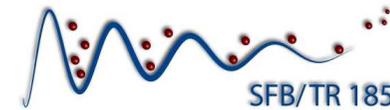


# Superfluidity in Strong Dipolar Quantum Gases

Axel Pelster



From few to many-body physics  
with dipolar quantum gases

DFG FLW

Hannover, Innsbruck, Kaiserslautern, Munich, Stuttgart

1. Introduction
2. Dipolar Fermi Gases
3. Dipolar Bose Gases
4. Outlook

# 1.1 Periodic Table of Elements

1 H Wasserstoff 1s <sup>1</sup> 0.0899 <sup>u</sup> -1.1																	2 He Helium 1s <sup>2</sup> 4.0026 <sup>u</sup> 0						
3 Li Lithium [He]2s <sup>1</sup> 0.53 1	4 Be Beryllium [He]2s <sup>2</sup> 1.85 2																	5 B Bor [He]2s <sup>2</sup> 2p <sup>1</sup> 2.46 3	6 C Kohlenstoff [He]2s <sup>2</sup> 2p <sup>2</sup> 2.26 2.4	7 N Stickstoff [He]2s <sup>2</sup> 2p <sup>3</sup> 1.25* -3.2,3,4,5	8 O Sauerstoff [He]2s <sup>2</sup> 2p <sup>4</sup> 1.43* -2-1	9 F Fluor [He]2s <sup>2</sup> 2p <sup>5</sup> 1.70* -1	10 Ne Neon [He]2s <sup>2</sup> 2p <sup>6</sup> 0.90*
11 Na Natrium [Ne]3s <sup>1</sup> 0.97 1	12 Mg Magnesium [Ne]3s <sup>2</sup> 1.74 2																	13 Al Aluminium [Ne]3s <sup>2</sup> 3p <sup>1</sup> 2.7 (1,2,3)	14 Si Silicium [Ne]3s <sup>2</sup> 3p <sup>2</sup> 2.34 -4.4	15 P Phosphor [Ne]3s <sup>2</sup> 3p <sup>3</sup> 1.83 -3,3,4,5	16 S Schwefel [Ne]3s <sup>2</sup> 3p <sup>4</sup> 2.07 -2,2,4,6	17 Cl Chlor [Ne]3s <sup>2</sup> 3p <sup>5</sup> 3.21* -1,1,3,5,7	18 Ar Argon [Ne]3s <sup>2</sup> 3p <sup>6</sup> 1.78*
19 K Kalium [Ar]4s <sup>1</sup> 0.86 1	20 Ca Calcium [Ar]4s <sup>2</sup> 1.55 2	21 Sc Scandium [Ar]3d <sup>1</sup> 4s <sup>2</sup> 2.99 3	22 Ti Titan [Ar]3d <sup>2</sup> 4s <sup>2</sup> 4.50 2.3,4	23 V Vanadium [Ar]3d <sup>3</sup> 4s <sup>2</sup> 6.11 2.3,4,5	24 Cr Chrom [Ar]3d <sup>5</sup> 4s <sup>1</sup> 7.14 2.3,6	25 Mn Mangan [Ar]3d <sup>5</sup> 4s <sup>2</sup> 7.43 1.2,3,4,6,7	26 Fe Eisen [Ar]3d <sup>6</sup> 4s <sup>2</sup> 7.87 2.3,4,6	27 Co Kobalt [Ar]3d <sup>7</sup> 4s <sup>2</sup> 8.90 2.3	28 Ni Nickel [Ar]3d <sup>8</sup> 4s <sup>2</sup> 8.91 2.3	29 Cu Kupfer [Ar]3d <sup>10</sup> 4s <sup>1</sup> 8.92 1.2	30 Zn Zink [Ar]3d <sup>10</sup> 4s <sup>2</sup> 7.14 2	31 Ga Gallium [Ar]3d <sup>10</sup> 4s <sup>1</sup> 4p <sup>1</sup> 5.90 3	32 Ge Germanium [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>2</sup> 5.72 -3,3,5	33 As Arsen [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>3</sup> 5.72 -3,3,5	34 Se Selen [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>4</sup> 4.82 -2,2,4,6	35 Br Brom [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>5</sup> 3.12 -1,1,3,5,7	36 Kr Krypton [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>6</sup> 3.75*						
37 Rb Rubidium [Kr]5s <sup>1</sup> 1.53 1	38 Sr Strontium [Kr]5s <sup>2</sup> 1.63 2	39 Y Yttrium [Kr]4d <sup>1</sup> 5s <sup>2</sup> 4.47 3	40 Zr Zirkonium [Kr]4d <sup>2</sup> 5s <sup>2</sup> 6.50 2.4	41 Nb Niob [Kr]4d <sup>4</sup> 5s <sup>1</sup> 8.57 2.5	42 Mo Molybdän [Kr]4d <sup>5</sup> 5s <sup>1</sup> 10.28 2.3,4,5,6	43 Tc Technetium [Kr]4d <sup>5</sup> 5s <sup>2</sup> 11.5 -3 bis 7	44 Ru Ruthenium [Kr]4d <sup>7</sup> 5s <sup>1</sup> 12.37 2.3,4,6,8	45 Rh Rhodium [Kr]4d <sup>8</sup> 5s <sup>1</sup> 12.38 1,2,3,4	46 Pd Palladium [Kr]4d <sup>10</sup> 5s <sup>0</sup> 11.99 0,2,4	47 Ag Silber [Kr]4d <sup>10</sup> 5s <sup>1</sup> 10.49 1,2,3	48 Cd Cadmium [Kr]4d <sup>10</sup> 5s <sup>2</sup> 8.65 2	49 In Indium [Kr]4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>1</sup> 7.31 (1,3)	50 Sn Zinn [Kr]4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>2</sup> 5.77 -4,(2,4)	51 Sb Antimon [Kr]4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>3</sup> 6.70 -3,3,5	52 Te Tellur [Kr]4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>4</sup> 6.24 -2,2,4,6	53 I Iod [Kr]4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>5</sup> 4.94 -1,1,3,5,7	54 Xe Xenon [Kr]4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>6</sup> 5.90*						
