

Bose-Einstein Condensates in Disorder Potentials: Perturbative and Non-perturbative approach

B. Nikolić

New Year Seminar, 2014



Outline

Introduction

Disorder

Mean-Field Hydrodynamic Approach

Weak Disorder

Perturbation Theory

Contact interaction

Strong disorder

Homogeneous system

Local density approximation

Outlook



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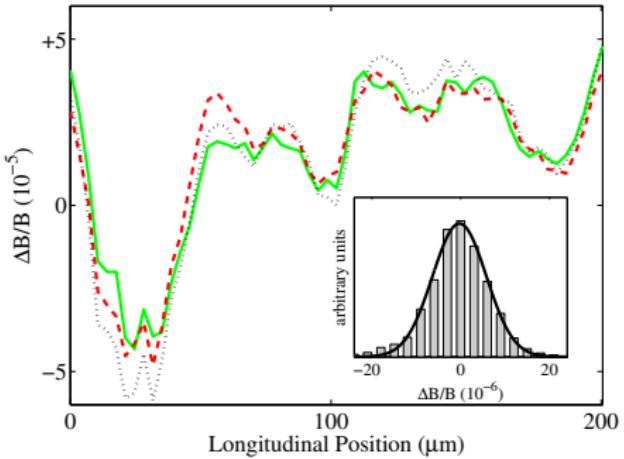
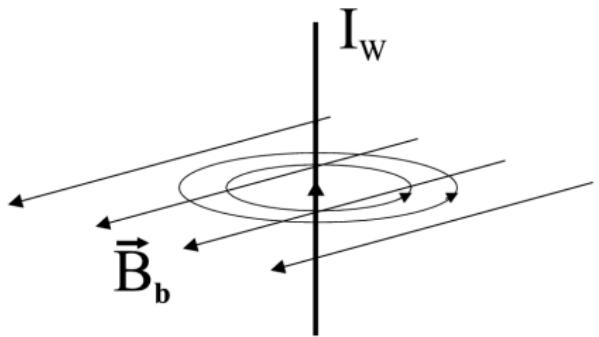
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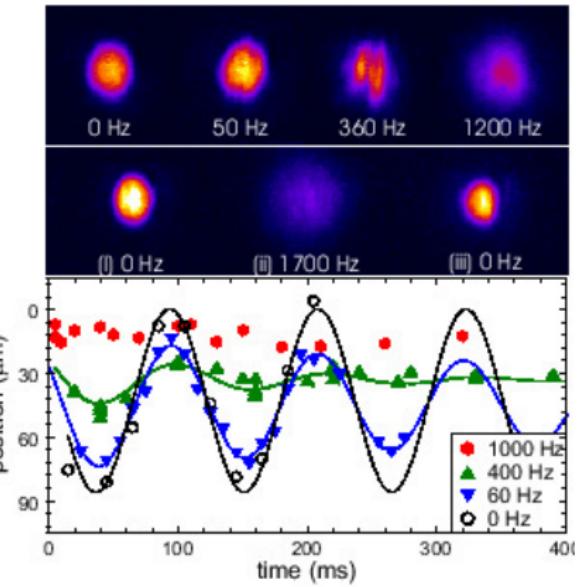
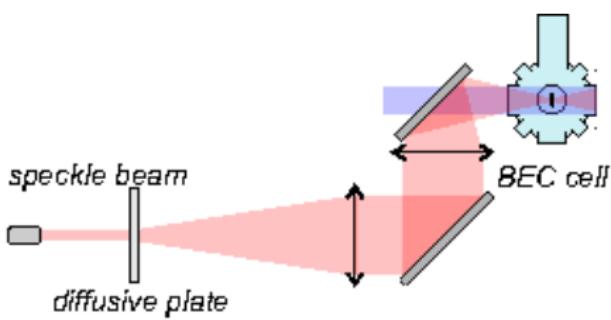
Unwanted Disorder



Fortágh, Zimmermann, RMP **79**, 235 (2007)
Krüger et al., PRA **76**, 063621 (2007)

Controllable Disorder

- ▶ Laser speckles



Inguscio et al., PRL **95**, 070401 (2005)



