Ultracold Quantum Gases – A Fascinating Playground for Basic Research in Physics

Axel Pelster

SFB/Transregio 49

Frankfurt – Kaiserslautern - Mainz

Condensed matter systems with variable many-body interactions

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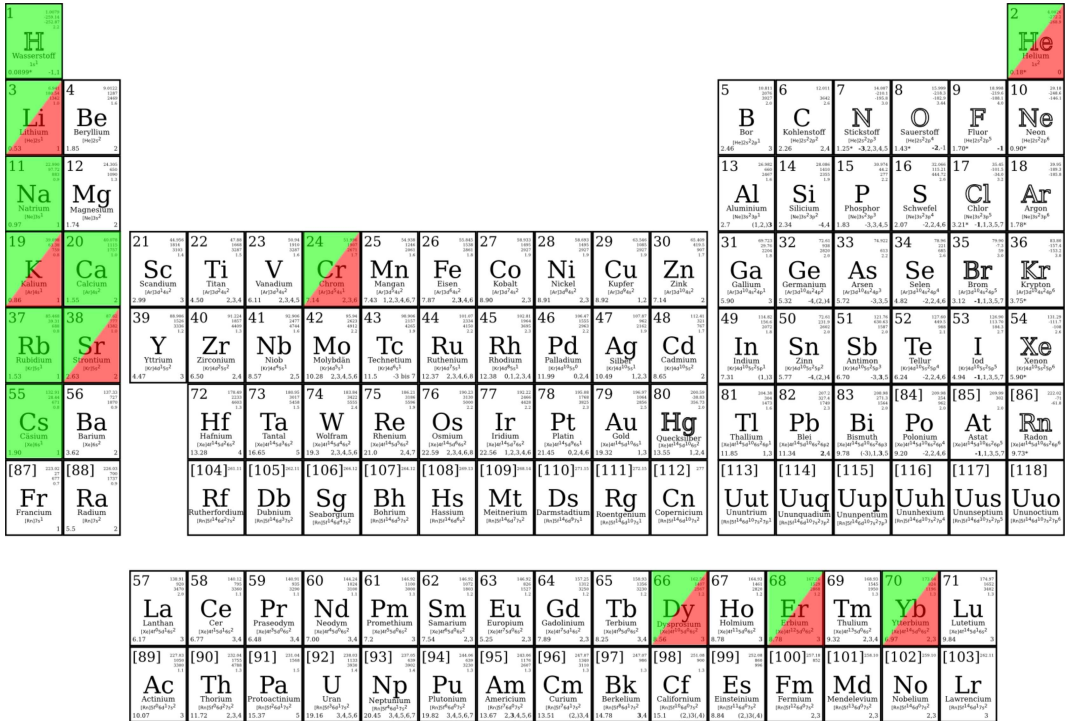

quantum degenerate **bosons** and **fermions**

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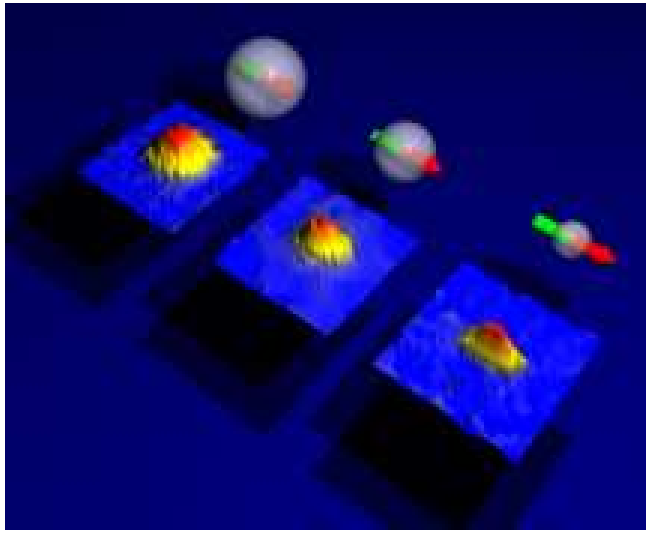
1. Dipolar Bose-Einstein Condensates
2. On the Dirty Boson Problem
3. Anisotropic Superfluidity
4. Bosons in Optical Lattices
5. Conclusion



