Dipolar Bose-Einstein Condensates in Weak Anisotropic Disorder Potentials

Master thesis

Branko Nikolić

Scientific Computing Laboratory, Institute of Physics Belgrade
Pregrevica 118, 11080 Belgrade, Serbia
http://www.scl.rs/

Faculty of Physics, University of Belgrade
Studentski Trg 12, 11000 Belgrade, Serbia

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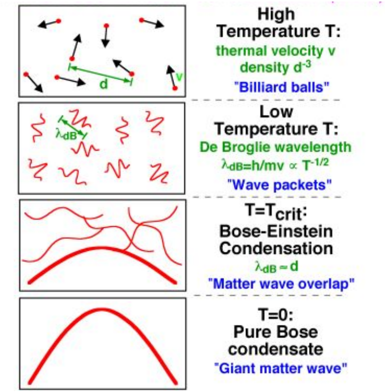
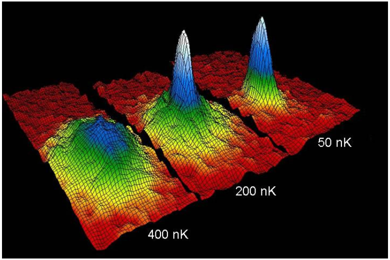
Overview

- Introduction
  - Bose-Einstein condensates
  - Dipolar interaction
  - Anisotropic disorder potentials
- Mean-field approach
  - Condensate density
  - Equation of state
  - Superfluidity
  - Sound velocity
- Model and results
  - Lorentz-correlated disorder and dipolar interaction
- Conclusions and outlook

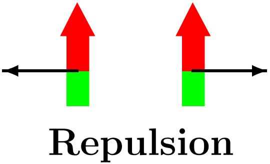
Bose-Einstein condensates

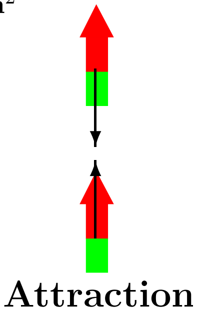




Dipolar interaction

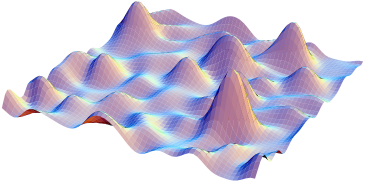
- Contact interaction $V_{\mathrm{cont}}(\mathbf{x}) = g\delta(\mathbf{x})$
- Dipolar interaction $V_{\mathrm{d}}(\mathbf{x}) = \frac{C_{\mathrm{d}}}{x^3}(1 - 3\cos\theta)$
  - Atoms: ^{52}Cr, ^{164}Dy, ^{168}Er, $C_{\mathrm{d}} = \mu_0 \mathbf{m}^2$
  - Molecules: ^{41}K^{87}Rb, $C_{\mathrm{d}} = \frac{\mathbf{d}^2}{\epsilon_0}$
  - $\epsilon = \frac{C_{\mathrm{d}}}{3g}$





Disorder potentials

- Uncontrolled disorder: Wire traps
  J. Fortagh and C. Zimmermann,
  Rev. Mod. Phys. **79**, 235 (2007)
- Controlled disorder: Laser speckles
  J. Billy et al. Nature **453**, 891 (2008)
- σ - hill's and valley's width



Mean-field approach

- Gross-Pitaevskii equation for a homogeneous disordered system:

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{r}) - \mu + \int d^3\mathbf{r}' V(\mathbf{r}' - \mathbf{r})\psi^*(\mathbf{r}')\psi(\mathbf{r}')\right)\psi(\mathbf{r}) = 0$$

- Statistical properties of the disorder potential: ensemble averages

$$\langle U(\mathbf{r})\rangle = 0, \qquad \langle U(\mathbf{r})U(\mathbf{r}')\rangle = R(\mathbf{r} - \mathbf{r}')$$

- Assumption: coarse-graining $\approx$ ensemble average

$$A_{\mathrm{mac}} = \frac{1}{l^3}\int_{l^3} d^3\mathbf{r}' A(\mathbf{r}') = \langle A\rangle\,,\ l \gg \sigma, \xi = \sqrt{\frac{\hbar^2}{2mng}}$$

