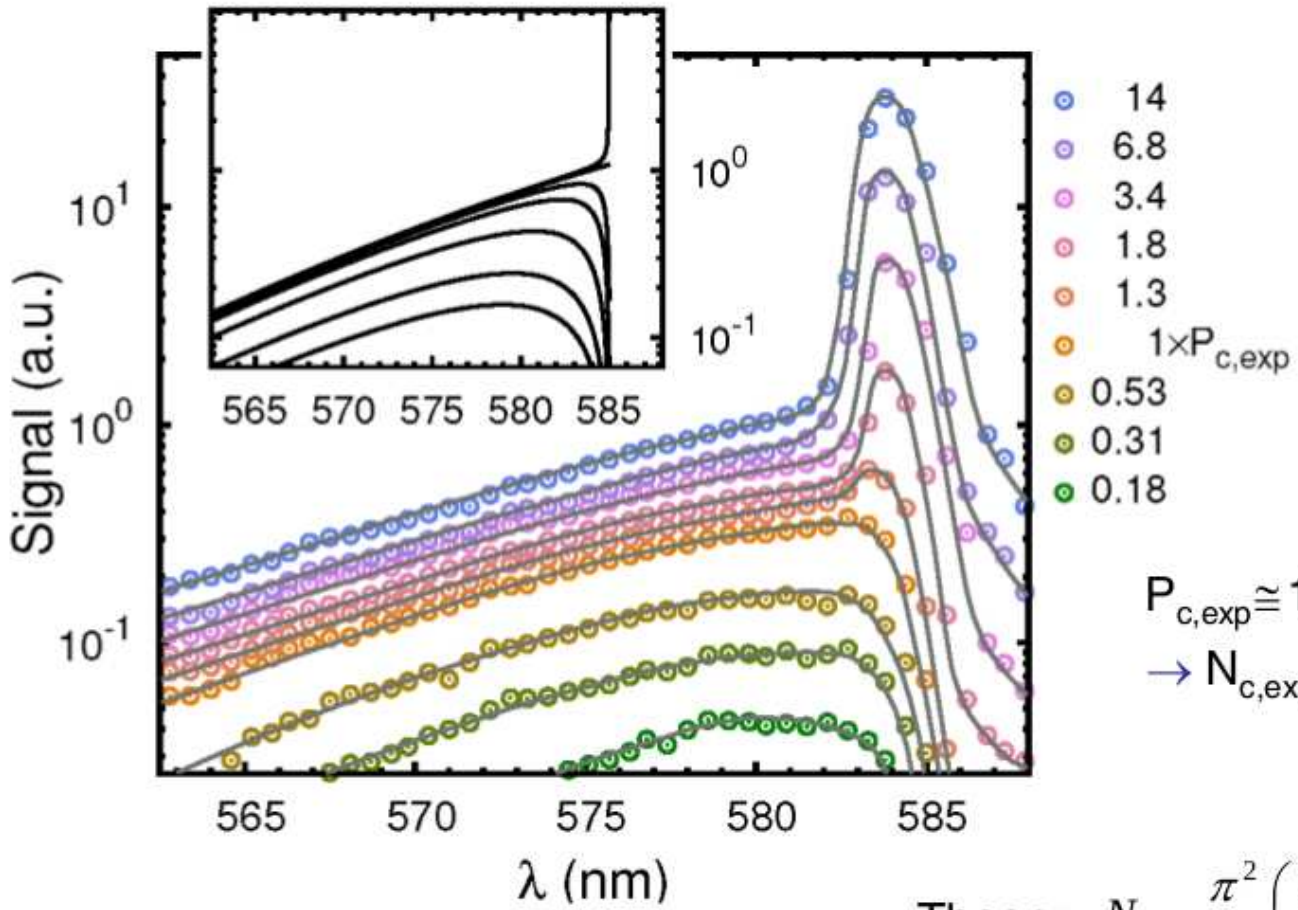




$N < N_{\mathrm{c}}$ $\qquad$ $N > N_{\mathrm{c}}$

Klärs, Schmitt, Vewinger, and Weitz, Nature **468**, 545 (2010)

2.8 Spectrum of Photon Gas Around Threshold



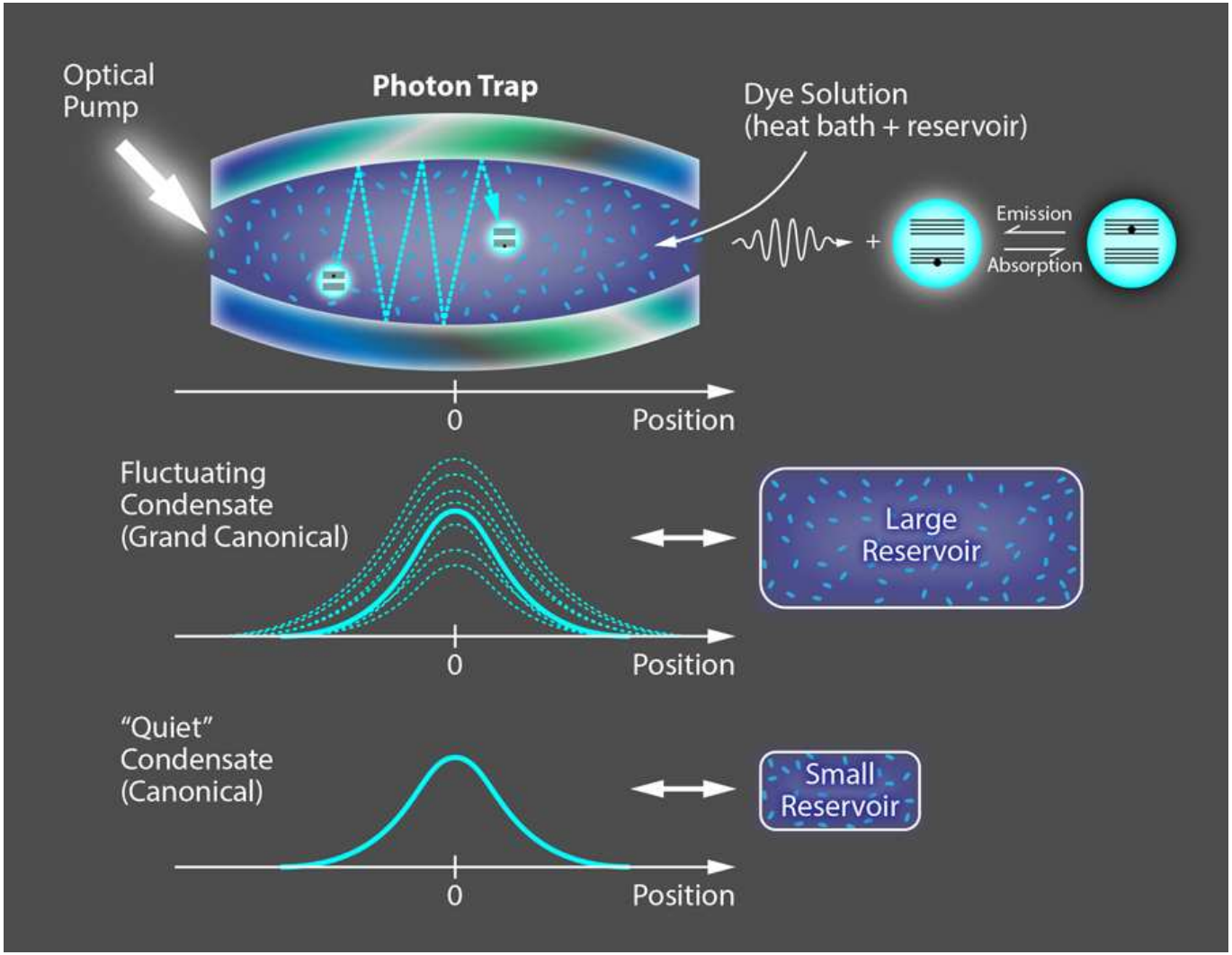
$P_{c,exp} \cong 1.55(60)$ W

$\rightarrow N_{c,exp} \cong (6.3 \pm 2{,}4) \cdot 10^4$

Theory: $N_c = \frac{\pi^2}{3}\left(\frac{k_B T}{\hbar\Omega}\right)^2 \cong 7.7 \cdot 10^4$

Klärs, Schmitt, Vewinger, and Weitz, Nature **468**, 545 (2010)

2.9 Canonical Versus Grand-Canonical Ensemble



Klaers, Schmitt, Damm, Vewinger, and Weitz, PRL **108**, 160403 (2012)
Schmitt, Damm, Dung, Vewinger, Klaers, and Weitz, PRL **112**, 030401 (2014)