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Mean-Field Approach

- ▶ External random potential $U(\mathbf{r})$
 - ▶ zero average $\overline{U(\mathbf{r})} = 0$
 - ▶ given correlation function $\overline{U(\mathbf{r})U(\mathbf{r}')}=R(\mathbf{r}-\mathbf{r}')$

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- ▶ Gross-Pitaevskii equation

$$\left[-\frac{\hbar^2}{2m} \nabla_{\mathbf{r}}^2 + U\left(\mathbf{r} - \frac{\hbar}{m} \mathbf{k}_U t\right) + \int d^3 \mathbf{r}' V(\mathbf{r} - \mathbf{r}') \psi^*(\mathbf{r}', t) \psi(\mathbf{r}', t) \right] \psi(\mathbf{r}, t) = i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t}$$

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- ▶ Hydrodynamic equation from $\mathbf{r} \rightarrow \mathbf{r} + \frac{\hbar}{m} \mathbf{k}_U t$ and

$$\psi(\mathbf{r}, t) = a(\mathbf{r}) e^{i\phi(\mathbf{r})} e^{i\mathbf{k}_S \cdot \mathbf{r}} e^{-\frac{i}{\hbar} \left(\mu + \frac{\hbar^2 k_S^2}{2m} \right) t} \text{ and } \mathbf{K} = \mathbf{k}_S - \mathbf{k}_U$$

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$$\begin{aligned} \left[-\frac{\hbar^2}{2m} (\nabla^2 - (\nabla\phi(\mathbf{r}))^2) + \frac{\hbar^2}{m} \mathbf{K} \cdot \nabla\phi(\mathbf{r}) + U(\mathbf{r}) - \mu + \int d^3 \mathbf{r}' V(\mathbf{r} - \mathbf{r}') a^2(\mathbf{r}') \right] a(\mathbf{r}) &= 0 \\ \nabla [a^2(\mathbf{r})(\nabla\phi(\mathbf{r}) + \mathbf{K})] &= 0 \end{aligned}$$

Two-Fluid Model

- ▶ Separate momentum and energy densities with $\mathbf{k}'_S = \mathbf{k}_S - \Delta\mathbf{k}_S$

$$\frac{m}{\hbar} \overline{\mathbf{j}(\mathbf{r})} = n_S \mathbf{k}'_S + n_N \mathbf{k}_U$$

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- Equating and solving gives

$$n_S = n \frac{1}{1 + \delta}, \quad n_N = n \frac{\delta}{1 + \delta}$$

$$\Delta \mathbf{k}_S = -\frac{1 + \delta}{n} \Delta \mathbf{j} - \delta \mathbf{K}, \text{ with } \delta = \frac{n \epsilon_s - \Delta \mathbf{j}^2}{(n \mathbf{K} + \Delta \mathbf{j})^2}$$

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- With hydrodynamics: $n = \overline{a^2}$, $\Delta \mathbf{j} = \overline{a^2 \nabla \phi}$, $\epsilon_s = \overline{a^2 (\nabla \phi)^2}$, $\epsilon_0 = \overline{(\nabla a)^2}$



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Perturbation Theory

- ▶ Expand amplitude and phase with small disorder

$$a = a_0 + a_1 + a_2 + \dots$$

$$\nabla\phi = \nabla\phi_0 + \nabla\phi_1 + \nabla\phi_2 + \dots$$

- ▶ 0th order: due to homogeneity, $a_0(\mathbf{r}) = a_0$ and $\phi_0(\mathbf{r}) = 0$

$$\mu = a_0^2 V(\mathbf{k} = 0)$$

note: $V(\mathbf{k} = 0)$ is angle dependent for dipolar interaction

Nikolić et al., PRA **88**, 013624 (2013)

- ▶ 1st order: the Fourier transforms of solutions

$$a_1(\mathbf{k}) = \frac{-a_0 U(\mathbf{k})}{\frac{\hbar^2 \mathbf{k}^2}{2m} - 2 \frac{\hbar^2}{m} \frac{(\mathbf{Kk})^2}{\mathbf{k}^2} + 2a_0^2 V(\mathbf{k})}$$

$$(\nabla\phi_1)(\mathbf{k}) = 2 \frac{(\mathbf{Kk})\mathbf{k}}{\mathbf{k}^2} \frac{U(\mathbf{k})}{\frac{\hbar^2 \mathbf{k}^2}{2m} - 2 \frac{\hbar^2}{m} \frac{(\mathbf{Kk})^2}{\mathbf{k}^2} + 2a_0^2 V(\mathbf{k})}$$

