Collective Motion of Polarized Dipolar Fermi Gases in the Hydrodynamic Regime

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- Semiclassical Hartree-Fock theory
- Equilibrium configuration
- Low-lying excitations
- Time-of-flight expansion

PRA **81**, 021606(R) (2010)
arXiv:1003.1881

Physical Motivation

- Dipole-Dipole Interaction (DDI) potential

$$V_{\rm dd}(\mathbf{x}) = \frac{C_{\rm dd}}{4\pi|\mathbf{x}|^3}\left[1 - 3\frac{z^2}{|\mathbf{x}|^2}\right]$$

- Magnetic systems: $C_{\rm dd} = \mu_0 m^2$, with $m \sim 10\ \mu_B$

Exp.: ^{53}Cr R. Chicireanu et al., PRA **73**, 053406 (2006)
^{173}Yb T. Fukuhara et al., PRL **98**, 030401 (2007)
^{163}Dy M. Lu et al., PRL **104**, 063001 (2010)

- Electric systems: $C_{\rm dd} = 4\pi d^2$, with $d \sim 1$ Debye

Exp.: ^{40}K^{87}Rb S. Ospelkaus et al., Science **32**, 231 (2008)

Variational Hartree-Fock

- Action $\mathcal{A} = \int_{t_1}^{t_2} \mathrm{d}t \langle \Psi | i\hbar \frac{\partial}{\partial t} - H | \Psi \rangle$ Slater determinant
- Common-phase factorization $\psi_i(x,t) = e^{iM\chi(x,t)/\hbar} |\psi_i(x,t)|$
- Time-even Slater determinant
  $\Psi_0(x_1, \cdots, x_N, t) = \mathrm{SD}\left[|\psi(x,t)|\right]$
- Time-even one-body density matrix
  $$\rho_0(x, x'; t) = \prod_{i=2}^{N} \int \mathrm{d}^3 x_i \Psi_0^*(x', \cdots, x_N, t) \Psi_0(x, \cdots, x_N, t)$$
- Equations of motion $\frac{\delta \mathcal{A}}{\delta \chi(x,t)} = 0, \quad \frac{\delta \mathcal{A}}{\delta \rho_0(x,x';t)} = 0$
- In this work, we switch to Wigner space

$$\nu_0(\mathbf{x}, \mathbf{k}; t) = \int \mathrm{d}^3 s \, \rho_0 \left(\mathbf{x} + \frac{\mathbf{s}}{2}, \mathbf{x} - \frac{\mathbf{s}}{2}; t \right) e^{-i\mathbf{k}\cdot\mathbf{s}}$$

Action

- Ansatz

$$\chi(x,t) = \frac{1}{2}\left[\alpha_x(t)x^2 + \alpha_y(t)y^2 + \alpha_z(t)z^2\right]$$

$$\nu_0\left(\mathbf{x},\mathbf{k};t\right) = \Theta\left(1 - \sum_i \frac{x_i^2}{R_i(t)^2} - \sum_i \frac{k_i^2}{K_i(t)^2}\right)$$

- Action

$$\mathcal{A} = -\int_{t_1}^{t_2} \mathrm{d}t \frac{\overline{R}^3\overline{K}^3}{3\cdot 2^7}\left\{\frac{M}{2}\sum_i\left[\dot{\alpha}_i R_i^2 + \alpha_i^2 R_i^2 + \omega_i^2 R_i^2\right] + \sum_i \frac{\hbar^2 K_i^2}{2M}\right.$$

$$\left. -c_0\overline{K}^3\left[f\left(\frac{R_x}{R_z},\frac{R_y}{R_z}\right) - f\left(\frac{K_z}{K_x},\frac{K_z}{K_y}\right)\right]\right\} - \int_{t_1}^{t_2} \mathrm{d}t\,\mu(t)\left(\frac{\overline{R}^3\overline{K}^3}{48} - N\right)$$

with $c_0 = \frac{2^{10}C_{\mathrm{dd}}}{3^4\cdot 5\cdot 7\cdot \pi^3} \approx 0.0116\, C_{\mathrm{dd}}$ and $\overline{\bullet} = \left(\bullet_x\, \bullet_y\, \bullet_z\right)^{\frac{1}{3}}$