55 Cs Cäsium [Xe]6s <sup>1</sup> 1.90 1	56 Ba Barium [Xe]6s <sup>2</sup> 3.62 2	72 Hf Hafnium [Xe]4f <sup>14</sup> 5d <sup>2</sup> 6s <sup>2</sup> 13.28 4	73 Ta Tantal [Xe]4f <sup>14</sup> 5d <sup>3</sup> 6s <sup>2</sup> 16.65 5	74 W Wolfram [Xe]4f <sup>14</sup> 5d <sup>4</sup> 6s <sup>2</sup> 19.3 2,3,4,5,6	75 Re Rhenium [Xe]4f <sup>14</sup> 5d <sup>5</sup> 6s <sup>2</sup> 21.0 -2,4,7	76 Os Osmium [Xe]4f <sup>14</sup> 5d <sup>6</sup> 6s <sup>2</sup> 22.59 2,3,4,6,8	77 Ir Iridium [Xe]4f <sup>14</sup> 5d <sup>7</sup> 6s <sup>2</sup> 22.56 1,2,3,4,6	78 Pt Platin [Xe]4f <sup>14</sup> 5d <sup>9</sup> 6s <sup>1</sup> 21.45 0,2,4,6	79 Au Gold [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>1</sup> 19.52 1,3	80 Hg Quecksilber [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup> 13.55 1,2,4	81 Tl Thallium [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup> 6p <sup>1</sup> 11.85 1,3	82 Pb Blei [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup> 6p <sup>2</sup> 11.34 2,4	83 Bi Bismuth [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup> 6p <sup>3</sup> 9.78 (-3),1,3,5	[84] Po Polonium [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup> 6p <sup>4</sup> 9.20 -2,2,4,6	[85] At Astat [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup> 6p <sup>5</sup> -1,1,3,5,7	[86] Rn Radon [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup> 6p <sup>6</sup> 9.73*							
[87] Fr Francium [Rn]7s <sup>1</sup> 1	[88] Ra Radium [Rn]7s <sup>2</sup> 5.5 2	[104] Rf Rutherfordium [Rn]5f <sup>14</sup> 6d <sup>2</sup> 7s <sup>2</sup>	[105] Db Dubnium [Rn]5f <sup>14</sup> 6d <sup>3</sup> 7s <sup>2</sup>	[106] Sg Seaborgium [Rn]5f <sup>14</sup> 6d <sup>4</sup> 7s <sup>2</sup>	[107] Bh Bohrium [Rn]5f <sup>14</sup> 6d <sup>5</sup> 7s <sup>2</sup>	[108] Hs Hassium [Rn]5f <sup>14</sup> 6d <sup>6</sup> 7s <sup>2</sup>	[109] Mt Meitnerium [Rn]5f <sup>14</sup> 6d <sup>7</sup> 7s <sup>2</sup>	[110] Ds Darmstadtium [Rn]5f <sup>14</sup> 6d <sup>8</sup> 7s <sup>1</sup>	[111] Rg Roentgenium [Rn]5f <sup>14</sup> 6d <sup>9</sup> 7s <sup>1</sup>	[112] Cn Copernicium [Rn]5f <sup>14</sup> 6d <sup>10</sup> 7s <sup>1</sup>	[113] Uut Ununtrium [Rn]5f <sup>14</sup> 6d <sup>10</sup> 7s <sup>2</sup> 7p <sup>1</sup>	[114] Uuq Ununquadium [Rn]5f <sup>14</sup> 6d <sup>10</sup> 7s <sup>2</sup> 7p <sup>2</sup>	[115] Uup Ununpentium [Rn]5f <sup>14</sup> 6d <sup>10</sup> 7s <sup>2</sup> 7p <sup>3</sup>	[116] Uuh Ununhexium [Rn]5f <sup>14</sup> 6d <sup>10</sup> 7s <sup>2</sup> 7p <sup>4</sup>	[117] Uus Ununseptium [Rn]5f <sup>14</sup> 6d <sup>10</sup> 7s <sup>2</sup> 7p <sup>5</sup>	[118] Uuo Ununoctium [Rn]5f <sup>14</sup> 6d <sup>10</sup> 7s <sup>2</sup> 7p <sup>6</sup>							
57 La Lanthan [Xe]4f <sup>1</sup> 5d <sup>1</sup> 6s <sup>2</sup> 6.17 3	58 Ce Cer [Xe]4f <sup>1</sup> 5d <sup>1</sup> 6s <sup>2</sup> 6.77 3,4	59 Pr Praseodym [Xe]4f <sup>3</sup> 6s <sup>2</sup> 6.48 3,4	60 Nd Neodym [Xe]4f <sup>4</sup> 6s <sup>2</sup> 7.00 3,4	61 Pm Promethium [Xe]4f <sup>5</sup> 6s <sup>2</sup> 7.2 3	62 Sm Samarium [Xe]4f <sup>6</sup> 6s <sup>2</sup> 7.54 2,3	63 Eu Europium [Xe]4f <sup>7</sup> 6s <sup>2</sup> 5.25 2,3	64 Gd Gadolinium [Xe]4f <sup>7</sup> 5d <sup>1</sup> 6s <sup>2</sup> 7.89 2,3	65 Tb Terbium [Xe]4f <sup>9</sup> 6s <sup>2</sup> 8.25 3,4	66 Dy Dysprosium [Xe]4f <sup>10</sup> 6s <sup>2</sup> 8.56 -3	67 Ho Holmium [Xe]4f <sup>11</sup> 6s <sup>2</sup> 8.78 3	68 Er Erbium [Xe]4f <sup>12</sup> 6s <sup>2</sup> 8.78 3	69 Tm Thulium [Xe]4f <sup>13</sup> 6s <sup>2</sup> 9.32 2,3,4	70 Yb Ytterbium [Xe]4f <sup>14</sup> 6s <sup>2</sup> 6.97 -2,3	71 Lu Lutetium [Xe]4f <sup>14</sup> 6s <sup>2</sup> 9.84 3									
[89] Ac Actinium [Rn]5f <sup>1</sup> 6s <sup>1</sup> 7s <sup>2</sup> 10.07 3	[90] Th Thorium [Rn]5f <sup>0</sup> 6d <sup>2</sup> 7s <sup>2</sup> 11.72 2,3,4	[91] Pa Protoactinium [Rn]5f <sup>0</sup> 6d <sup>1</sup> 7s <sup>2</sup> 15.37 5	[92] U Uran [Rn]5f <sup>3</sup> 6d <sup>1</sup> 7s <sup>2</sup> 19.16 3,4,5,6	[93] Np Neptunium [Rn]5f <sup>4</sup> 6d <sup>1</sup> 7s <sup>2</sup> 20.45 3,4,5,6,7	[94] Pu Plutonium [Rn]5f <sup>6</sup> 6d <sup>1</sup> 7s <sup>2</sup> 19.82 3,4,5,6,7	[95] Am Americium [Rn]5f <sup>7</sup> 6d <sup>1</sup> 7s <sup>2</sup> 13.67 2,3,4,5,6	[96] Cm Curium [Rn]5f <sup>8</sup> 6d <sup>1</sup> 7s <sup>2</sup> 13.51 (2),3,4	[97] Bk Berkelium [Rn]5f <sup>9</sup> 6d <sup>1</sup> 7s <sup>2</sup> 14.78 3,4	[98] Cf Californium [Rn]5f <sup>10</sup> 6d <sup>1</sup> 7s <sup>2</sup> 15.1 (2),3,4	[99] Es Einsteinium [Rn]5f <sup>11</sup> 6d <sup>1</sup> 7s <sup>2</sup> 8.84 (2),3,4	[100] Fm Fermium [Rn]5f <sup>12</sup> 6d <sup>1</sup> 7s <sup>2</sup> 2.3	[101] Md Mendelevium [Rn]5f <sup>13</sup> 6d <sup>1</sup> 7s <sup>2</sup> 2.3	[102] No Nobelium [Rn]5f <sup>14</sup> 6d <sup>1</sup> 7s <sup>2</sup> 2.3	[103] Lr Lawrencium [Rn]5f <sup>14</sup> 6d <sup>1</sup> 7s <sup>2</sup> 3									