Perturbation Theory

- ▶ Auxiliary integrals

$$I_{cd} = \frac{\bar{a}_1^2}{\bar{a}_0^2} = \int \frac{d^d \mathbf{k}' R(\mathbf{k}')}{(2\pi)^d} \frac{1}{\left[\frac{\hbar^2 \mathbf{k}'^2}{2m} - 2 \frac{\hbar^2}{m} \frac{(\mathbf{K}\mathbf{k}')^2}{\mathbf{k}'^2} + 2\bar{a}_0^2 V(\mathbf{k}') \right]^2}$$

$$I_{ai} = -2 \frac{\overline{a_1 \nabla \phi_1}}{\bar{a}_0} = \int \frac{d^d \mathbf{k}' R(\mathbf{k}')}{(2\pi)^d} \frac{4 \mathbf{k}' \frac{(\mathbf{K}\mathbf{k}')}{\mathbf{k}'^2}}{\left[\frac{\hbar^2 \mathbf{k}'^2}{2m} - 2 \frac{\hbar^2}{m} \frac{(\mathbf{K}\mathbf{k}')^2}{\mathbf{k}'^2} + 2\bar{a}_0^2 V(\mathbf{k}') \right]^2},$$

$$I_{uv} = \int \frac{d^d \mathbf{k}' R(\mathbf{k}')}{(2\pi)^d} \frac{\mathbf{k}'^2}{\left[\frac{\hbar^2 \mathbf{k}'^2}{2m} - 2 \frac{\hbar^2}{m} \frac{(\mathbf{K}\mathbf{k}')^2}{\mathbf{k}'^2} + 2\bar{a}_0^2 V(\mathbf{k}') \right]^2}$$

Perturbation theory

- ▶ 2nd order, with $\mathbf{I}_1 = \mathbf{I}_{ai} - \mathbf{K}\mathbf{I}_{cd}$, $I_2 = \frac{\hbar^2}{2m}(I_{uv} - \mathbf{K}\mathbf{I}_{ai})$

$$\overline{a_2(\mathbf{k})} = (2\pi)^d \delta(\mathbf{k}) a_0 \frac{I_2 - I_{cd} a_0^2 V(\mathbf{k} \rightarrow 0) - \frac{\hbar^2}{m} \frac{(\mathbf{Kk})(\mathbf{kl}_1)}{\mathbf{k}^2}}{2 \left(a_0^2 V(\mathbf{k} \rightarrow 0) - \frac{\hbar^2}{m} \frac{(\mathbf{Kk})^2}{\mathbf{k}^2} \right)},$$

$$\overline{\nabla \phi_2(\mathbf{k})} = (2\pi)^d \delta(\mathbf{k}) \left(-\frac{\mathbf{k}(\mathbf{kK})}{\mathbf{k}^2} \frac{I_2 - I_{cd} a_0^2 V(\mathbf{k} \rightarrow 0) - \frac{\hbar^2}{m} \frac{(\mathbf{Kk})(\mathbf{kl}_1)}{\mathbf{k}^2}}{a_0^2 V(\mathbf{k} \rightarrow 0) - \frac{\hbar^2}{m} \frac{(\mathbf{Kk})^2}{\mathbf{k}^2}} + \frac{\mathbf{k}(\mathbf{kl}_1)}{\mathbf{k}^2} \right)$$

- ▶ Change in physical observables

$$n = a_0^2 (1 + I_{sd} + 2\overline{a_2}/a_0 + \dots)$$

$$\Delta \mathbf{j} = a_0^2 (-\mathbf{I}_{ai} + \overline{\nabla \phi_2} + \dots)$$

$$n_N = a_0^2 \delta_2 + \dots = a_0^2 \frac{\mathbf{Kl}_{iu}}{\mathbf{K}^2} + \dots$$

$$\Delta \mathbf{k}_S = \mathbf{k}_S - \mathbf{k}'_S = \mathbf{I}_{ai} - \frac{\mathbf{I}_{ai}\mathbf{K}}{\mathbf{K}^2}\mathbf{K} - \overline{\nabla \phi_2} \stackrel{\text{isotropy}}{=} -\overline{\nabla \phi_2}$$