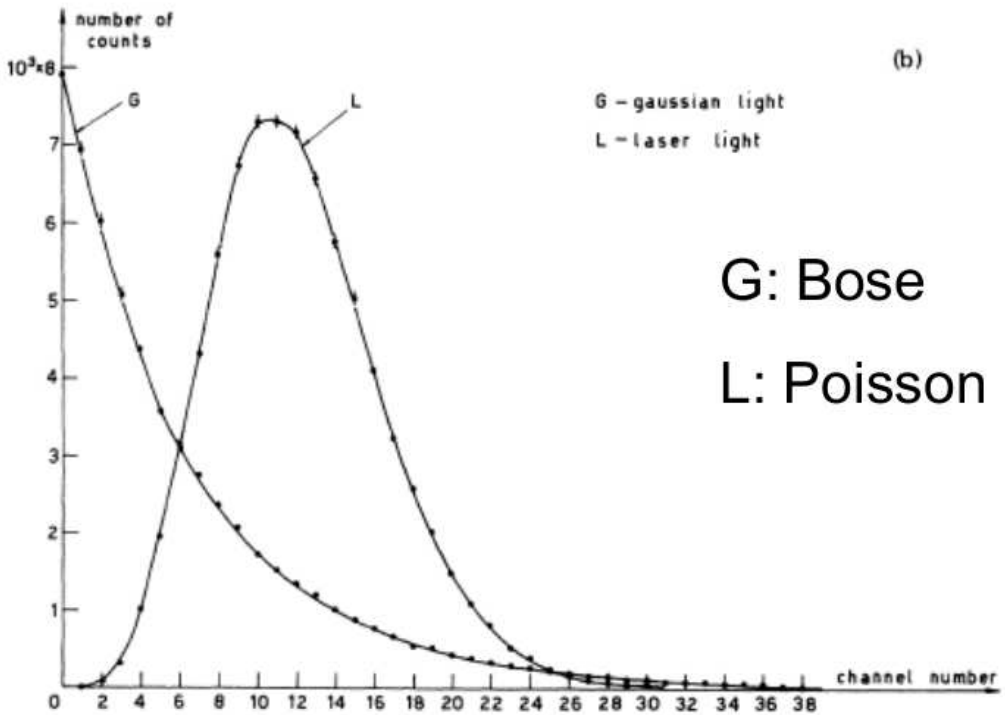
2.10 Comparison

Photon BEC

- $N > N_{\mathrm{c}}$
- thermal equilibrium

Laser

- gain $>$ loss
- non-equilibrium



Arecchi, PRL **15**, 912 (1965)

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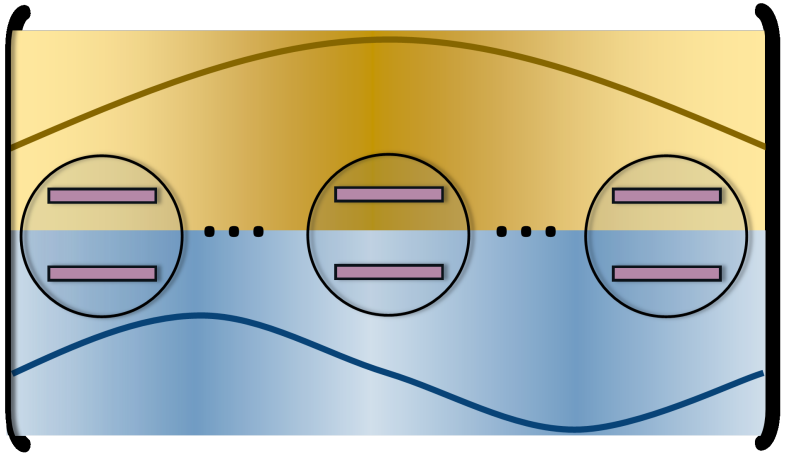


Kopylov, Radonjić, Brandes, Balaž, and Pelster, arXiv:1507.01811

3.1 Tavis-Cummings Model With Two Modes

- Hamilton operator:

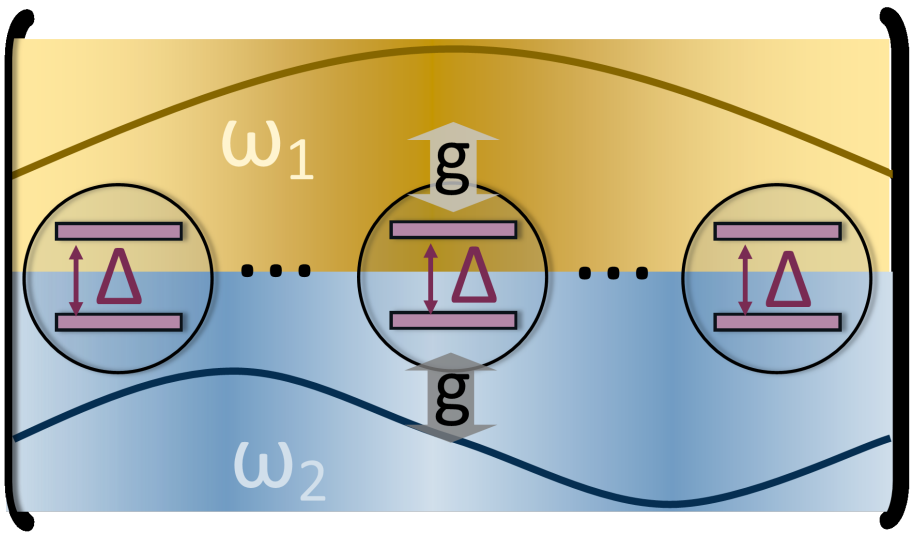
$$\hat{H} = \sum_{i=1}^{2} \hbar\omega_i \hat{a}_i^\dagger \hat{a}_i + \Delta \hat{J}_z + \frac{g}{\sqrt{N}} \sum_{i=1}^{2} (\hat{a}_i \hat{J}^+ + \hat{a}_i^\dagger \hat{J}^-)$$

- Population inversion:

$$\hat{J}_z = \frac{1}{2} \sum_{k=1}^{N} \sigma_k^z$$

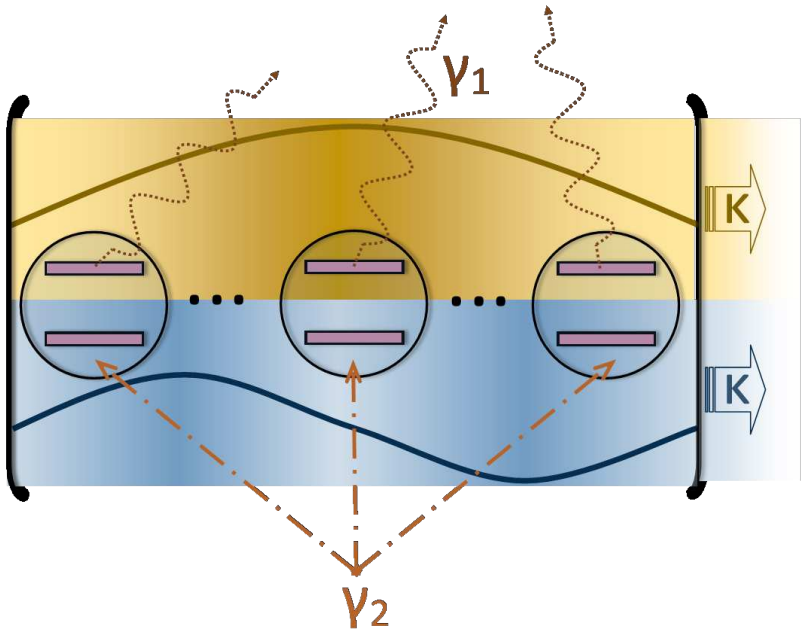
Dipole moment:

$$\hat{J}^\pm = \sum_{k=1}^{N} \sigma_k^\pm$$

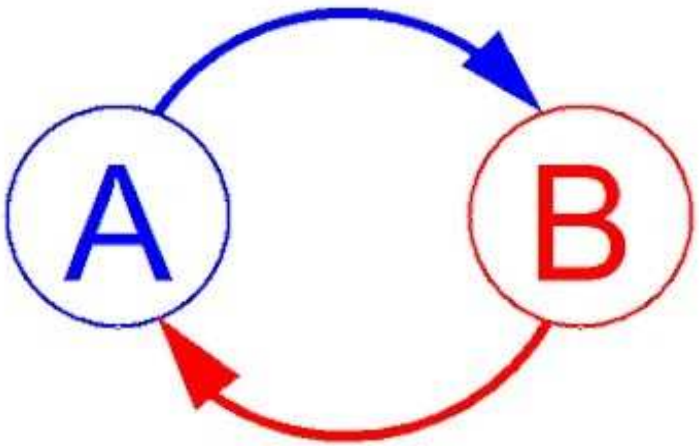


3.2 Lindblad-Master Equation

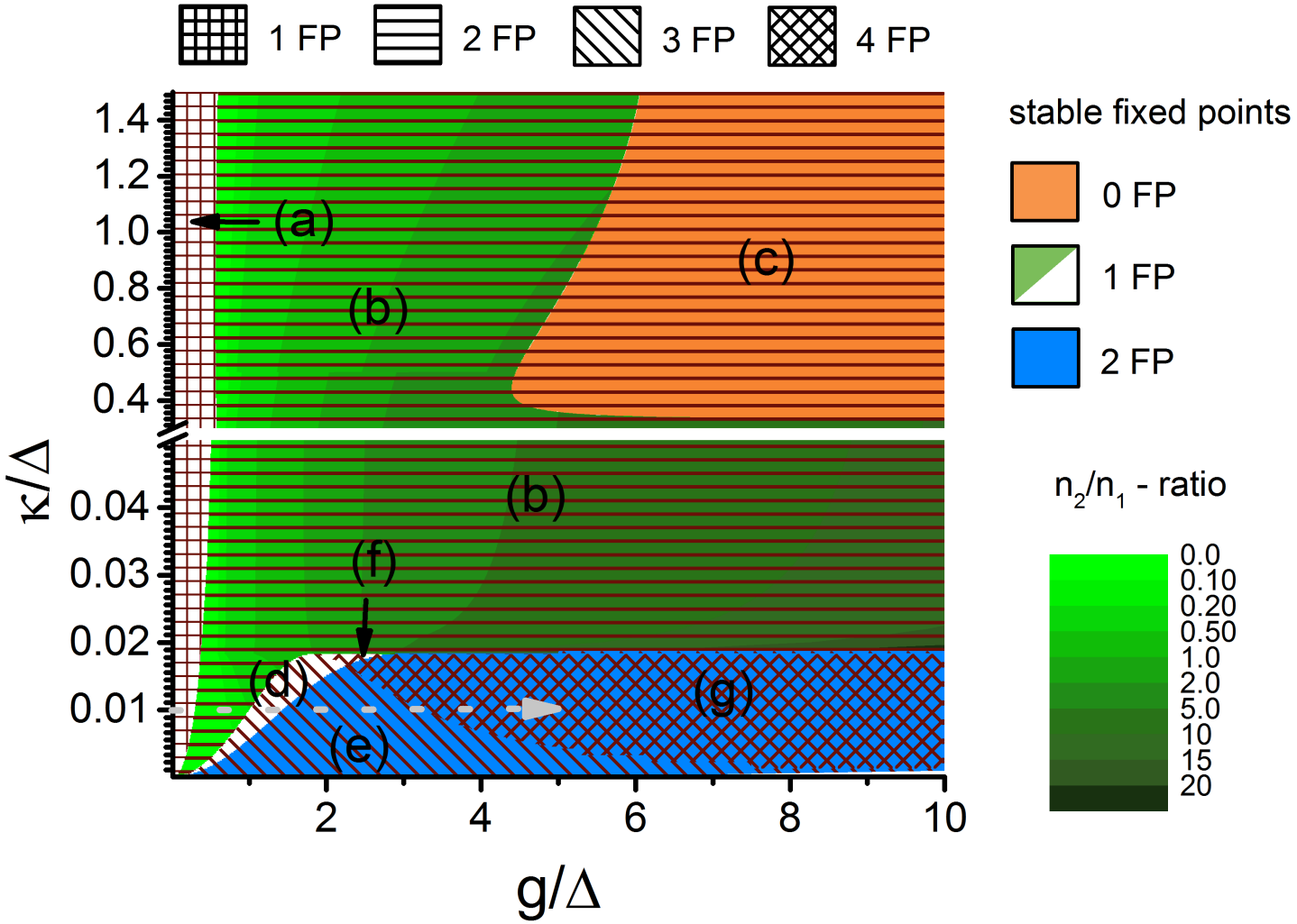
- Exchange with environment:
  - decay of cavity modes: κ
  - spontaneous emission: γ_1
  - pumping: γ_2



- Theoretical description:
  - thermodynamic limit $N \to \infty$
  - semiclassical equations for light and matter