quantum degenerate **bosons** and **fermions**

## 1.2 Magnetic versus Electric Dipolar Systems

- **Magnetic systems:**  $C_{\text{dd}}^{\mathcal{B}} = \mu_0 m^2$ , with  $m \sim 1$  to  $10 \mu_B$ 
  - Realized samples
    - Boson:  $^{52}\text{Cr}$  Griesmaier *et al.*, PRL **94**, 160401 (2005)
    - Boson:  $^{87}\text{Rb}$  Vengalattore *et al.*, PRL **100**, 170403 (2008)
    - Fermion:  $^{53}\text{Cr}$  Chicireanu *et al.*, PRA **73**, 053406 (2006)
    - Both: Dy Lu *et al.*, PRL **104**, 063001 (2010); PRL **107**, 190401 (2011)
    - Boson:  $^{168}\text{Er}$  Aikawa *et al.*, PRL **108**, 210401 (2012)
    - Fermion:  $^{167}\text{Er}$  Aikawa *et al.*, Science **345**, 1484 (2014)
  - Bose-nova explosion (Cr), Fermi surface deformation (Er)
- **Electric systems:**  $C_{\text{dd}}^{\mathcal{E}} = d^2/\epsilon_0$ , with  $d \sim 1$  Debye
  - Realized samples (**STIRAP: STImulated Raman Adiabatic Passage**)
    - Fermion:  $^{40}\text{K}^{87}\text{Rb}$  Ospelkaus *et al.*, Science **32**, 231 (2008)
    - Boson:  $^{41}\text{K}^{87}\text{Rb}$  Aikawa *et al.*, NJP **11**, 055035 (2009)
  - Quantum degeneracy ( $^{40}\text{K}^{87}\text{Rb}$ ) De Marco *et al.*, Science **363**, 853 (2019)
- **Ratio:**  $C_{\text{dd}}^{\mathcal{B}}/C_{\text{dd}}^{\mathcal{E}} \approx \alpha^2 \approx 10^{-4}$  ,  $\alpha = e^2/(4\pi\epsilon_0\hbar c) \approx 1/137$

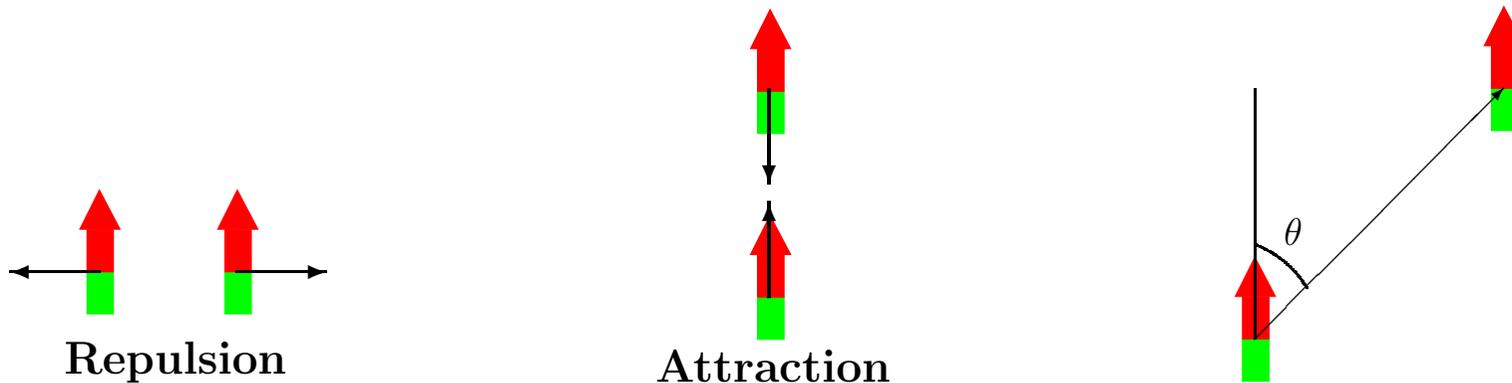
# 1.3 Trapping and Interaction Potentials

- **Harmonic trap:**

$$U_{\text{trap}}(\mathbf{x}) = \frac{M}{2} [\omega_{\perp}^2 (x^2 + y^2) + \omega_z^2 z^2]$$

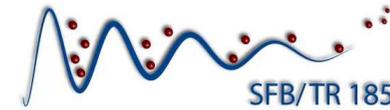
- **Interaction potential:**

$$V_{\text{int}}(\mathbf{x} - \mathbf{x}') = g\delta(\mathbf{x} - \mathbf{x}') + \frac{C_{\text{dd}}}{4\pi|\mathbf{x} - \mathbf{x}'|^3} (1 - 3\cos^2\theta), \quad g = \frac{4\pi\hbar^2 a_s}{M}$$



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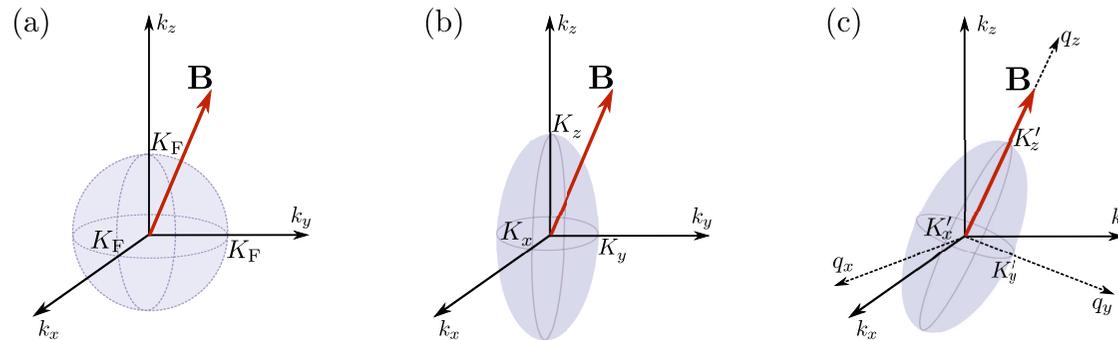
DFG FLW