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Superfluid and Condensate Depletion

- ▶ Contact interaction $V(\mathbf{k}) = g$ and δ -correlated disorder $R(\mathbf{k}) = R$

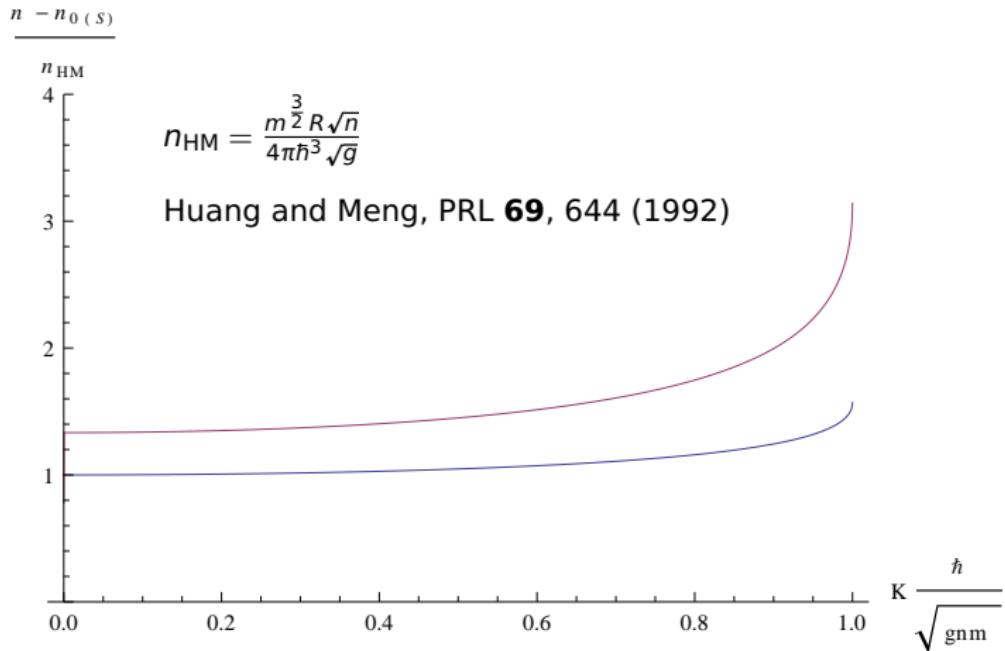


Figure : For $\mathbf{K} = 0$ the ratio between them is $4/3$ and increases to 2 when the superfluid velocity reaches sound velocity $\sqrt{\frac{gn}{m}}$

Regularization problem

- Divergent I_{uv} for vanishing correlation length: $R(\mathbf{k}) = \frac{R}{1+k^2\sigma^2}$

$$\begin{aligned}
 I_{uv} &= \int \frac{d^3 \mathbf{k}' R(\mathbf{k}')}{(2\pi)^3} \frac{\mathbf{k}'^2}{\left[\frac{\hbar^2 \mathbf{k}'^2}{2m} - 2 \frac{\hbar^2}{m} \frac{(\mathbf{K}\mathbf{k}')^2}{\mathbf{k}'^2} + 2a_0^2 V(\mathbf{k}') \right]^2} \\
 &\approx \frac{R m^2 k_c}{\pi \hbar^4} \left[\frac{1}{\sigma k_c} - \frac{3}{2} \left(\sqrt{1 - \left(\frac{K}{k_c} \right)^2} + \frac{k_c}{K} \arcsin \frac{K}{k_c} \right) + \dots \right] \\
 \text{with } k_c &= \frac{\sqrt{mn\sigma}}{\hbar} \text{ being the sound wavevector}
 \end{aligned}$$