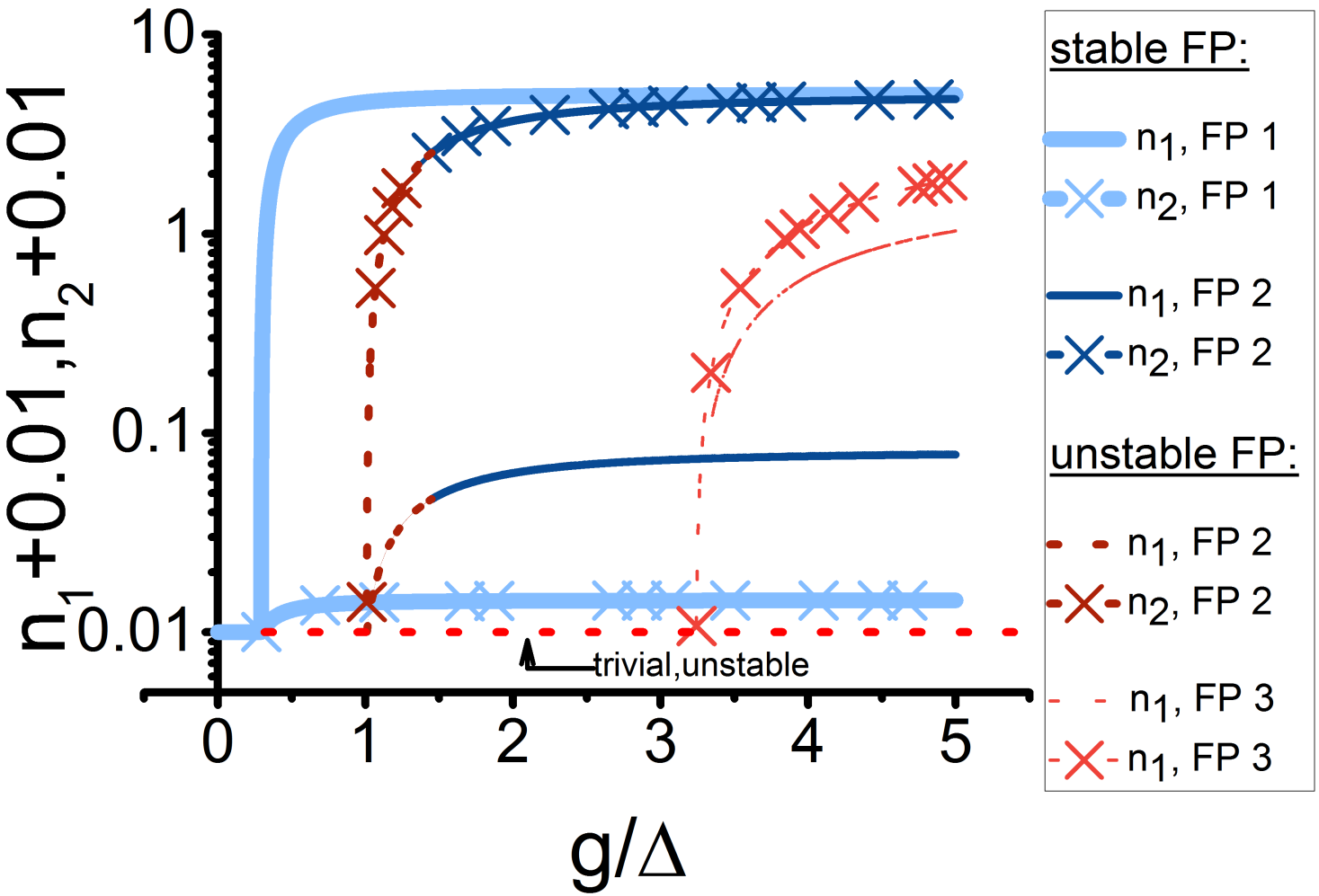


3.3 Steady States



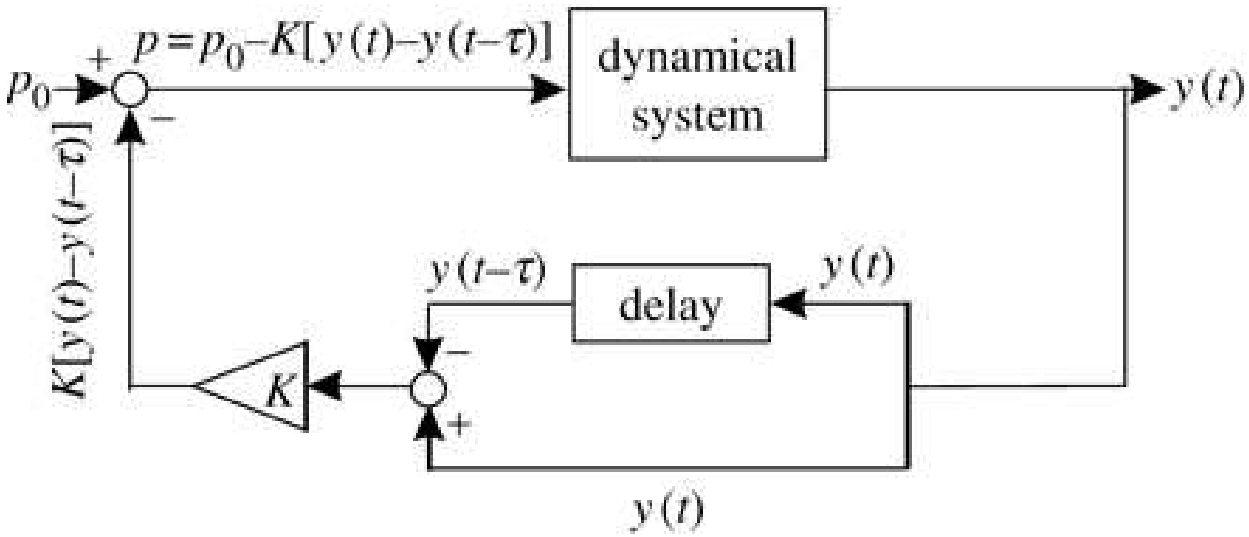
$$\hbar\omega_1 = 2\Delta,\ \hbar\omega_2 = 4\Delta,\ \gamma_1 = 0.1\Delta,\ \gamma_2 = 0.2\Delta$$

3.4 Bifurcation Scenario



$g > 1.5\Delta$: two stable fixed points with their respective attraction regions

3.5 Time-Delayed Feedback Control



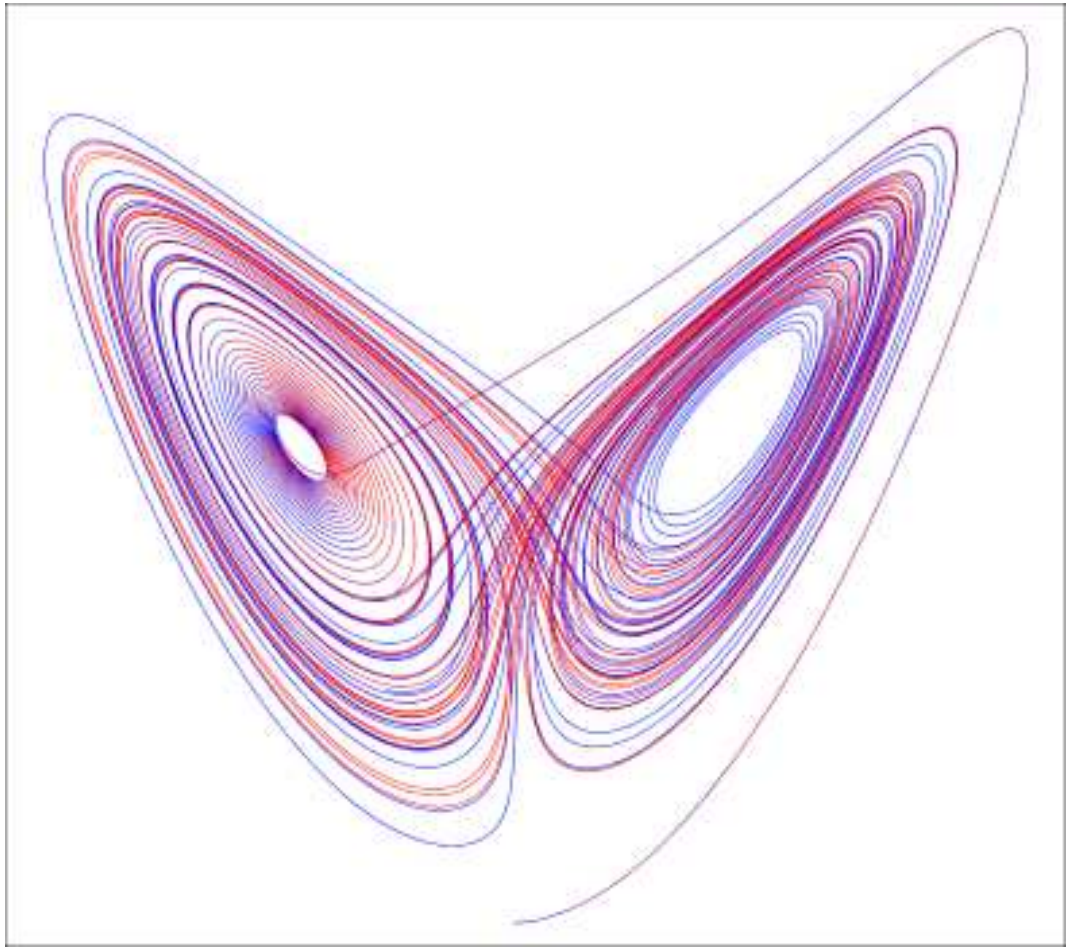
- **Fixed point / Limit cycle not changed**
- **Stability changed**
- **Application: chaos control**

Pyragas, Phys. Lett. A **170**, 421 (1992)

Just, Pelster, Schanz, and Schöll, Phil. Trans. Roy. Soc. A **368**, 303 (2009)

SFB 910: Control of Self-Organizing Nonlinear Systems (TU Berlin)

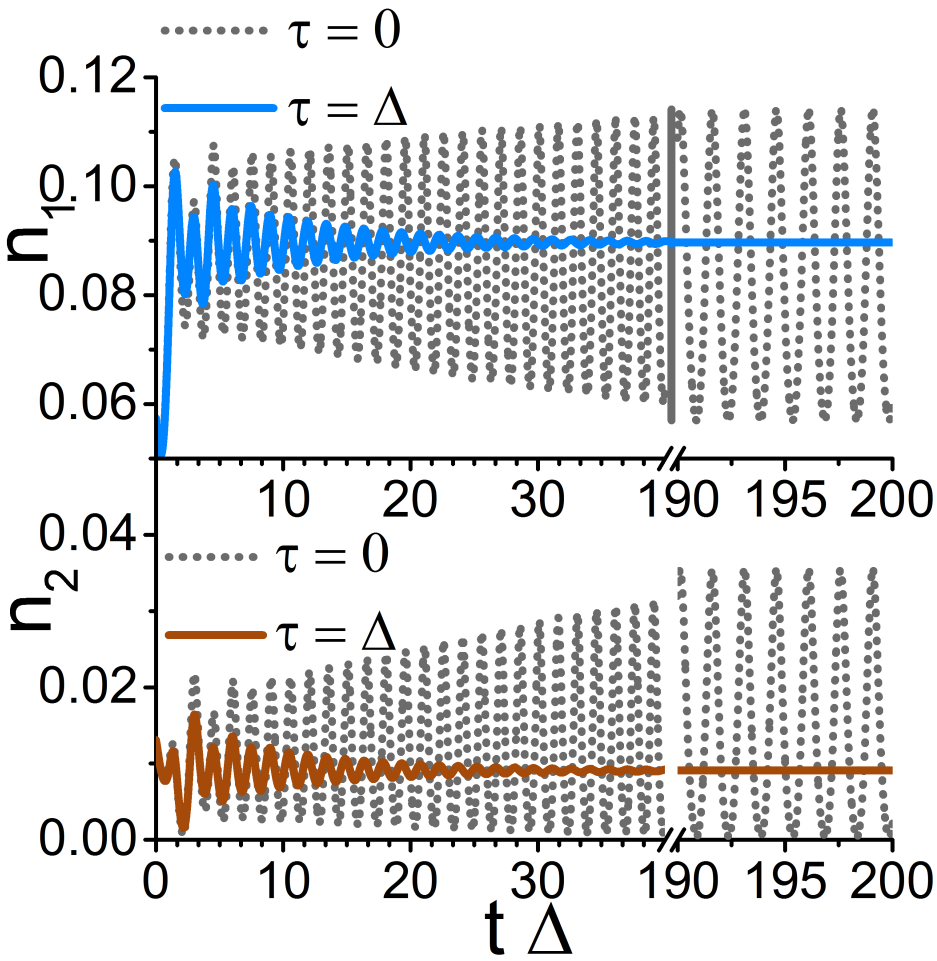
3.6 Lorenz Attractor



- Rayleigh-Bénard convection: Lorenz, J. Atmos. Sci. **20**, 130 (1963)
- Laser model: Haken, Phys. Lett. A **53** (1975)
- Time-delayed feedback control: Pyragas, Phys. Lett. A **170**, 421 (1992)