Equations of motion (unitless)

- Auxiliary equations of motion $\alpha_i = \dot{R}_i / R_i$
- Particle conservation $\overline{R}^3 \overline{K}^3 = 1$
- Momentum deformation* $K_z^2 - K_x^2 = \frac{3c_3\epsilon_{\rm dd}}{\overline{R}^3}\left[-1 + \frac{(2K_x^2+K_z^2)}{2(K_x^2-K_z^2)} f_s\left(\frac{K_z}{K_x}\right)\right]$

  with $c_3 = \frac{2^{\frac{38}{3}}}{3^{\frac{23}{6}}\cdot 5\cdot 7\cdot \pi^2} \approx 0.2791$ and $\epsilon_{\rm dd} = \frac{C_{\rm dd}}{4\pi}\left(\frac{M^3\overline{\omega}}{\hbar^5}\right)^{\frac{1}{2}} N^{\frac{1}{6}}$

- Finally $\frac{1}{\omega_i^2}\frac{d^2 R_i}{dt^2} = -R_i + \sum_j \frac{K_j^2}{3R_i} - \epsilon_{\rm dd} Q_i(\mathbf{R}, \mathbf{K})$

$$Q_x(\mathbf{r},\mathbf{k}) = \frac{c_3}{x^2 y z}\left[f\left(\frac{x}{z},\frac{y}{z}\right) - \frac{x}{z} f_1\left(\frac{x}{z},\frac{y}{z}\right) - f_s\left(\frac{k_z}{k_x}\right)\right]$$

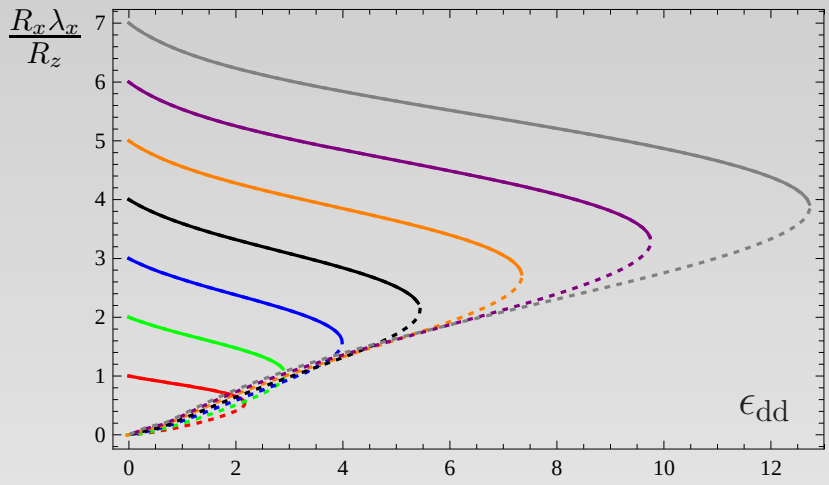
$$Q_y(\mathbf{r},\mathbf{k}) = \frac{c_3}{x y^2 z}\left[f\left(\frac{x}{z},\frac{y}{z}\right) - \frac{y}{z} f_2\left(\frac{x}{z},\frac{y}{z}\right) - f_s\left(\frac{k_z}{k_x}\right)\right]$$