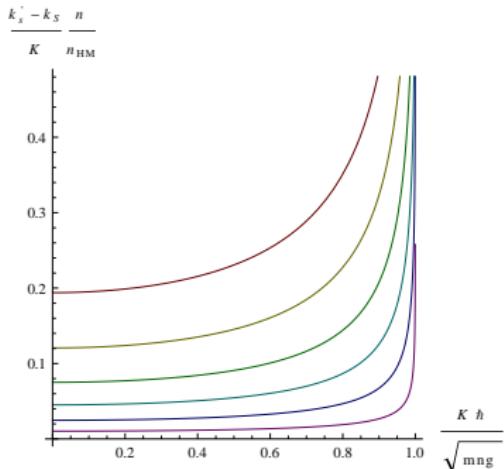
- Schwinger trick: $A^{-n} = \Gamma^{-1}(n) \int_0^\infty dt t^{n-1} e^{-At}$

$$I_{uv} = - \frac{3R m^2 k_c}{2\pi \hbar^4} \left(\sqrt{1 - \left(\frac{K}{k_c} \right)^2} + \frac{k_c}{K} \arcsin \frac{K}{k_c} \right)$$

- Note: in 3D change in the sign accelerates superfluid

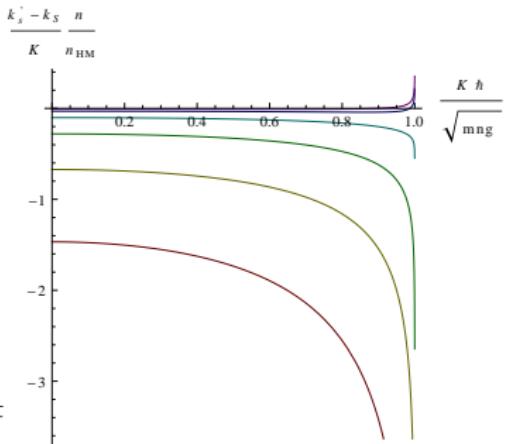
Regularizable quantities

- ▶ Change in the superfluid velocity:



with regularization

$$\sigma \frac{\sqrt{mng}}{\hbar} = 0, \frac{1}{5}, \frac{1}{2}, 1, 2, 5$$



without regularization

$$\sigma \frac{\sqrt{mng}}{\hbar} = \frac{1}{32}, \frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1$$



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Gaussian ansatz

- ▶ Assumption: cumulants of 3rd and higher order are zero
- ▶ Second cumulant in homogeneous systems is

$$G_{aU}(\mathbf{r}) = \overline{a(\mathbf{r} + \mathbf{r}')U(\mathbf{r}')} - \bar{a}\bar{U}$$

- ▶ Example of expansion (with $\bar{U} = 0$)

$$\overline{a(\mathbf{r}_1)a(\mathbf{r}_2)U(\mathbf{r}_3)} = \bar{a}(G_{aU}(\mathbf{r}_2 - \mathbf{r}_3) + G_{aU}(\mathbf{r}_1 - \mathbf{r}_3))$$

- ▶ Restrict to $\mathbf{K} = 0$ and $V(\mathbf{k}) = g$ case

$$GP : \quad \left[-\frac{\hbar^2}{2m} \nabla^2 + U(\mathbf{r}) - \mu + ga^2(\mathbf{r}) \right] a(\mathbf{r}) = 0$$

$$\overline{GP} : \quad G_{aU}(0) - \mu a + gna + 2gG_{aa}(0)a = 0$$

$$\overline{GP \ U(\mathbf{r})} : \quad \left(\frac{\hbar^2 \mathbf{k}^2}{2m} - \mu + 3ng \right) G_{aU}(\mathbf{k}) = -aR(\mathbf{k})$$

$$\overline{GP \ a(\mathbf{r})} : \quad \left(\frac{\hbar^2 \mathbf{k}^2}{2m} - \mu + 3ng \right) G_{aa}(\mathbf{k}) = -aG_{Ua}(\mathbf{k})$$



Self-consistency equations

- ▶ Condensate density a and equation of state $\mu(n)$:

$$a^2 = \frac{n}{1 + I_{aa}}$$

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$$I_{aU} + \mu = gn \frac{1 + 3I_{aa}}{1 + I_{aa}}, \quad I_{aU} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{R(\mathbf{k})}{\frac{\hbar^2\mathbf{k}^2}{2m} - \mu + 3ng}$$