1.1 Magnetic versus Electric Dipolar Systems

- **Magnetic systems:** $C_{\rm dd}^{\mathcal{B}} = \mu_0 m^2$, with $m \sim 1$ to $10\ \mu_B$
  - Realized samples

    Boson: ^{52}Cr Griesmaier *et al.*, PRL **94**, 160401 (2005)

    Boson: ^{87}Rb Vengalattore *et al.*, PRL **100**, 170403 (2008)

    Fermion: ^{53}Cr Chicireanu *et al.*, PRA **73**, 053406 (2006)

    Both: Dy Lu *et al.*, PRL **104**, 063001 (2010); PRL **107**, 190401 (2011)

    Boson: ^{168}Er Aikawa *et al.*, PRL **108**, 210401 (2012)

    Fermion: ^{167}Er Aikawa *et al.*, Science **345**, 1484 (2014)
  - Effects: Bose-nova explosion (Cr), Fermi surface deformation (Er)

- **Electric systems:** $C_{\rm dd}^{\mathcal{E}} = d^2/\epsilon_0$, with $d \sim 1$ Debye
  - Realized samples **(STIRAP: STImulated Raman Adiabatic Passage)**

    Fermion: ^{40}K^{87}Rb Ospelkaus *et al.*, Science **32**, 231 (2008)

    Boson: ^{41}K^{87}Rb Aikawa *et al.*, NJP **11**, 055035 (2009)
  - Effects: thermalization (^{40}K^{87}Rb)

- **Ratio:** $C_{\rm dd}^{\mathcal{B}}/C_{\rm dd}^{\mathcal{E}} \approx \alpha^2 \approx 10^{-4}$ (α: Sommerfeld fine-structure constant)

1.2 Trapping and Interaction Potentials

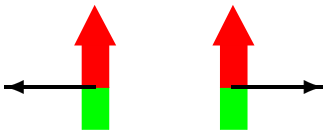
- **Harmonic trap:**

$$U_{\text{trap}}(\mathbf{x}) = \frac{M}{2}\omega^2\left(x^2 + y^2 + \lambda^2 z^2\right)$$

- **Interaction potential:**

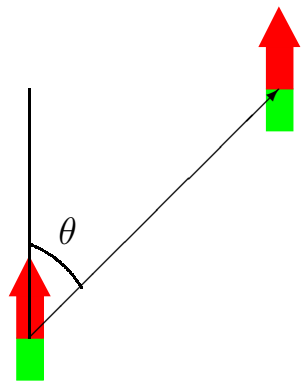
$$V_{\text{int}}(\mathbf{x}-\mathbf{x}') = g\left[\delta(\mathbf{x}-\mathbf{x}') + \frac{3\epsilon_{\text{dd}}}{4\pi|\mathbf{x}-\mathbf{x}'|^3}\left(1 - 3\cos^2\theta\right)\right]$$

$$g = \frac{4\pi\hbar^2 a}{M}, \qquad \epsilon_{\text{dd}} = \frac{C_{\text{dd}}}{3g}$$