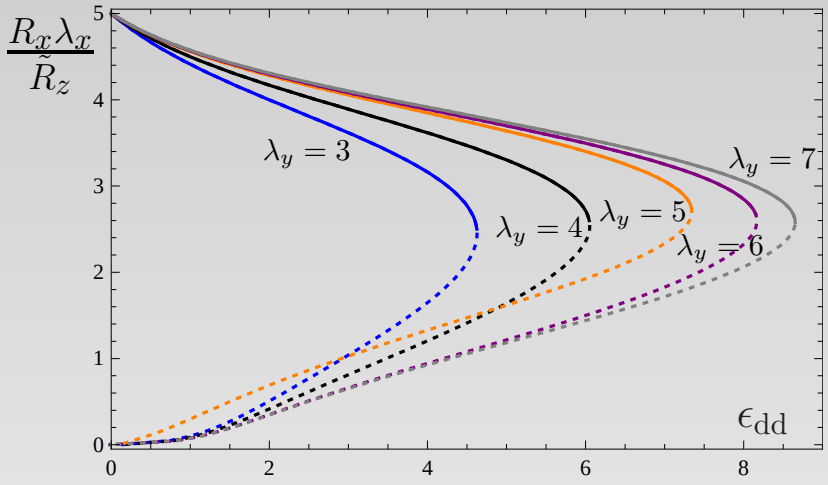
$$Q_z(\mathbf{r},\mathbf{k}) = \frac{c_3}{x y z^2}\left[f\left(\frac{x}{z},\frac{y}{z}\right) + \frac{x}{z} f_1\left(\frac{x}{z},\frac{y}{z}\right) + \frac{y}{z} f_2\left(\frac{x}{z},\frac{y}{z}\right) - f_s\left(\frac{k_z}{k_x}\right)\right]$$

*T. Miyakawa et al, PRA **77**, 061603(R) (2008)