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- ▶ Bose-glass phase:

$$a^b = 0, \quad I_{aa}^b = \infty, \quad \mu^b = 3ng$$

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consistent with renormalization procedure

$$0 = I_{aU}^b = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{R(\mathbf{k})}{\frac{\hbar^2\mathbf{k}^2}{2m}} = \frac{C}{\sigma}$$

Results

- ▶ Plots for condensate density and chemical potential

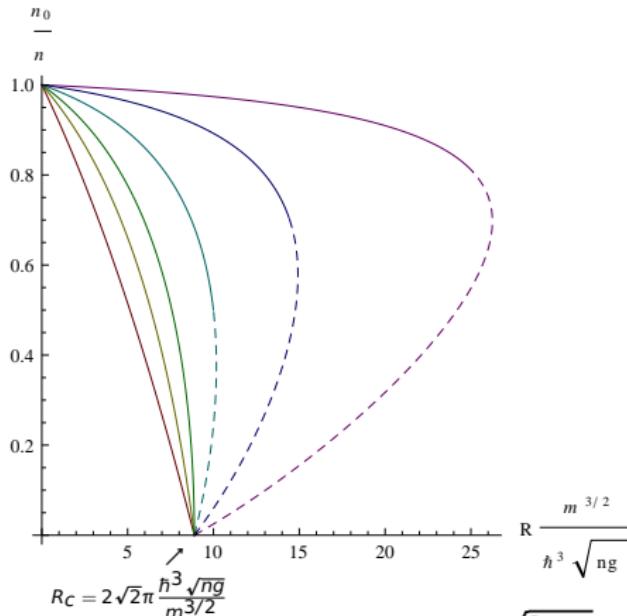
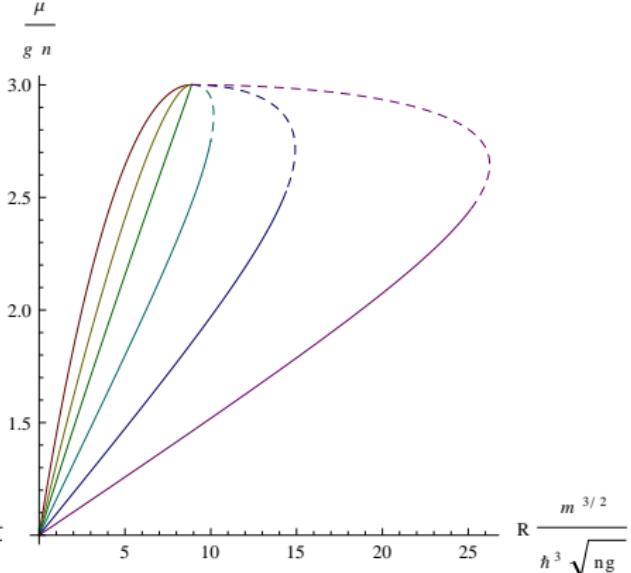