Repulsion

Attraction

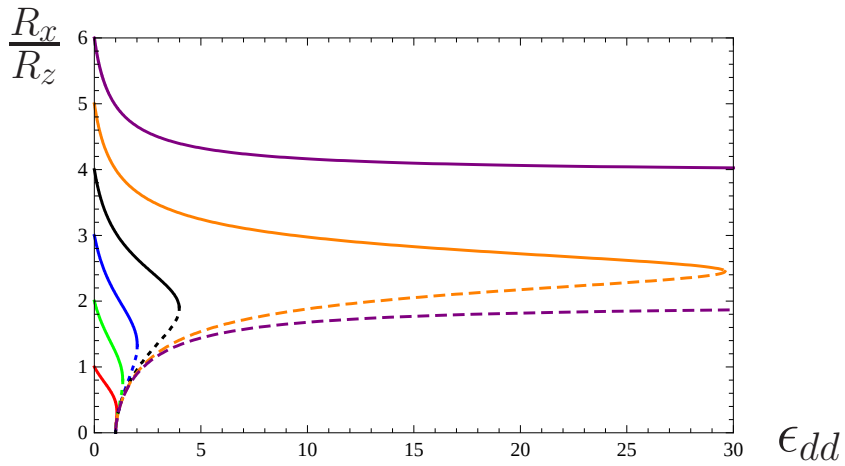


1.3 Mean-Field Results (T=0)

- **Aspect Ratio:**

  Eberlein *et al.*,

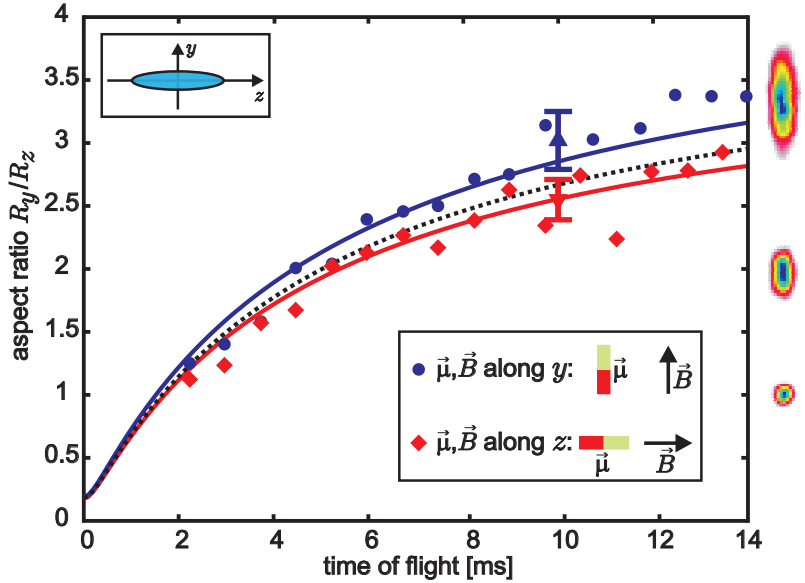
  PRA **71**, 033618 (2005)



- **Time-of-Flight:**

  Stuhler *et al.*,

  PRL **95**, 150406 (2005)

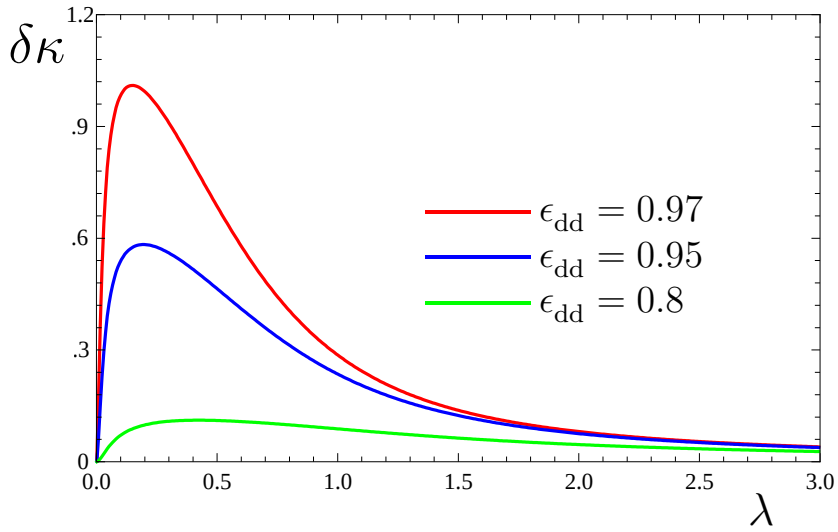


see also: Glaum, Pelster, Kleinert, and Pfau, PRL **98**,080407 (2007)