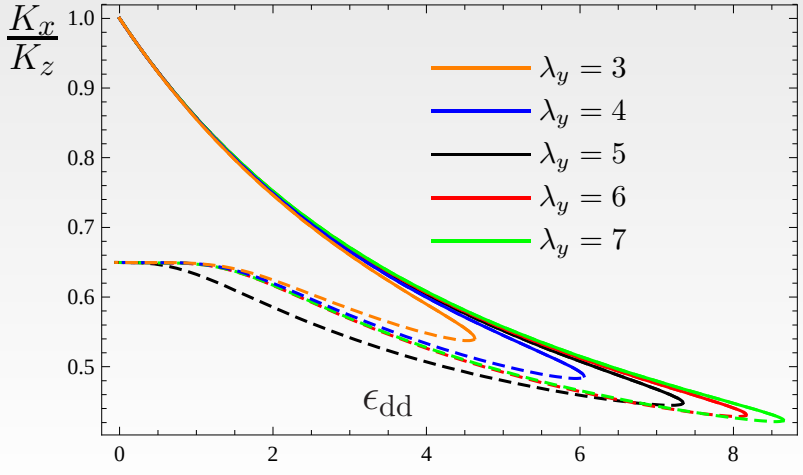
Static properties

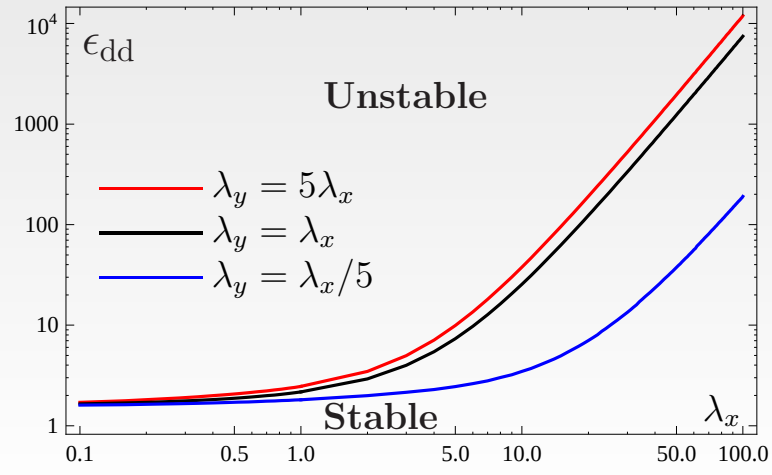
- Aspect ratio in real space





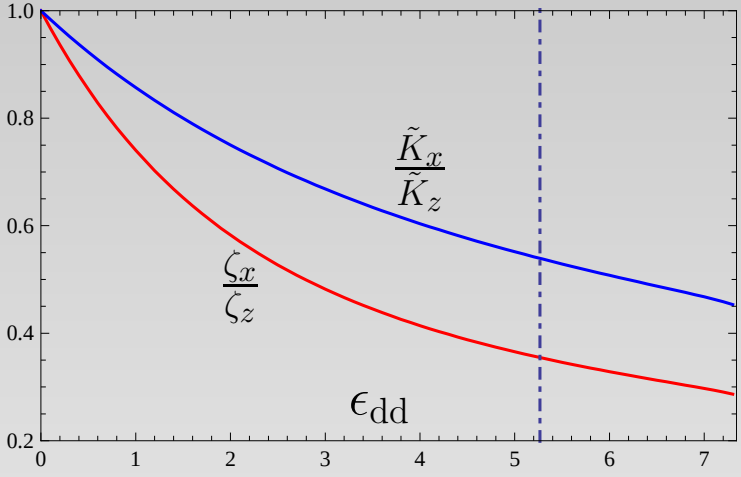
- Aspect ratio in momentum space and stability diagram





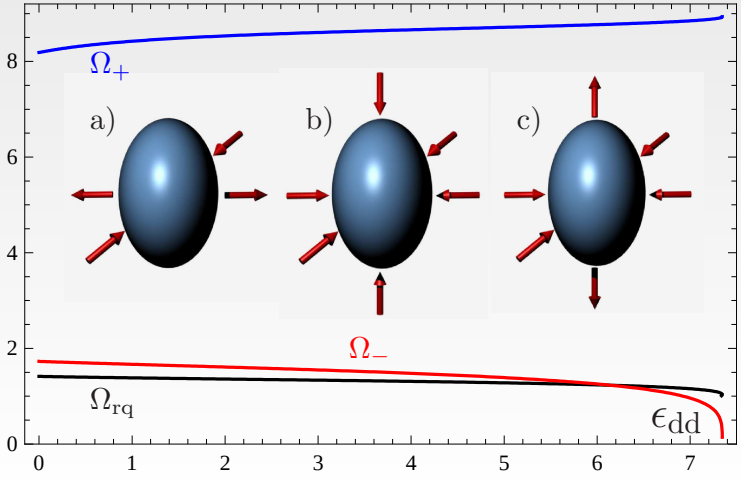
Low-lying excitations

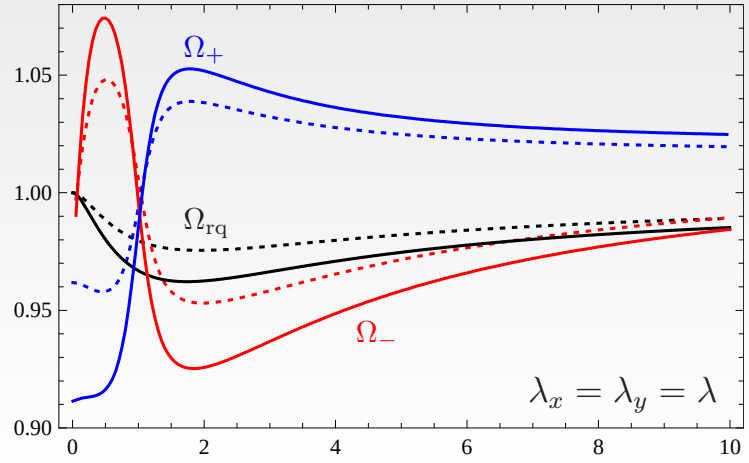
- Momentum anisotropic breathing oscillations $\lambda_x = \lambda_y = 5$