Figure : $\sigma \frac{\sqrt{2mn}g}{\hbar} = 0, 1/4, 1/2, 1, 2$, and 4



Results

- Density and condensate density vs. chemical potential

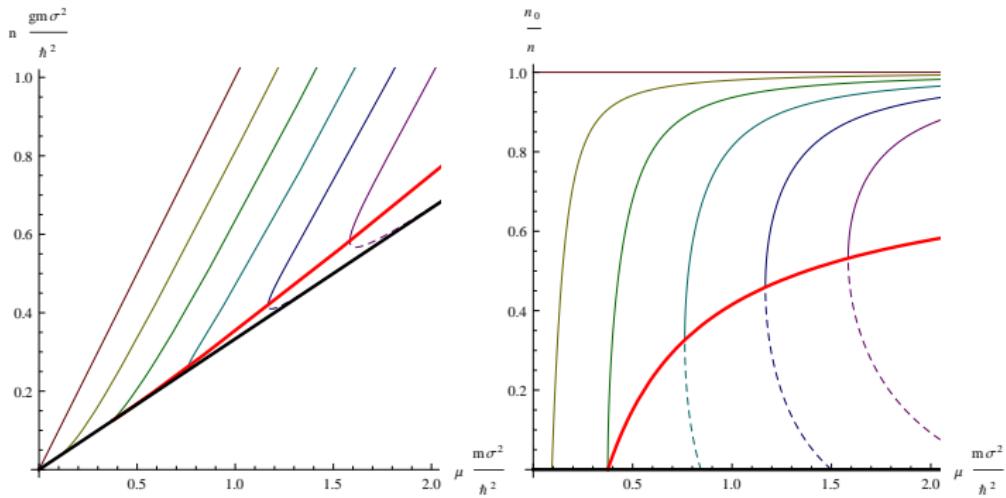


Figure : $R \frac{m^2\sigma}{\hbar^4} = 0, \pi/2, \pi, 3\pi/2, 2\pi$, and $5\pi/2$. Thick black and red lines show Bose glass phase and first-order phase transition, respectively.

- Border of first/second order transition: $R_{1-2}\sigma_{1-2} = \frac{\pi\hbar^4}{m^2}$



Outline

Introduction

Disorder

Mean-Field Hydrodynamic Approach

Weak Disorder

Perturbation Theory

Contact interaction

Strong disorder

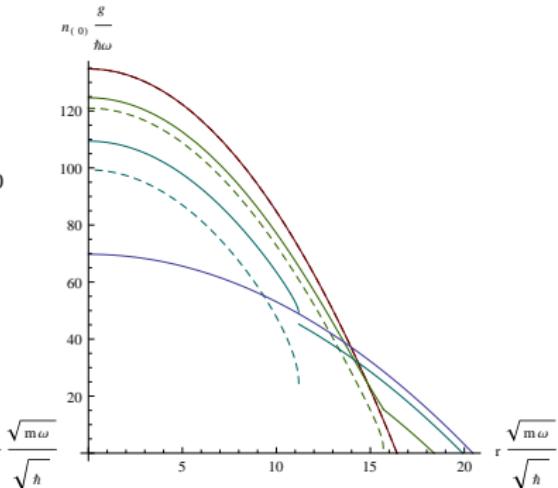
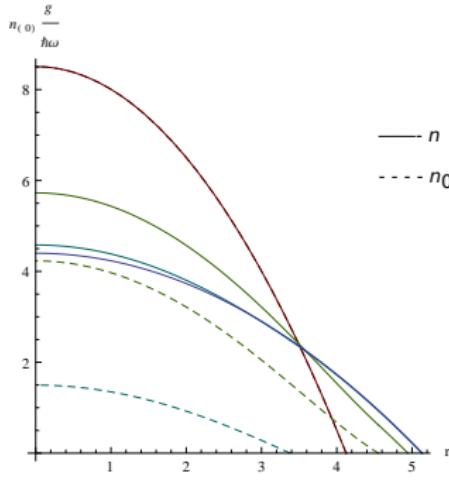
Homogeneous system

Local density approximation

Outlook

Local density approximation

- External potential $V_t(\mathbf{r}) = \frac{1}{2}m\omega^2\mathbf{r}^2$
- $\mu(\mathbf{r}) = \mu_0 - V_t(\mathbf{r})$
- Density (solid) and condensate density (dashed) profiles:



$$\frac{\sigma\sqrt{m\omega}}{\sqrt{\hbar}} = 0, \quad \frac{Ng\sqrt{m^3\omega}}{\sqrt{\hbar^5}} = 1000, \quad \frac{R\sqrt{m^3}}{\sqrt{\hbar^7}\omega} = 0, 7, 14, 21 \quad - \quad \frac{\sigma\sqrt{m\omega}}{\sqrt{\hbar}} = \frac{1}{10}, \quad \frac{Ng\sqrt{m^3\omega}}{\sqrt{\hbar^5}} = 10^6, \quad \frac{R\sqrt{m^3}}{\sqrt{\hbar^7}\omega} = 0, 35, 70, 105$$



- ▶ 1D ring system - (non) perturbatively
- ▶ Superfluidity in strong disorder ($\kappa \neq 0$)
- ▶ Numerical calculation of dipolar BEC in disorder