1.4 Beyond Mean-Field Results (T=0)

- **Aspect Ratio:**

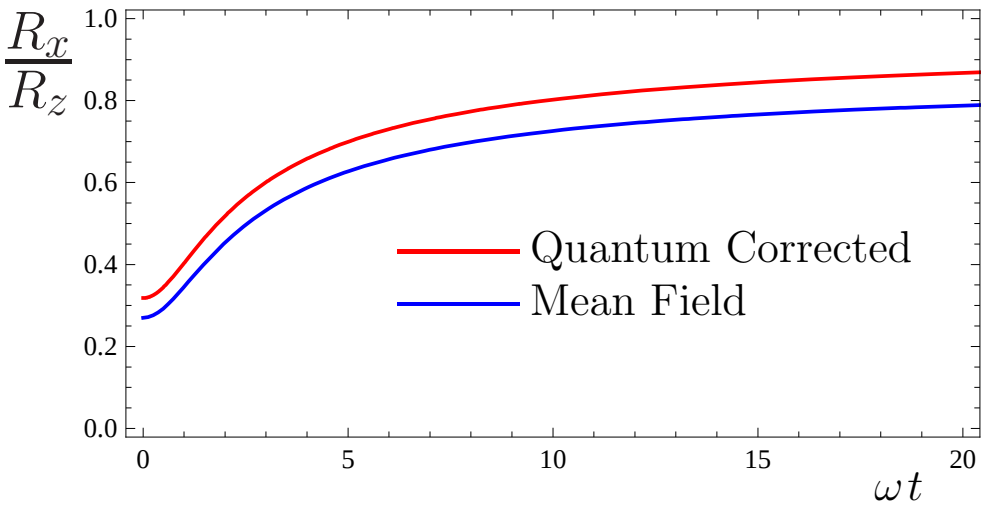
$$\frac{R_x}{R_z} = \kappa_{\mathrm{MF}}\left(1 + \delta\kappa\right)$$



- **Time-of-Flight:**

expected for Dy

$\epsilon_{dd} = 0.9$



Lima and Pelster, PRA **84**, 041604(R) (2011); PRA **86**, 063609 (2012)

Rosensweig instability in Dy BEC, Pfau group: `arXiv:1508.05007`

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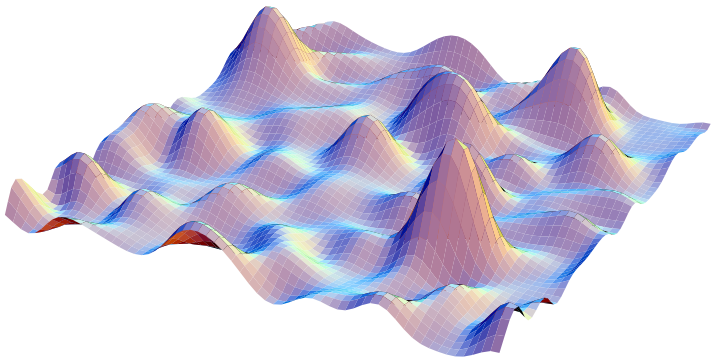
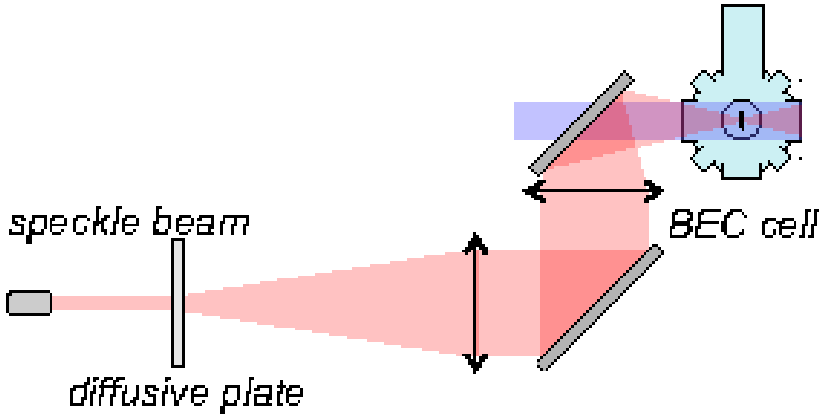
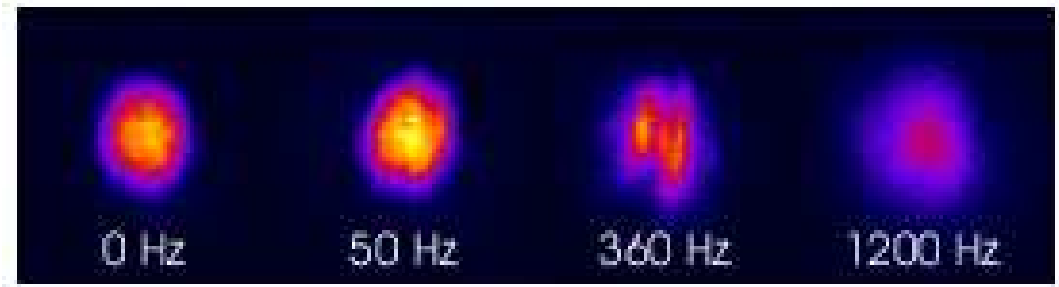