Condensate density

- Density matrix: $\rho(\mathbf{r},\mathbf{r}') = \langle \psi(\mathbf{r})\psi(\mathbf{r}')\rangle$
- Fluid density: $n = \left\langle \psi(\mathbf{r})^2 \right\rangle$
- Condensate density:

$$n_0 = \lim_{|\mathbf{r}-\mathbf{r}'|\to\infty} \rho(\mathbf{r},\mathbf{r}') = \langle \psi(\mathbf{r})\rangle^2$$

- Condensate depletion due to disorder:

$$n - n_0 = n \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{R(\mathbf{k})}{\left[\frac{\hbar^2\mathbf{k}^2}{2m} + 2nV(\mathbf{k})\right]^2} + \ldots$$

Equation of state

- Chemical potential from $\langle \psi^2(\mu) \rangle = n$

$$\mu_b = nV(\mathbf{k}=0) - \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{\frac{\hbar^2 k^2}{2m} R(\mathbf{k})}{\left[\frac{\hbar^2 k^2}{2m} + 2nV(\mathbf{k})\right]^2} + \dots$$

- Renormalization, G.M. Falco, A. Pelster, and R. Graham, Phys. Rev. A **75**, 063619 (2007)

$$\begin{aligned} \mu(n) &= \mu_b(n) - \mu_b(0) = \mu_b + \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{R(\mathbf{k})}{\frac{\hbar^2 k^2}{2m}} + \dots \\ &= nV(\mathbf{k}=0) + 4n \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{V(\mathbf{k}) R(\mathbf{k}) \left(\frac{\hbar^2 k^2}{2m} + nV(\mathbf{k})\right)}{\frac{\hbar^2 k^2}{2m} \left[\frac{\hbar^2 k^2}{2m} + 2nV(\mathbf{k})\right]^2} + \dots \end{aligned}$$

Superfluidity (1)

- Moving disorder, time-dependent GP equation:

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{r} - \mathbf{k}_U \frac{\hbar}{m} t) + \int d^3\mathbf{r}' V(\mathbf{r} - \mathbf{r}')\Psi^*(\mathbf{r}', t)\Psi(\mathbf{r}', t)\right] \Psi(\mathbf{r}, t)$$

$$= i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t}$$

- Perturbed boosted solution:

$$\Psi(\mathbf{r}, t) = e^{i\mathbf{k}_S \mathbf{r}}(\psi_0 + \psi_1(\mathbf{r}, t) + \ldots)e^{-\frac{i}{\hbar}\left(\mu + \frac{\hbar^2 k_S^2}{2m}\right)t}.$$

- Total fluid wave-vector:

$$\mathbf{k}_{\mathrm{tot}} = \frac{1}{i}\nabla \ln \frac{\Psi}{|\Psi|}$$

Superfluid density

- Energy and momentum densities, arbitrary macroscopic superfluid velocity $\mathbf{k}'_S$

$$\langle n(\mathbf{r})\mathbf{k}_{\text{tot}}(\mathbf{r})\rangle = \hat{n}_S\mathbf{k}'_S + \hat{n}_N\mathbf{k}_U ,$$

$$\left\langle n(\mathbf{r})\mathbf{k}^2_{\text{tot}}(\mathbf{r})\right\rangle = \mathbf{k}'_S\hat{n}_S\mathbf{k}'_S + \mathbf{k}_U\hat{n}_N\mathbf{k}_U ,$$

- Density of the normal component ($\hat{n}_S = n\hat{I} - \hat{n}_N$):

$$\hat{n}_N = 4\psi_0^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\mathbf{k}\otimes\mathbf{k}}{k^2}\frac{R(\mathbf{k})}{\left[\frac{\hbar^2k^2}{2m} + 2nV(\mathbf{k})\right]^2} + \ldots$$

- Cylindrical symmetry: $\hat{n}_S = \begin{pmatrix} n_{S\rho} & 0 & 0 \\ 0 & n_{S\rho} & 0 \\ 0 & 0 & n_{Sz} \end{pmatrix}$
- Spherical symmetry: $n - n_S = \frac{4}{3}(n - n_0)$