Hannover, Innsbruck, Kaiserslautern, Munich, Stuttgart

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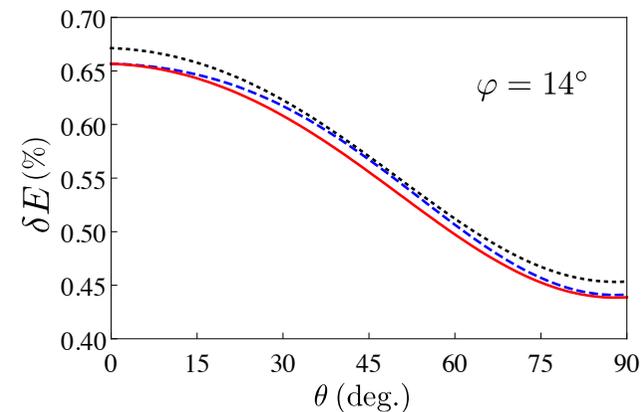
## 2.1 Fermi Surface Deformation (T=0)

- Possible Fermi Surfaces:



- Minimisation of Hartree-Fock energy:

**Fermi surface deformation  
from sphere to ellipsoid  
due to Fock energy**



- Consequences for TOF and collective excitations:

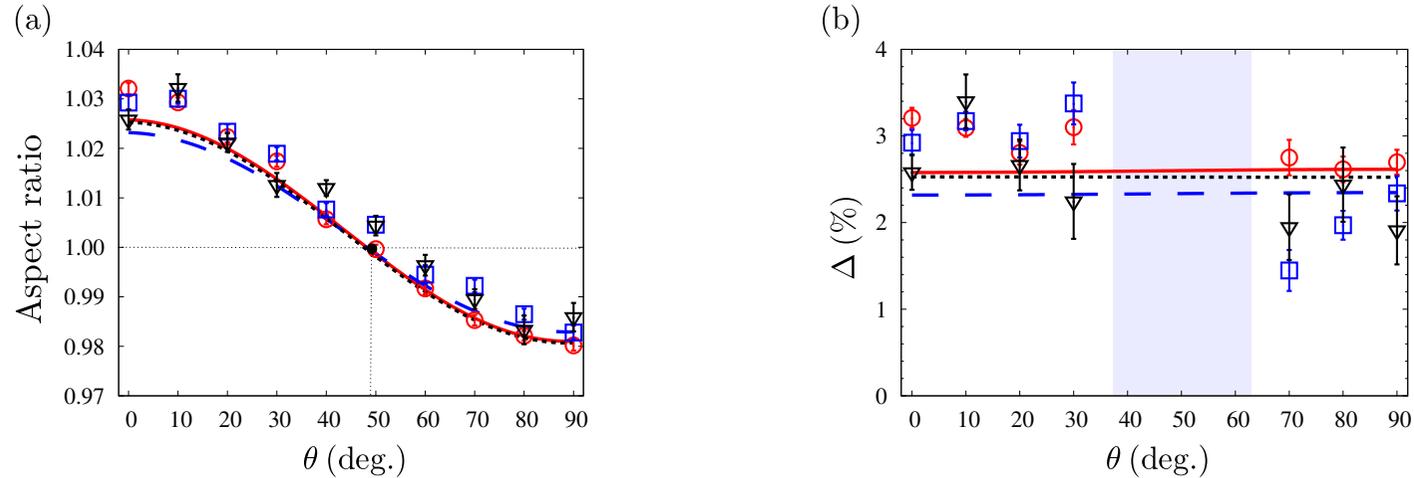
Lima and Pelster, PRA **81**, 021606(R) (2010); PRA **81**, 063629 (2010)

Veljić, Balaž, and Pelster, PRA **95**, 053635 (2017)

Wächtler, Lima, and Pelster, PRA **96**, 043608 (2017)

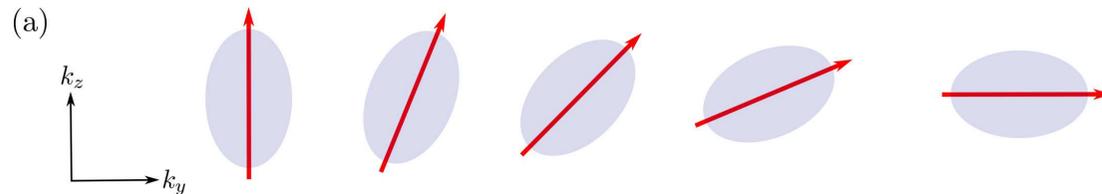
## 2.2 Comparison with Innsbruck Experiment

- Theoretical analysis of Erbium TOF data:

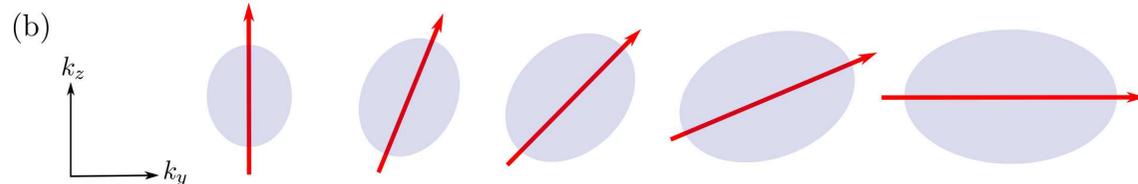


- Schematic Fermi surface deformation:

Magnetic dipolar atoms



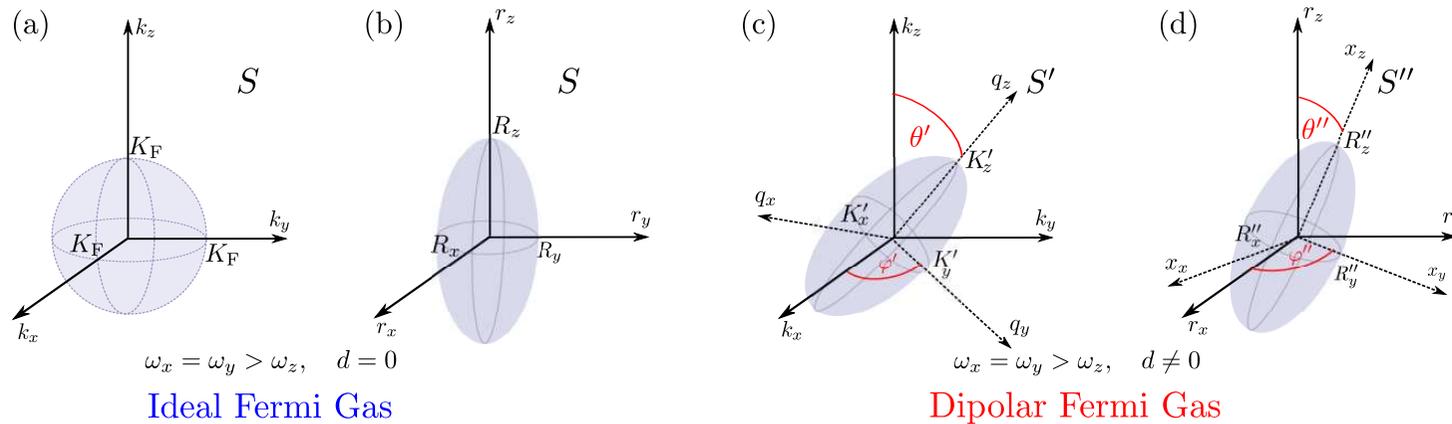
Electric dipolar molecules



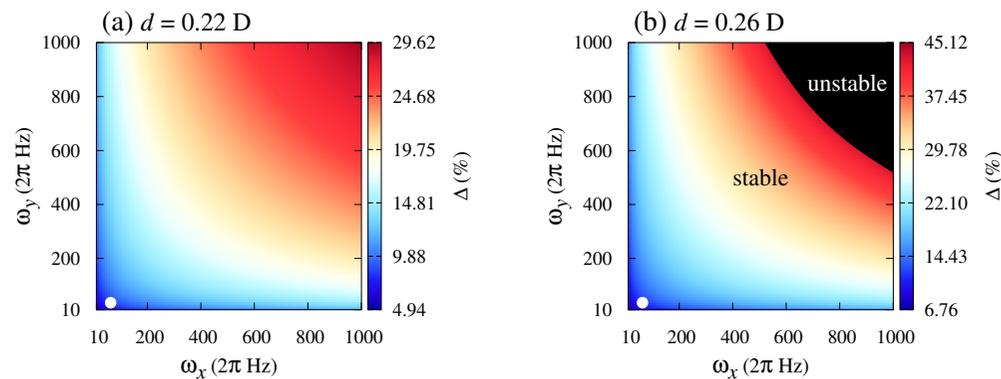
Veljić, Lima, Chomaz, Baier, Mark, Ferlaino, Pelster, and Balaž, NJP **20**, 093016 (2018)

## 2.3 Fermionic Dipolar Molecules

- **Parameters of JILA experiment:** De Marco *et al.*, Science **363**, 853 (2019)  
 $^{40}\text{K}^{87}\text{Rb}$ ,  $N = 3 \cdot 10^4$ ,  $\omega_x = 2\pi \cdot 63 \text{ Hz}$ ,  $\omega_y = 2\pi \cdot 36 \text{ Hz}$ ,  $\omega_z = 2\pi \cdot 200 \text{ Hz}$
- **Rotation of spatial and momentum ellipsoid:**



- **Huge Fermi surface deformation:**



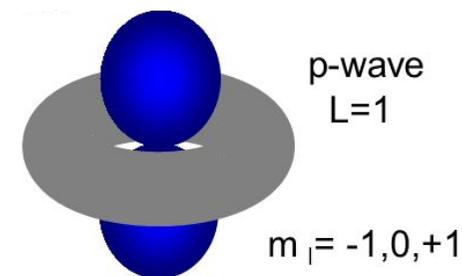
Veljić, Pelster, and Balaž, PRR **1**, 012009 (2019)

## 2.4 Cooper Pairing

- **One component: p-wave superfluidity**

- **Anisotropic order parameter**
- **Hartree-Fock-Bogoliubov theory**

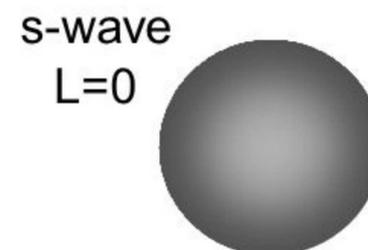
Zhao, Jiang, Liu, Liu, Zou, and Pu, PRA **81**, 063642 (2010)



- **Two components: s-wave superfluidity**

- **Isotropic order parameter**
- **BCS theory**

Shi, Zhang, Sun, and Yi, PRA **82**, 033623 (2010)

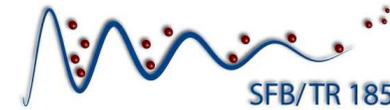


- **Anisotropic superfluidity:**

- **Extend uniform to trapped case**
- **Impact of Fermi surface deformation**
- **Tunability via trap geometry and dipolar orientation**

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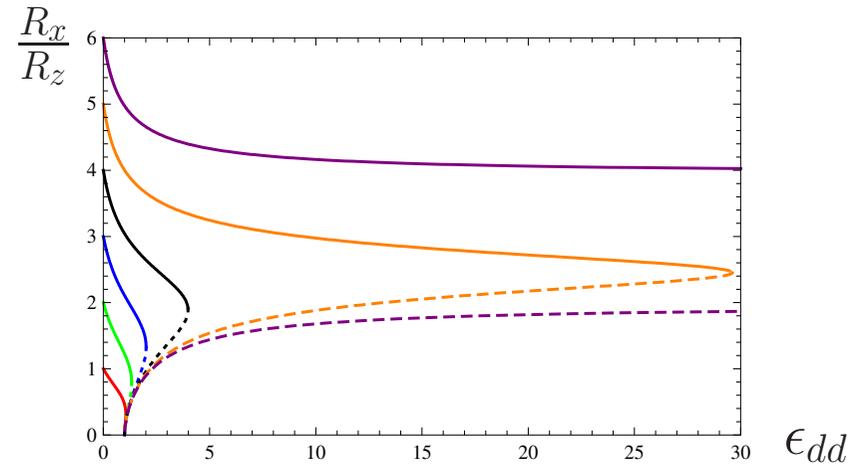
Hannover, Innsbruck, Kaiserslautern, Munich, Stuttgart

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# 3.1 BEC Mean-Field Results (T=0)

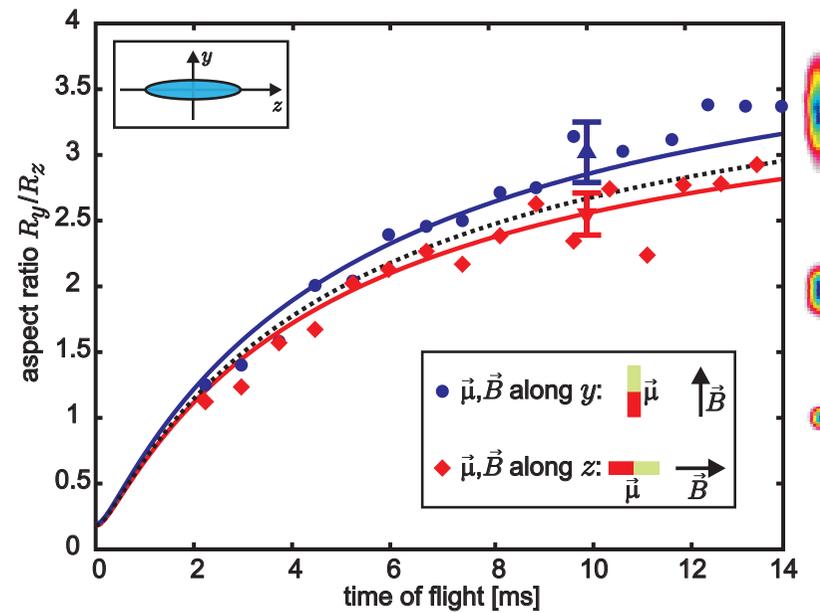
- **Aspect ratio:**

O'Dell *et al.*,  
PRL **92**, 205401 (2004)