3.7 Stabilization of Fixed Points

$$\dot{J}_z \to \dot{J}_z - \lambda\big[J_z(t-\tau) - J_z(t)\big]$$



$$\kappa = 0.5\Delta,\ \ g = 5\Delta,\ \ \hbar\omega_1 = 2\Delta,\ \ \hbar\omega_2 = 4\Delta,\ \ \gamma_1 = 0.1\Delta,\ \ \gamma_2 = 0.2\Delta$$

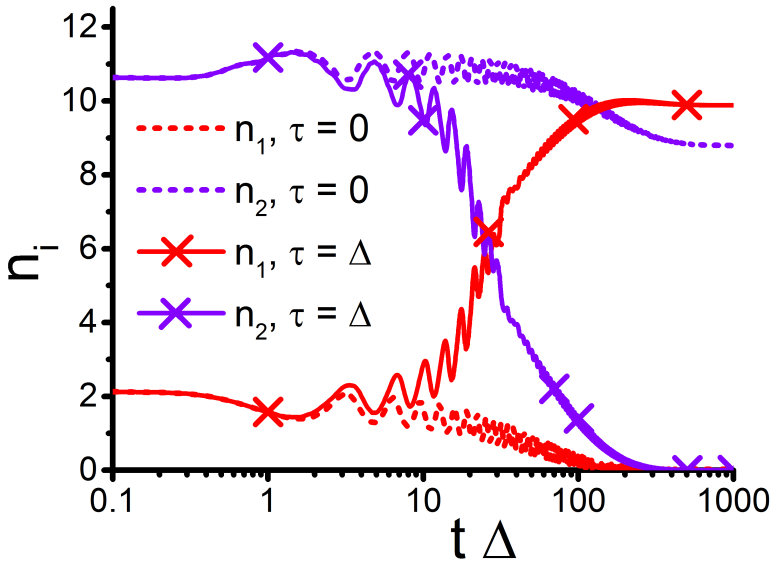
3.8 Selection of Fixed Points

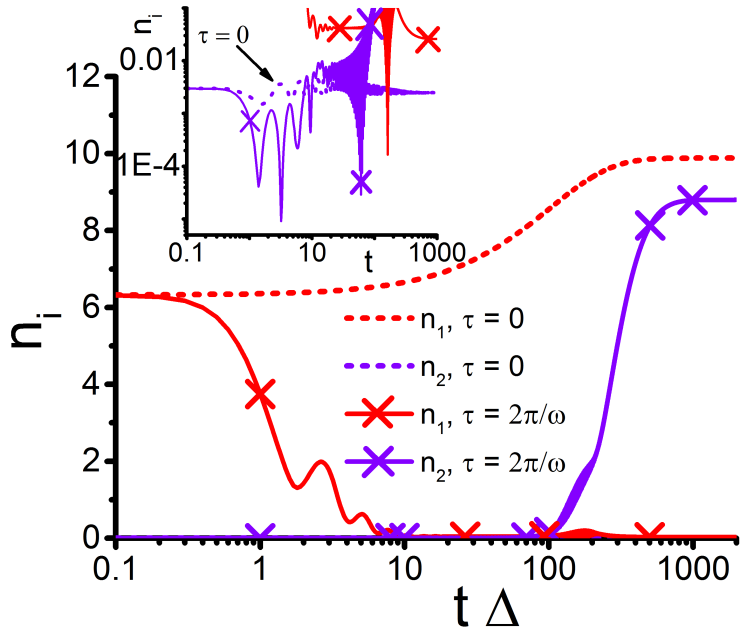
measurement-based feedback:
mean photon flux

$$\omega_1 \to \omega_1 + \lambda\big[n_2(t-\tau) - n_2(t)\big]$$

coherent feedback:
back coupling with mirror

$$\dot{a}_1 \to \dot{a}_1 - \lambda\big[a_1(t-\tau) - a_1(t)\big]$$





$$\kappa = 0.005\Delta,\ g = 2\Delta,\ \hbar\omega_1 = \Delta,\ \hbar\omega_2 = 4\Delta,\ \gamma_1 = 0.1\Delta,\ \gamma_2 = 0.2\Delta$$

3.9 Summary

- **Two-mode laser model:**
  - semiclassical equation in thermodynamic limit
  - complex phase diagram due to multiple stable fixed points

- **Time-delayed feedback control:**
  - steady-state selection and stabilization
  - affects stability of all fixed points

- **Farther away from fixed point:**
  - appearance of limit cycles or chaotic solutions
  - typical for nonlinear dynamical systems with time delay

Wischert, Wunderlin, Pelster, Olivier, and Groslambert, PRE **49**, 203 (1994)

Grigorieva, Haken, Kashchenko, and Pelster, Physica D **125**, 123 (1999)

Simmendinger, Wunderlin, and Pelster, PRE **59**, 5344 (1999)

Schanz and Pelster, SIAM J. Appl. Dyn. Syst. **2**, 277 (2003)

Schanz and Pelster, PRE **67**, 056205 (2003)

3.10 Outlook

- **Semiclassical approximation:**
  should be justified in thermodynamic limit $N \to \infty$

- **Quantum fluctuations for two-mode laser model:**
  Haken, *Laser Theory*, Springer (1970)
  Sargent, Scully, and Lamb, *Laser Physics*, Addison-Wesley (1976)

- **Quantum version of Pyragas control:**
  - unsolved yet
  - entanglement control and light bunching by structured environment
    Hein, Schulze, Carmele, and Knorr, PRA **91**, 052321 (2015)

- **Thermal fluctuations for two-mode laser model:**
  Kirton and Keeling, PRL **111**,100404 (2013)
  Kirton and Keeling, PRA **91**, 033826 (2015)

Laser $\Longleftrightarrow$ **Photon BEC**

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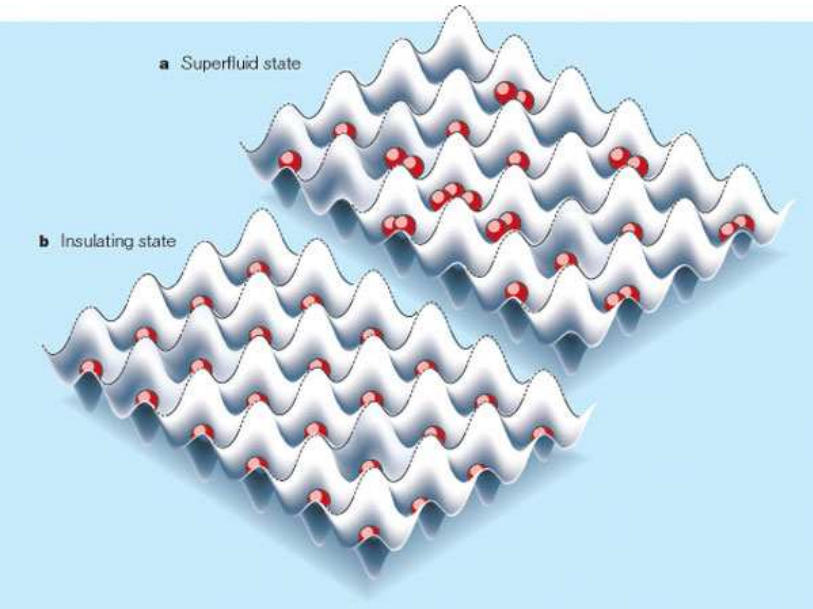
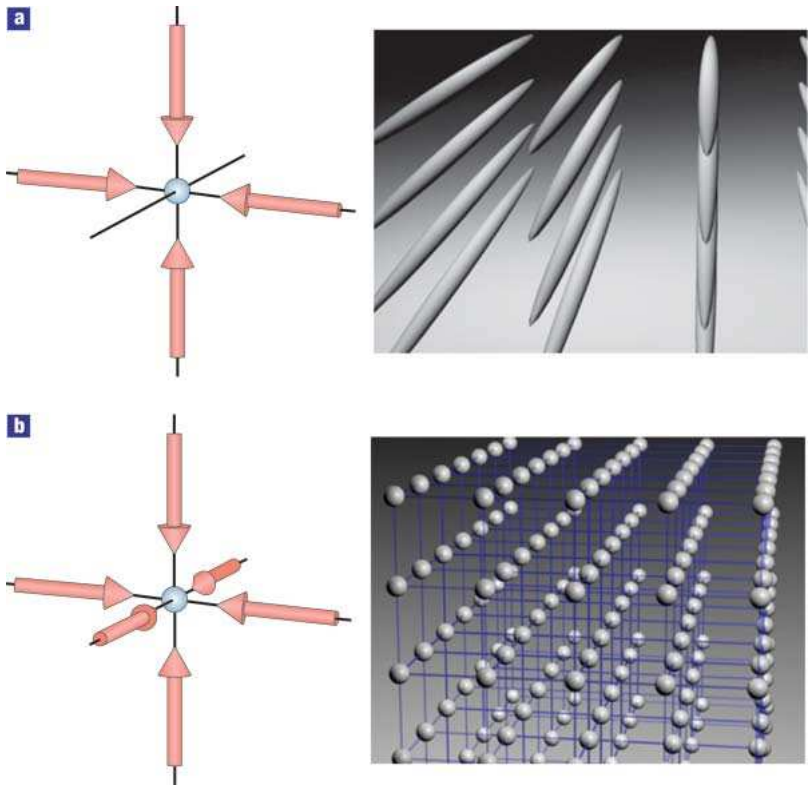