Sound velocity

- Hydrodynamic equations for macroscopic (averaged) quantities:

$$\frac{\partial n(\mathbf{x},t)}{\partial t} + \nabla(\hat{n}_S(\mathbf{x},t)\mathbf{v}'_S(\mathbf{x},t)) = 0$$

$$m\frac{\partial \mathbf{v}'_S(\mathbf{x},t)}{\partial t} + \nabla\left(\frac{m\mathbf{v}'_S(\mathbf{x},t)^2}{2} + \mu(n(\mathbf{x},t))\right) = \mathbf{0}$$

- Small variation from the equilibrium:

$$c_{\mathbf{e}}^2 = \frac{1}{m}\frac{\partial \mu}{\partial n}\mathbf{e}^T\hat{n}_S\mathbf{e}$$

- Measurable by Bragg spectroscopy

Anisotropic disorder and dipolar interaction

- Modeling disorder and interaction:

$$R(\mathbf{k}) = \frac{R}{1+\sigma_\rho^2 k_\rho^2 + \sigma_z^2 k_z^2},\ V(\mathbf{k}) = g + \frac{C_d}{3}\left(3\cos^2\phi(\mathbf{m},\mathbf{k}) - 1\right)$$

Gaussian correlation function, C. Krumnow and A. Pelster Phys. Rev. A **84**, 021608(R) (2011)

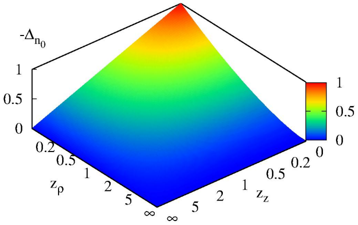
- Corrections are expressed as:

$$\Delta_A = \lim_{R\to 0} \frac{\frac{A(R)}{A(0)} - A_d}{\frac{n_{\mathrm{HM}}}{n}},\ n_{\mathrm{HM}} = \frac{m^{\frac{3}{2}} R\sqrt{n}}{4\pi\hbar^3\sqrt{g}}$$

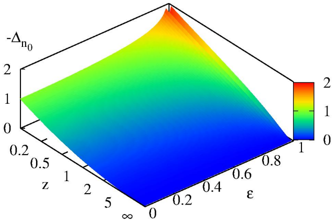
K. Huang and H. F. Meng, Phys. Rev. Lett. **69**, 644 (1992)