- **Time-of-flight:**

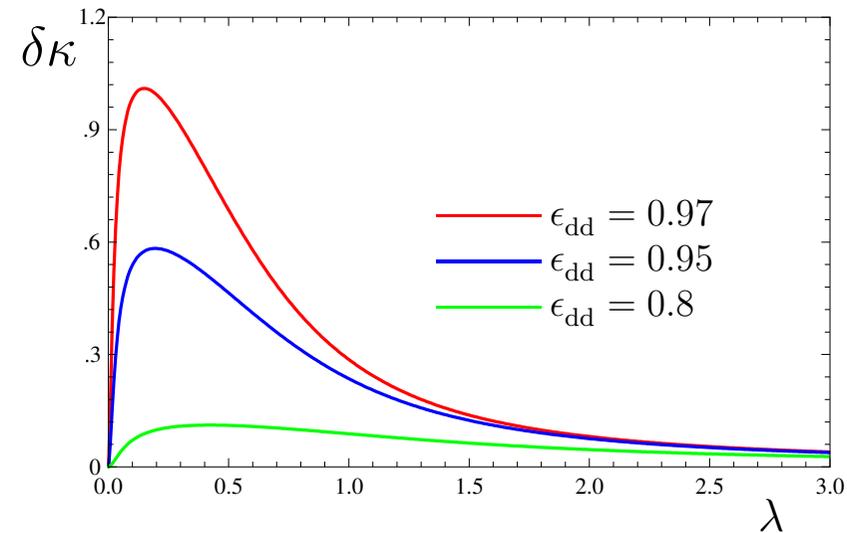
Stuhler *et al.*,  
PRL **95**, 150406 (2005)



## 3.2 BEC Beyond Mean-Field Results (T=0)

- **Aspect ratio:**

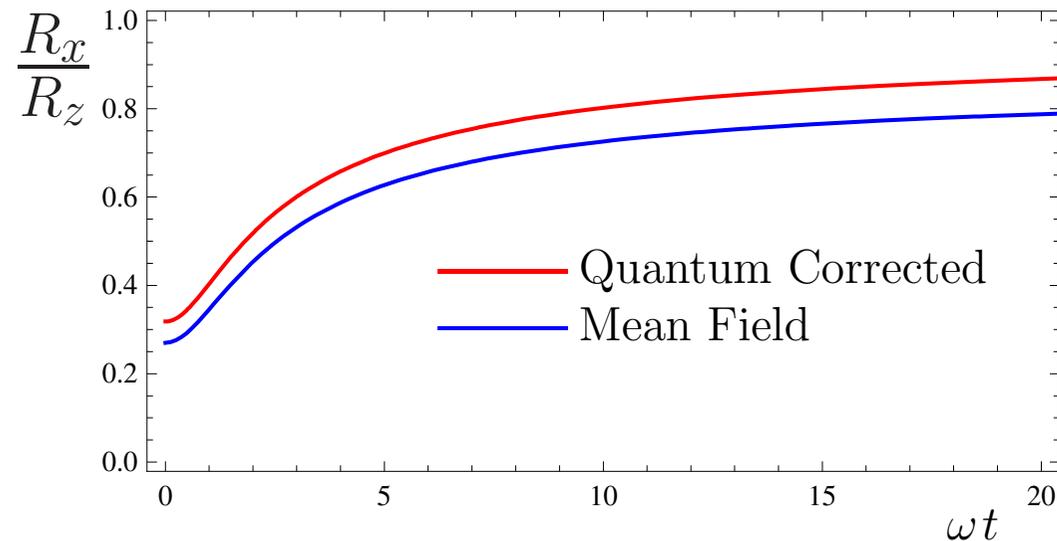
$$\frac{R_x}{R_z} = \kappa_{\text{MF}} (1 + \delta\kappa)$$



- **Time-of-flight:**

expected for Dy

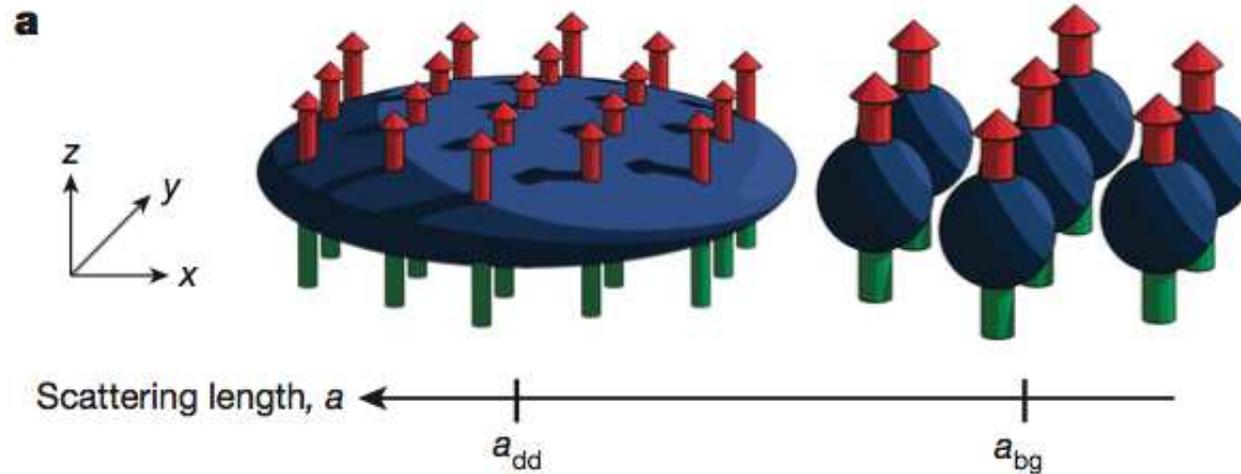
$$\epsilon_{\text{dd}} = 0.9$$



Lima and Pelster, PRA **84**, 041604(R) (2011); PRA **86**, 063609 (2012)

## 3.3 Quantum Droplets - A New State of Matter

- Spontaneous transition from BEC to quantum droplets:



Kadavil *et al.*, Nature **530**, 194 (2016)

- **Three-body interaction?**

Xi and Saito, PRA **93**, 011604(R) (2016)

Bisset and Blakie, PRA **92**, 061603(R) (2015); Blakie, PRA **93**, 033644 (2016)

- **Quantum fluctuations!**

Wächtler and Santos, PRA **93**, 061603(R) (2016)