4.1 Optical Lattice

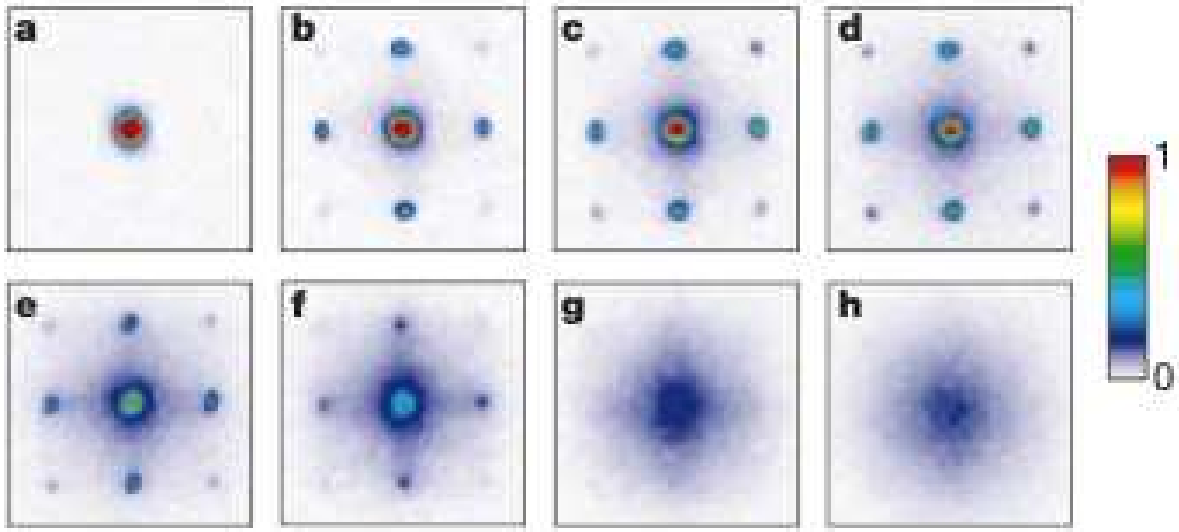
- Counter-propagating laser beams create periodic potential
- Different possible topologies at 1D, 2D, and 3D
- Hopping and interactions are highly controllable



4.2 Time-of-Flight Absorption Pictures

- **Superfluid phase:**
  delocalization in space, localization in Fourier space

- **Mott phase:**
  localization in space, delocalization in Fourier space



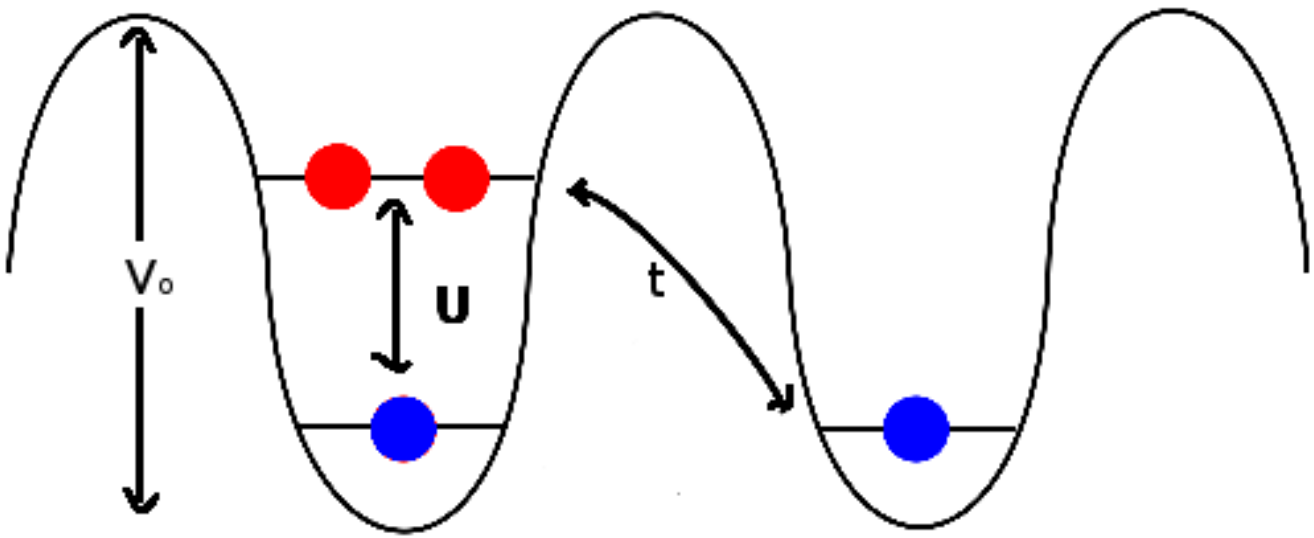
Greiner, Mandel, Esslinger, Hänsch, and Bloch, Nature **415**, 39 (2002)

4.3 Theoretical Description

Bose-Hubbard Hamiltonian:

$$\hat{H}_{\mathrm{BH}} = -t \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \left[\frac{U}{2} \hat{n}_i (\hat{n}_i - 1) - \mu \hat{n}_i \right], \qquad \hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$$

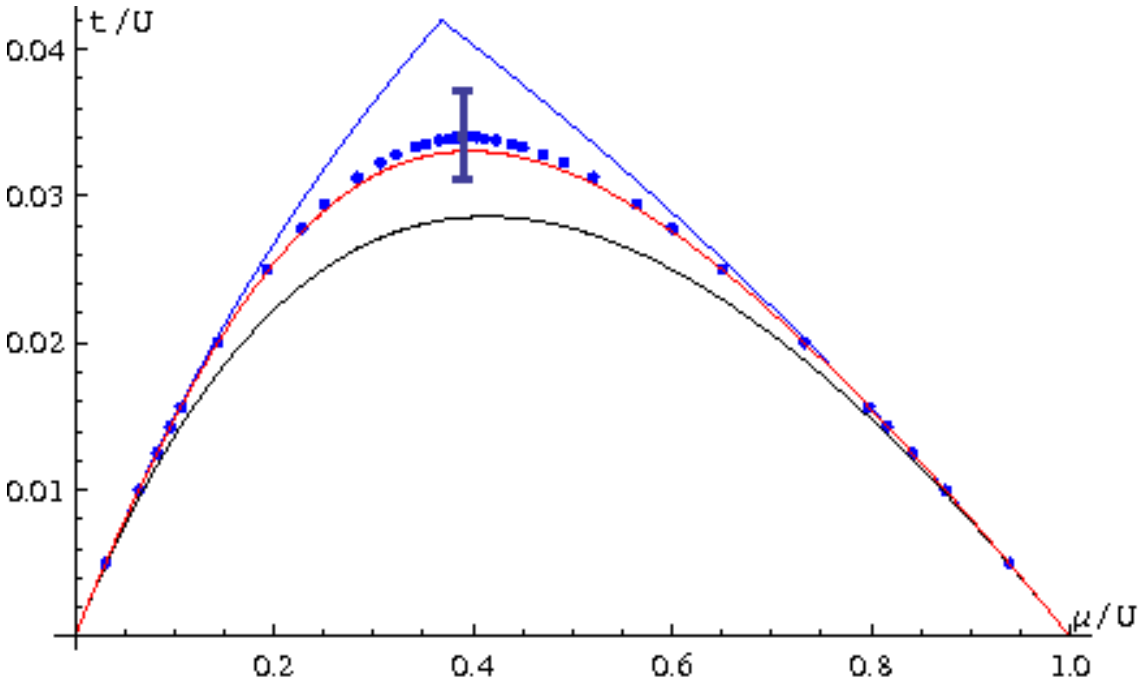
System Parameters:



Fisher *et al.*, PRB **40**, 546 (1989)

4.4 Landau Theory

Quantum Phase Diagram (T=0):