Condensate depletion

$\epsilon = 0$



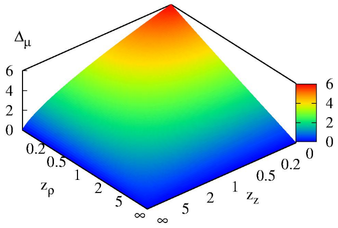
$z_\rho = z_z = z$



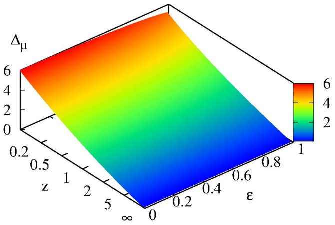
$$z = \frac{\sqrt{2}\sigma}{\xi}$$

Equation of state

$\epsilon = 0$

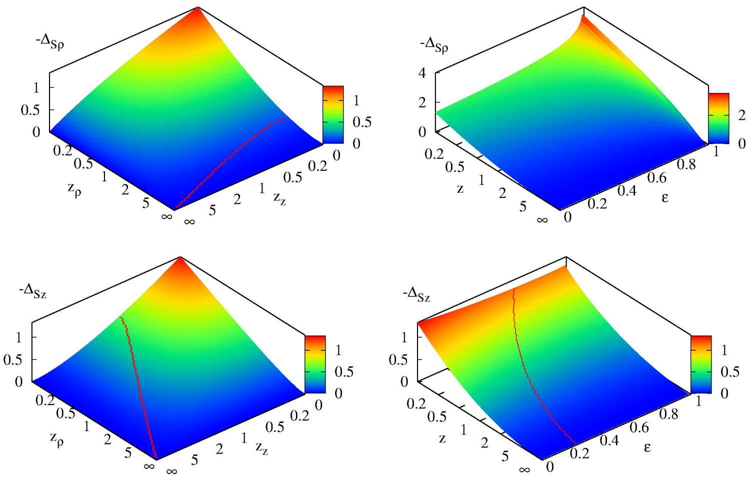


$z_\rho = z_z = z$

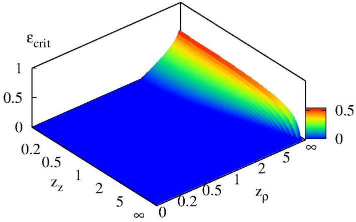


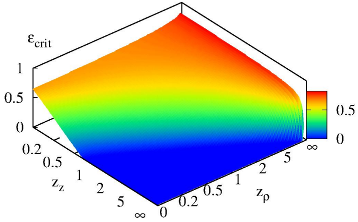
Superfluid depletion

$\epsilon = 0$ $\qquad$ $z_\rho = z_z = z$



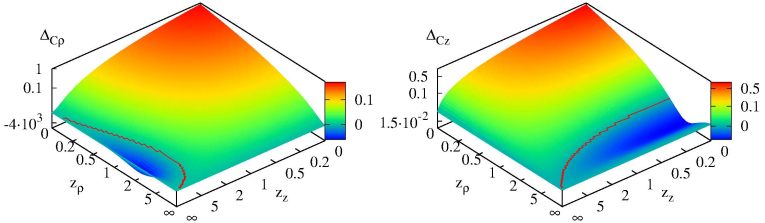
Critical values of ϵ





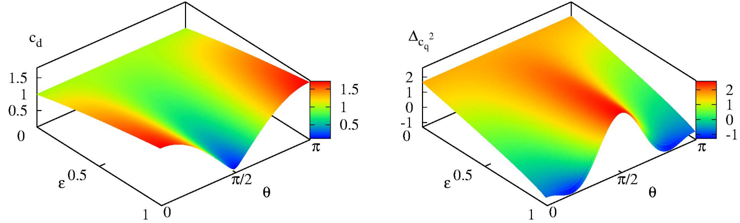
R. Graham and A. Pelster, Int. J. Bif. Chaos **19**, 2745 (2009)

Sound velocity for $\epsilon = 0$



$$c_{\mathbf{q}}^2 = c_\rho^2 \sin^2 \phi(\mathbf{q}, \mathbf{e}_z) + c_z^2 \cos^2 \phi(\mathbf{q}, \mathbf{e}_z)$$

Sound velocity for $\mathbf{z}_\rho = \mathbf{z}_\mathbf{z} = 0$



$$c_{\mathbf{q}}^2 = c_0^2 \left(c_d(\mathbf{q})^2 + \frac{n_{\mathrm{HM}}}{n} \Delta_{c_{\mathbf{q}}^2} \right), \ \theta = \phi(\mathbf{q}, \mathbf{e}_z)$$

Conclusions and outlook

- Summary
  - Consistent mean-field model
  - Interesting anisotropic effects
    - Superfluid depletion
    - Sound velocity
  - All results are measurable
- Further research
  - Automatization of higher order calculation
  - Numeric simulations for correlation function of laser speckles
  - Superfluid definition for finite temperatures:
    - Change of T_c and possible new phases

Appendix: Spatial average

- Assumption: Spatial averaging over large volumes coincides with the disorder average.
- Order parameter

$$\begin{aligned} \langle \psi(\mathbf{r})\psi(\mathbf{r}')\rangle &\approx \frac{1}{V_0^2}\int_{V_0} d^3\mathbf{r}_1 d^3\mathbf{r}_2 \, \langle \psi(\mathbf{r}+\mathbf{r}_1)\psi(\mathbf{r}'+\mathbf{r}_2)\rangle \\ &= \langle \langle \psi(\mathbf{r})\rangle \, \langle \psi(\mathbf{r}')\rangle\rangle = \langle \psi(\mathbf{r})\rangle^2 \end{aligned}$$

- Macroscopic hydrodynamic equations should be independent on the microscopic realization, therefore they relate local-spatially averaged quantities. The above assumption justifies the equations.