## 3.4 Theory for Quantum Droplets

- **Extended Gross-Pitaevskii Equation:**

$$i\hbar \frac{\partial \psi(\mathbf{x}, t)}{\partial t} = \left[ -\frac{\hbar^2}{2m} \Delta + U_{\text{trap}}(\mathbf{x}) + \int d^3 x' V_{\text{int}}(\mathbf{x} - \mathbf{x}') |\psi(\mathbf{x}', t)|^2 + V_{\text{QF}}(\mathbf{x}, t) \right] \psi(\mathbf{x}, t)$$

Wächtler and Santos, PRA **93**, 061603(R) (2016)

- **Quantum fluctuations with Local Density Approximation (LDA):**

$$V_{\text{QF}}(\mathbf{x}, t) = \frac{32}{3} g \sqrt{\frac{a_s^3}{\pi}} \mathcal{Q}_5(\epsilon_{\text{dd}}) |\psi(\mathbf{x}, t)|^3$$

Lima and Pelster, PRA **84**, 041604(R) (2011); PRA **86**, 063609 (2012)

- **Supersolidity: coherence between quantum droplets**

**Pisa group:** Tanzi *et al.*, PRL **122**, 130405 (2019)

**Stuttgart group:** Böttcher *et al.*, PRX **9**, 011051 (2019)

**Innsbruck group:** Chomaz *et al.*, PRX **9**, 021012 (2019)

- **Supersolidity: excitation spectra**

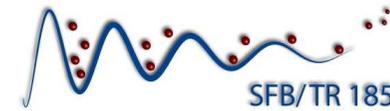
**Pisa group:** Tanzi *et al.*, Nature, 9. September 2019

**Stuttgart group:** Guo *et al.*, Nature, 9. September 2019

**Innsbruck group:** Natale *et al.*, PRL **123**, 050402 (2019)

# Superfluidity in Strong Dipolar Quantum Gases

Axel Pelster



From few to many-body physics  
with dipolar quantum gases

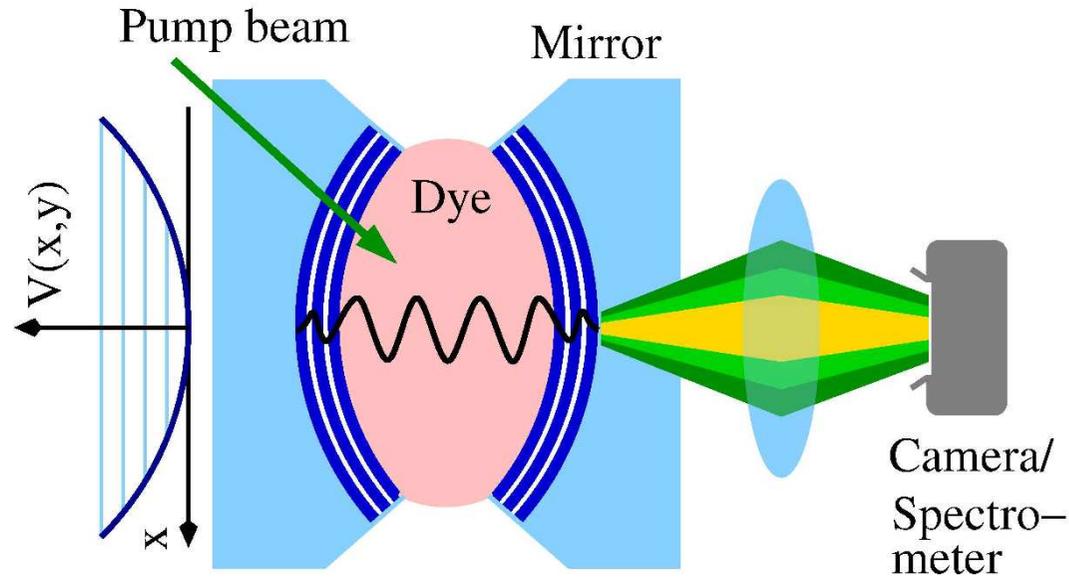
DFG FLW

Hannover, Innsbruck, Kaiserslautern, Munich, Stuttgart

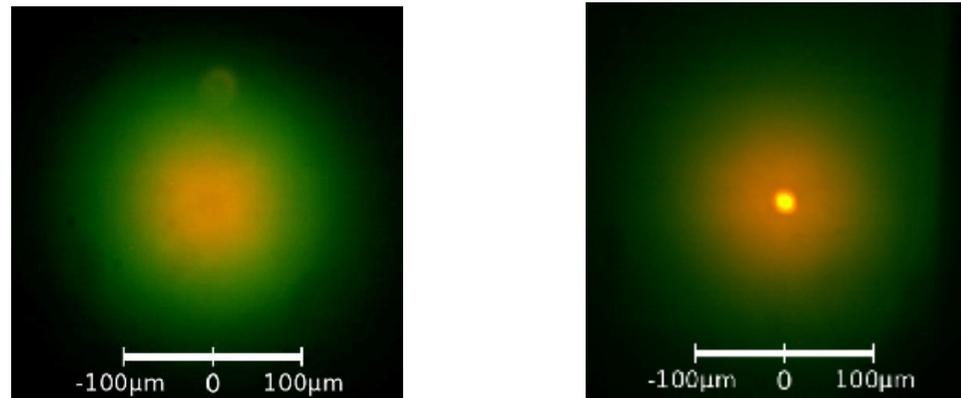
1. Introduction
2. Dipolar Fermi Gases
3. Dipolar Bose Gases
4. Outlook

# 4.1 Bose-Einstein Condensation of Light

Set-Up



Result

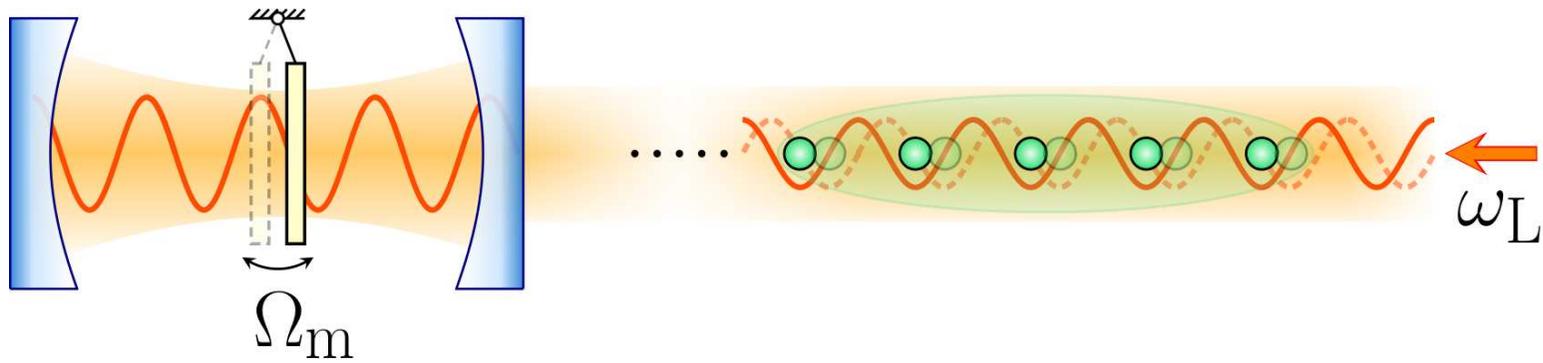


Klaers, Schmitt, Vewinger, and Weitz, *Nature* **468**, 545 (2010)