Blue line: 3rd strong-coupling order

Blue dots: Monte-Carlo data

Red line: Landau theory

Black line: Mean-field

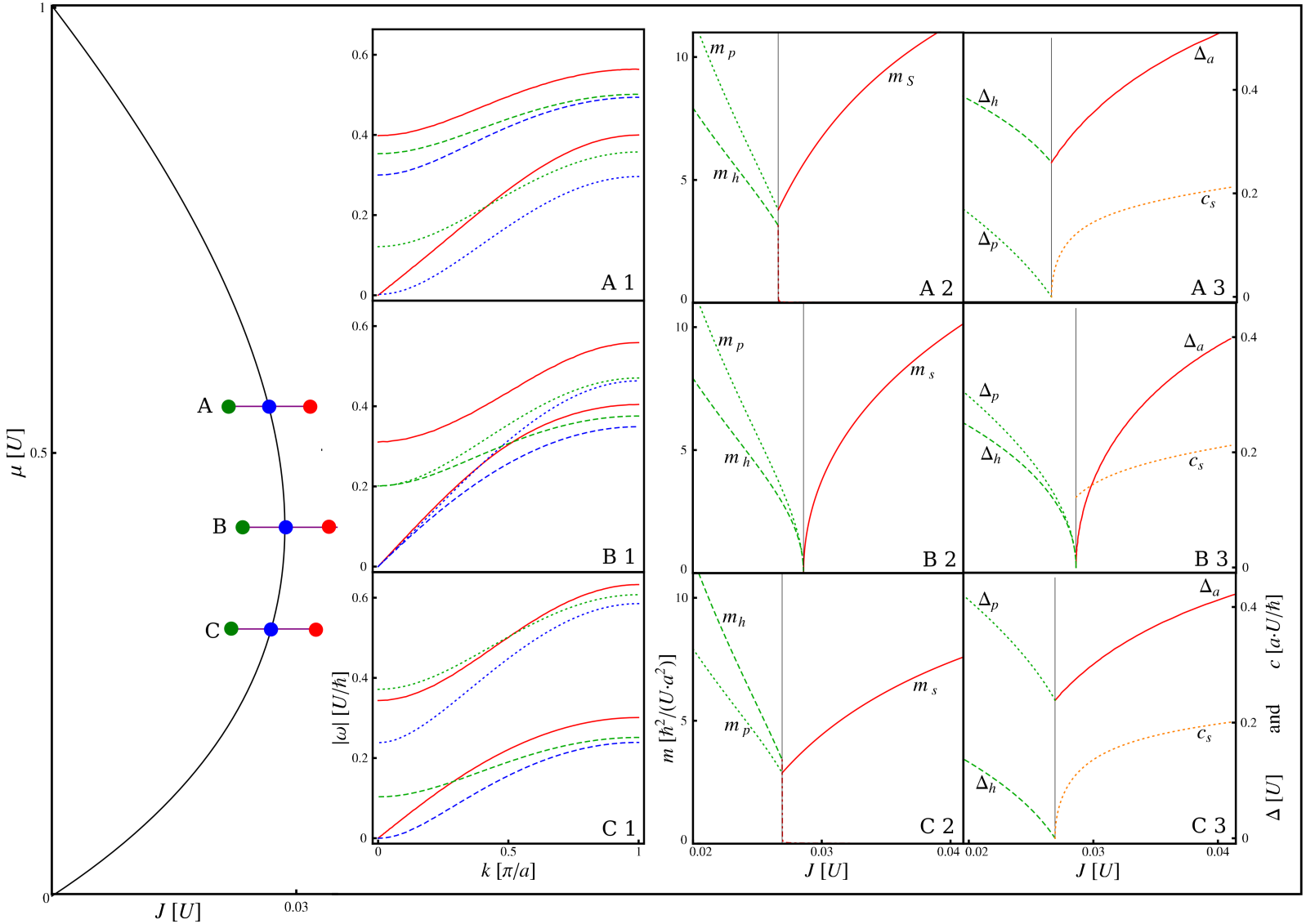
Santos and Pelster, PRA **79**, 013614 (2009)

Extension to Higher Orders:

Teichmann, Hinrichs, Holthaus, and Eckardt, PRB **79**, 100503(R) (2009)

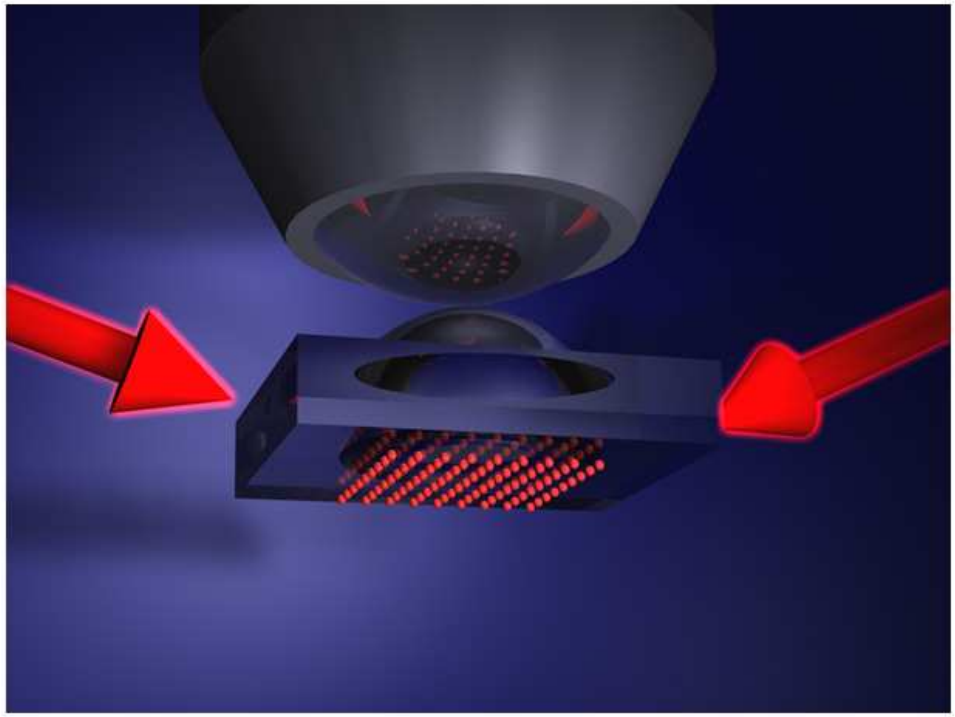
Hinrichs, Pelster, and Holthaus, APB **113**, 57 (2013)

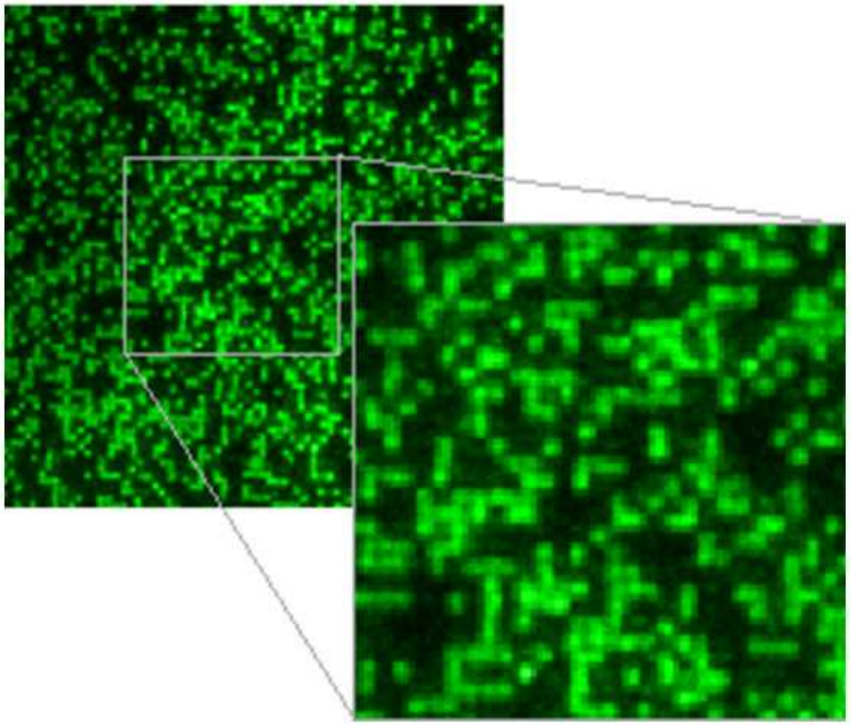
4.5 Excitation Spectra



Graß, Santos, and Pelster, PRA **84**, 013613 (2011)

4.6 Single-Site Adressability





Bakr, Gillen, Pengh, Fölling, and Greiner, Nature **462**, 74 (2009)