- Oscillation modes in real space

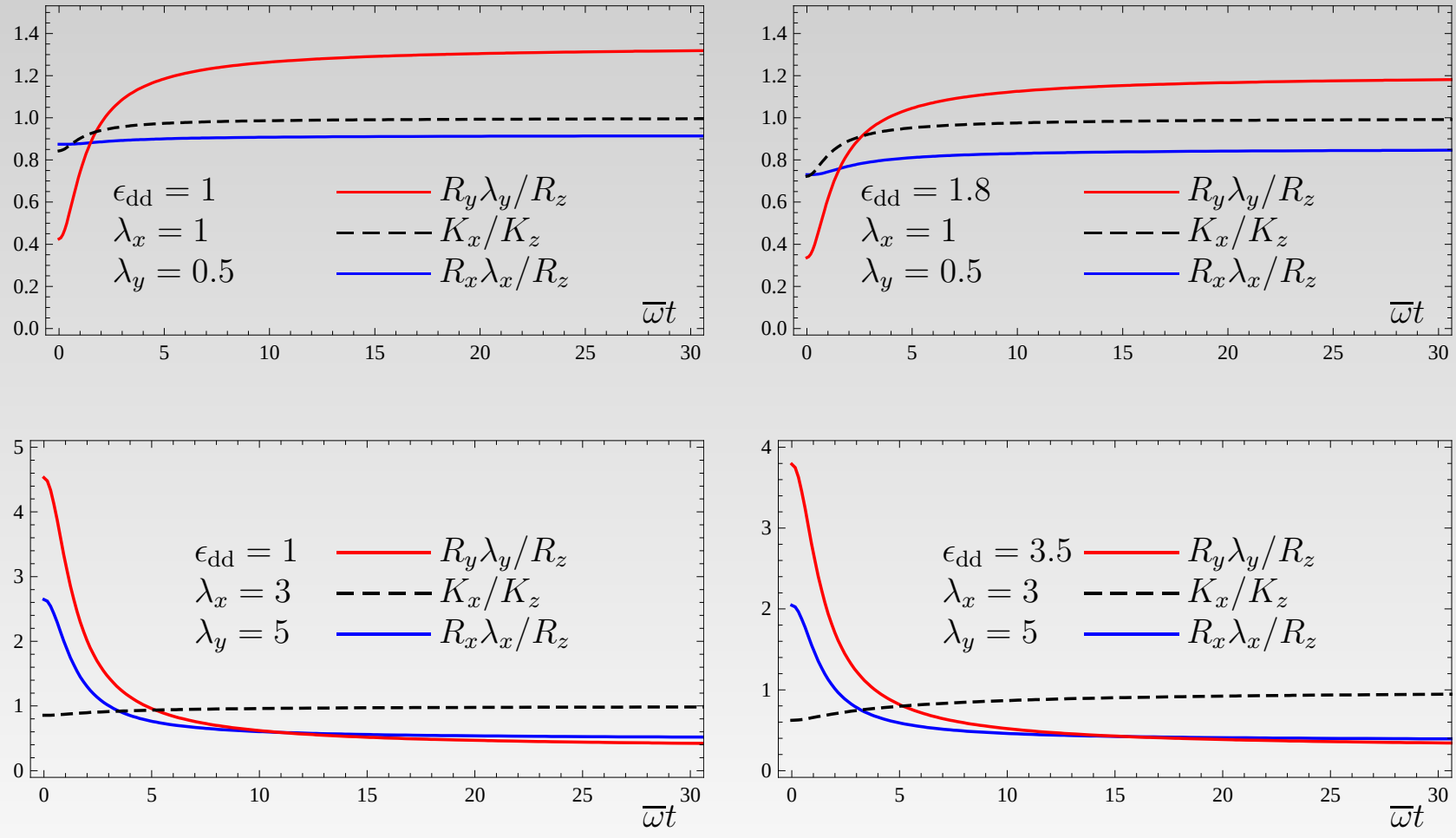
$$\lambda_x = \lambda_y = 5 \qquad \epsilon_{\rm dd} = \begin{cases} 0.8; & \text{dotted} \\ 1.2; & \text{solid} \end{cases}$$