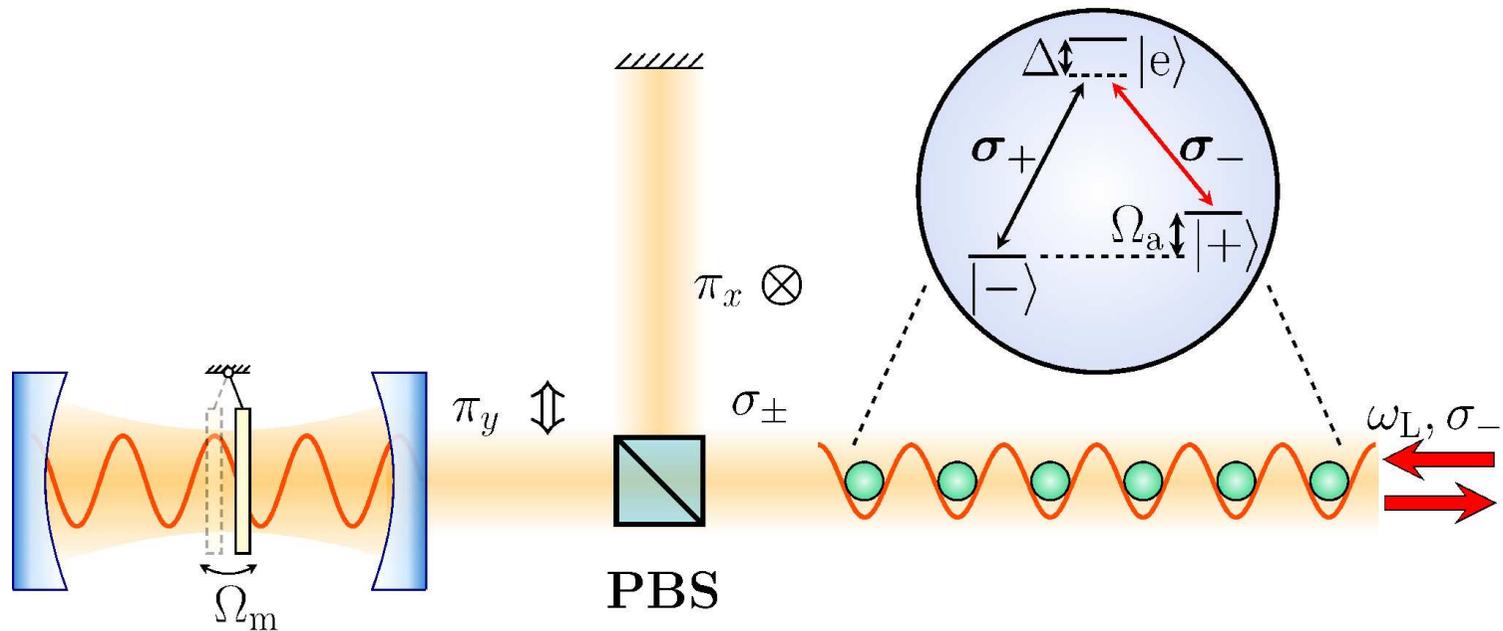
Radonjić, Kopylov, Balaž, and Pelster, *NJP* **20**, 055014 (2018)

Stein, Vewinger, and Pelster, *arXiv:1906.06214*

## 4.2 Hybrid Atom-Optomechanical System



Mann, Bakhtiari, Pelster, and Thorwart, PRL **120**, 063605 (2018)



Mann, Pelster, and Thorwart, arXiv:1810.12846

## 4.3 Mapping Between Quantum Gas Experiments

- **Time transformation:**  $\frac{d\tau(t)}{dt} = \lambda^2(t)$

Jackiw, Ann. Phys. **129**, 183 (1980)

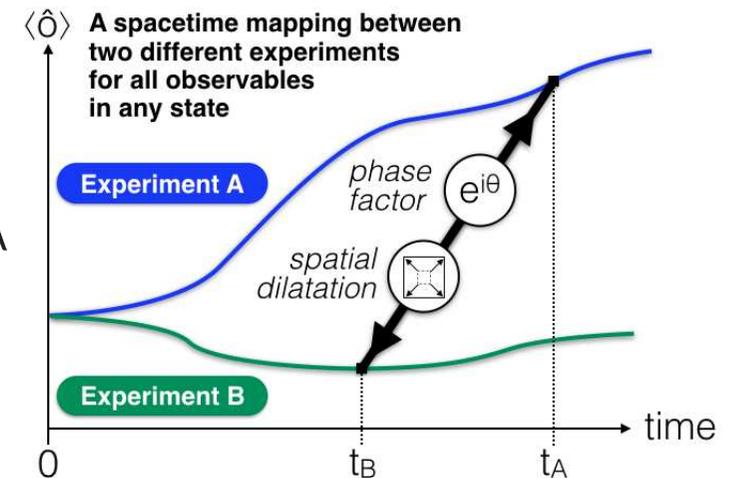
Cai, Inomata, and Wang, PLA **91**, 331 (1982)

- **Transformation formulas:**

$$\hat{\tilde{\psi}}(\mathbf{r}, t) = e^{-\frac{iMn\dot{\lambda}}{2\hbar}r^2} \lambda^{D/2} \hat{\psi}(\lambda\mathbf{r}, \tau(t))$$

$$\tilde{V}(\mathbf{r}, t) = \lambda^2 V(\lambda\mathbf{r}, \tau(t)) + \frac{Mr^2}{2} \lambda^3 \left( \frac{1}{\lambda^2} \frac{d}{dt} \right)^2 \lambda$$

$$\tilde{U}(\mathbf{r}, \mathbf{r}', t) = [\lambda(t)]^2 U(\lambda(t)\mathbf{r}, \lambda(t)\mathbf{r}', \tau(t))$$



Wamba, Pelster, and Anglin, PRA **94**, 043628 (2016)

## 4.4 Master of Science in Advanced Quantum Physics



- **Module topics:**

- Quantum technologies
- Many-body quantum systems
- Laboratory courses
- Research and master thesis

- **Application deadlines:**

	<b>Winter Term</b>	<b>Summer Term</b>
if visa required	April 30	October 31
if no visa required	July 15	Januar 15

- **Further information:**

[www.physik.uni-kl.de/quantum-master](http://www.physik.uni-kl.de/quantum-master)