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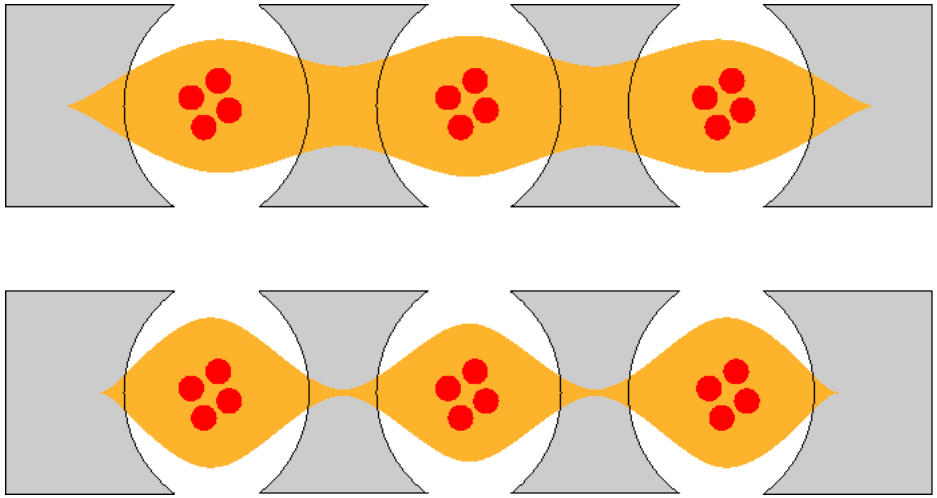
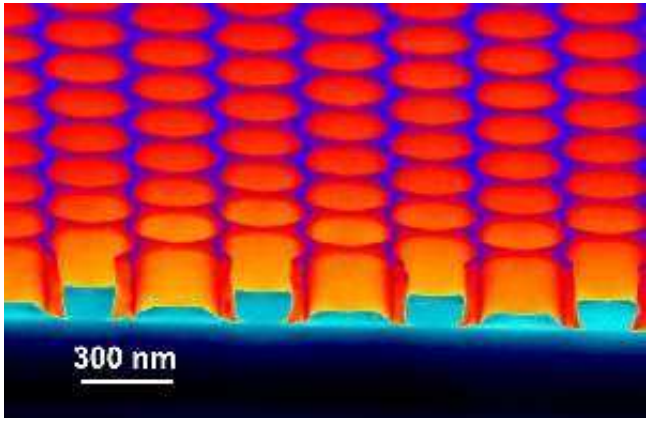


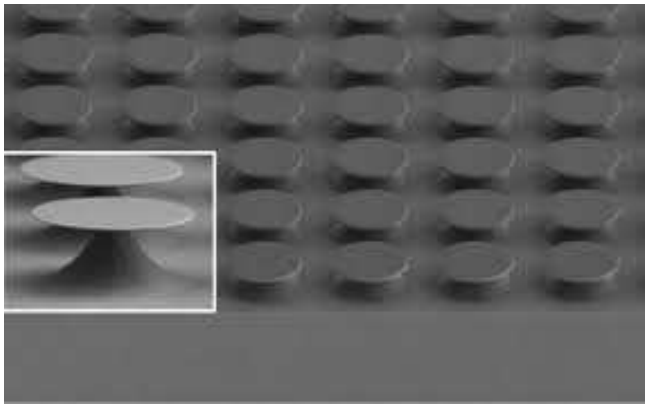
Hartmann, Brandão, and Plenio
Laser & Photon. Rev. 2, 527 (2008)

5.1 Experimental Set-Ups

- **Photonic crystal (IBM, New York):**
  periodic array of holes etched in silicon slab



- **Micro-disk array (CMM-FBK, Povo, Italy):**
  integrated resonators with embedded light emitting Si quantum dots



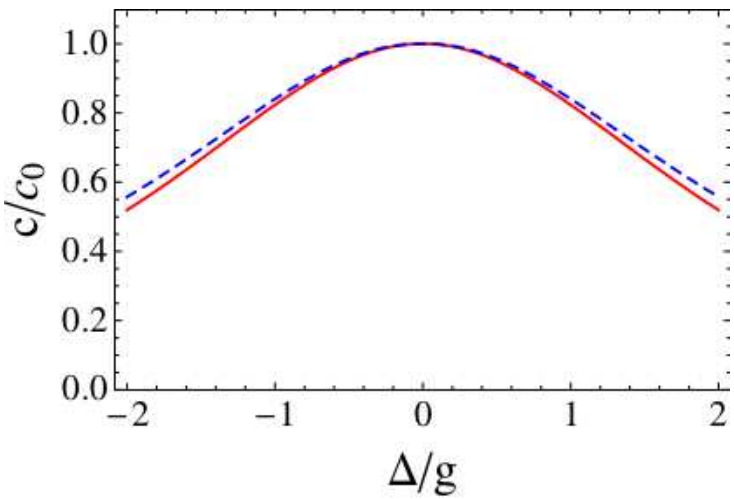
5.2 Highlights

- **Huge distance between cavities:**
  local in-situ control and accessability

- **Atoms isolated in cavities:**
  room temperature

- **Strongly coupled light and matter:**
  - reduced spontaneous emission
  - polaritons as quasi-particles

- **Superfluidity of light:**
  Sound velocity tunable via detuning
  Nietner and Pelster, PRA **85**, 043831 (2012)



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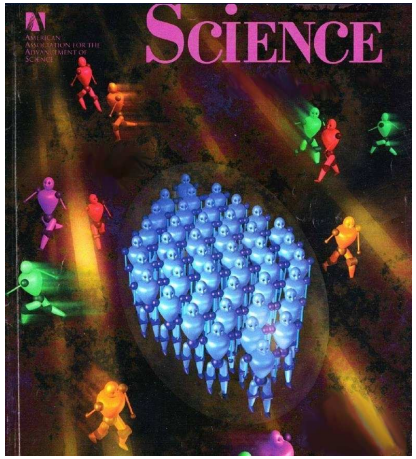
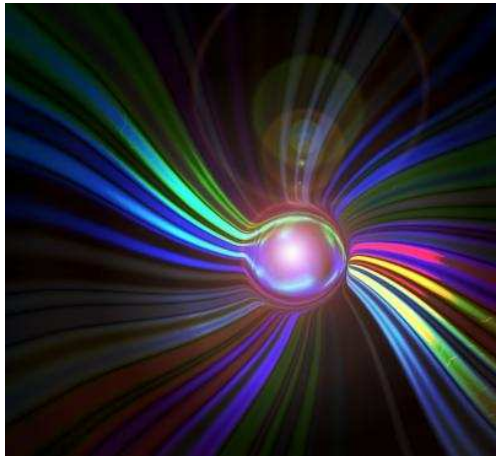
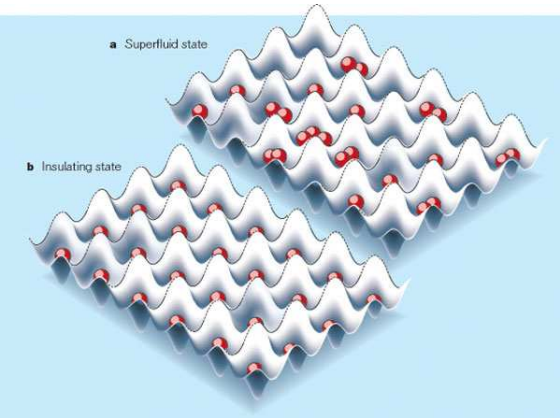
6.1 Summary and Outlook



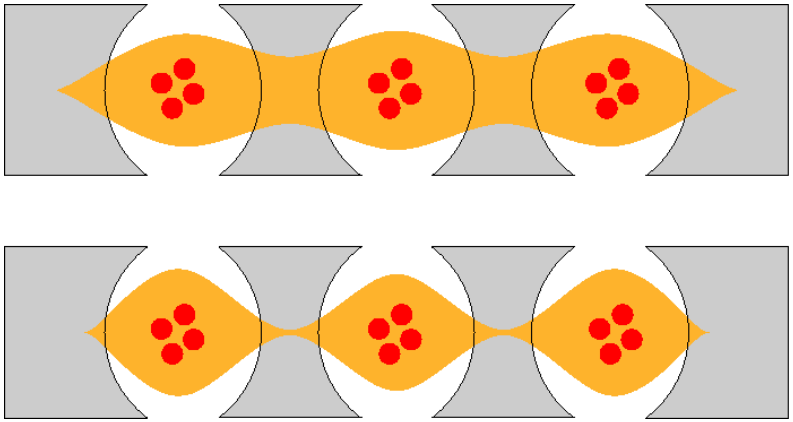
quantum simulation



equilibrium/nonequilibrium



driven optical lattices



superfluidity of light

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- Tobias Rexin (Potsdam)
- Falk Wächtler (Potsdam)

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