Time-of-flight expansion

- Aspect ratio

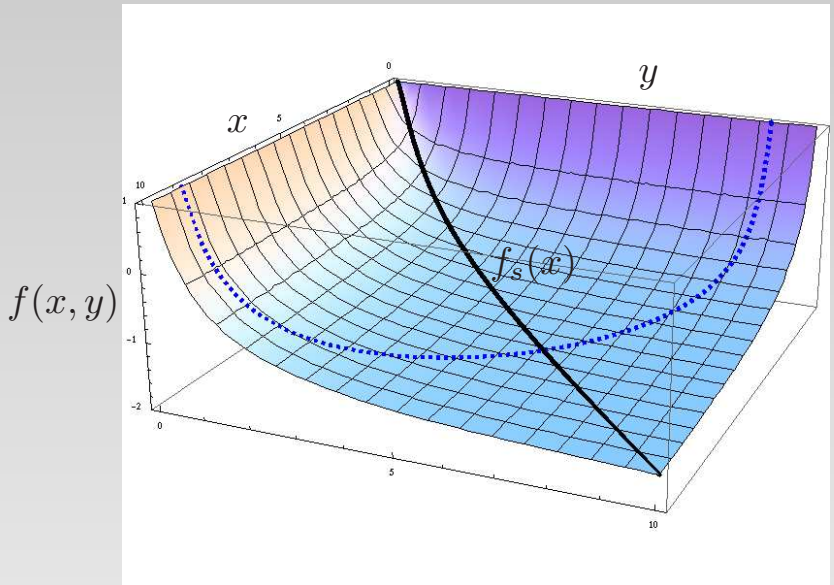


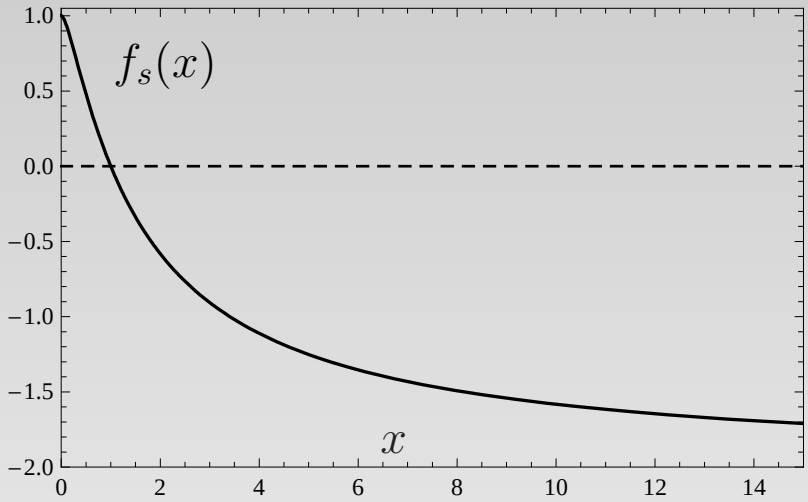
Estimation of τ_R

- Boltzmann-Vlasov eq. for DDI remains very hard to solve, due to anisotropy
- Approximation of the DDI through an effective contact interaction with scattering length $a_{\rm dd} = MC_{\rm dd}/(4\pi\hbar^2)$ gives $\frac{1}{\overline{\omega}\tau_R} = (N^{1/3} a_{\rm dd}\sqrt{M\overline{\omega}/\hbar})^2 F(T/T_{\rm F})$, with $F(T/T_{\rm F}) \sim 0.1$ in quantum regime
  (L.Vichi and S. Stringari PRA, **60**, 4734 (1999))
- Thus, for $N = 4 \times 10^4$ dipoles, one obtains
  - $\overline{\omega}\tau_R \approx 0.01$ for KRb ($d = .57\ D$) and
    $\overline{\omega}\tau_R \approx 2 \times 10^{-7}$ for LiCs ($d = 5.53\ D$) polar molecules
  - $\overline{\omega}\tau_R \approx 75$ for ^{53}Cr ($m = 6\ \mu_B$) and
    $\overline{\omega}\tau_R \approx 4 \times 10^3$ for ^{163}Dy ($m = 10\ \mu_B$) atoms

Equations of motion

- Anisotropy function





$$f(x,y) = 1 + 3xy\frac{E(\varphi,q)-F(\varphi,q)}{(1-y^2)\sqrt{1-x^2}}\star$$

$F(\varphi,q)$, $E(\varphi,q)$: elliptic integrals of first and second kind

$\varphi = \arcsin\sqrt{1-x^2}$, $q^2 = (1-y^2)/(1-x^2)$

$\star$K. Glaum and Axel Pelster, PRA **76**, 023604 (2007)