2.1 Laser Speckles: Controlled Randomness

Experimental Set-Up:

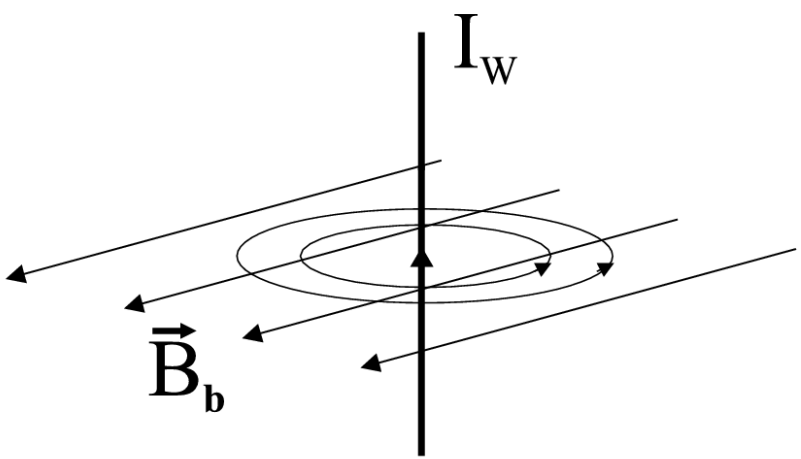


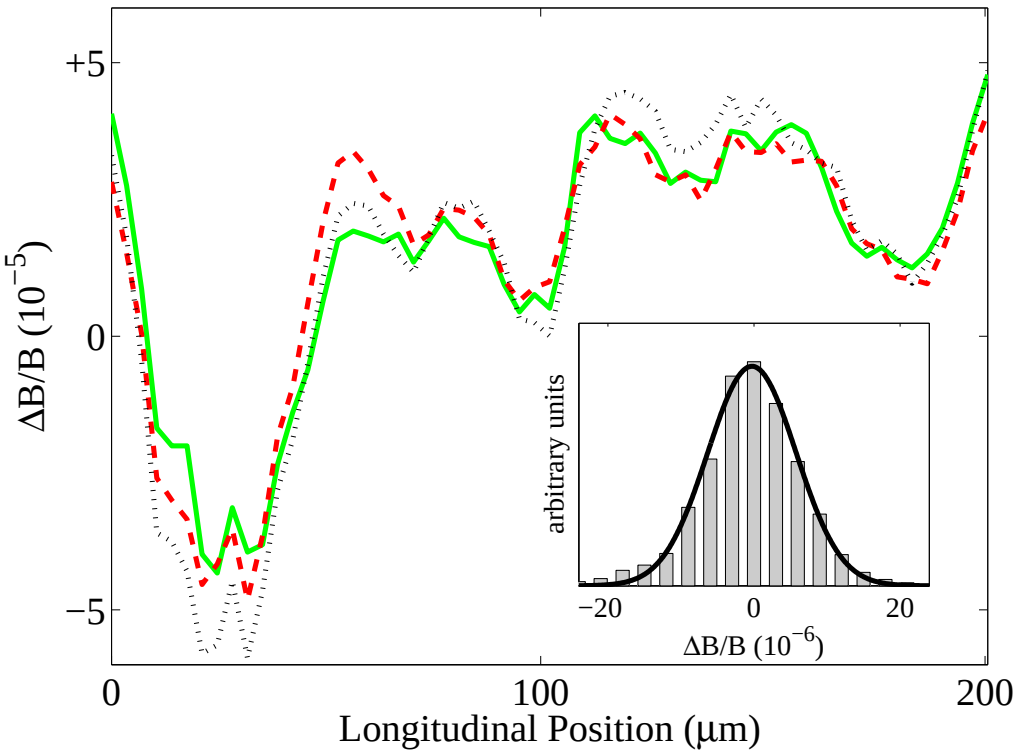
Fragmentation:



Lye *et al.*, PRL **95**, 070401 (2005)

2.2 Wire Trap: Undesired Randomness





Distance: $d = 10\ \mu$m

Wire Width: $100\ \mu$m

Magnetic Field: 10 G, 20 G, 30 G

Deviation: $\Delta B/B \approx 10^{-4}$

Krüger *et al.*, PRA **76**, 063621 (2007)

Fortàgh and Zimmermann, RMP **79**, 235 (2007)

2.3 Bogoliubov Theory of Dirty Bosons

Assumptions:

homogeneous Bose gas: $U(\mathbf{x}) = 0$

δ-correlated disorder: $R(\mathbf{x}) = R\,\delta(\mathbf{x})$

Condensate Depletion:

$$n_0 = n - \frac{8}{3\sqrt{\pi}}\sqrt{a\,n_0}^{\,3} - \frac{M^2 R}{8\pi^{3/2}\hbar^4}\sqrt{\frac{n_0}{a}}$$

Superfluid Depletion:

$$n_s = n - n_n = n - \frac{4}{3}\frac{M^2 R}{8\pi^{3/2}\hbar^4}\sqrt{\frac{n_0}{a}}$$

Huang and Meng, PRL **69**, 644 (1992)

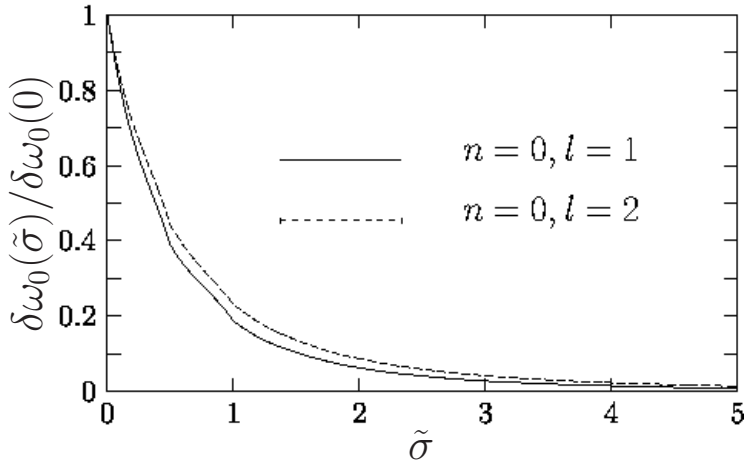
Falco, Pelster, and Graham, PRA **75**, 063619 (2007)

2.4 Collective Excitations

Typical Values:

Lye *et al.*, PRL **95**, 070401 (2005)

$$\left.\begin{array}{rcl} \sigma & = & 10\ \mu\text{m} \\ R_{\text{TF}} & = & 100\ \mu\text{m} \\ l_{\text{HO}} & = & 10\ \mu\text{m} \end{array}\right\} \quad \tilde{\sigma} = \frac{\xi R_{\text{TF}}}{l_{\text{HO}}^2\sqrt{2}} \approx 7$$



$\Longrightarrow$ **Disorder effect vanishes in laser speckle experiment**

Improvement:

laser speckle setup with correlation length $\sigma = 1\ \mu$m

Aspect *et al.*, NJP **8**, 165 (2006)

$\Longrightarrow$ **Disorder effect should be measurable**

Falco, Pelster, and Graham, PRA **76**, 013624 (2007)

2.5 Hartree-Fock Mean-Field Theory: Replica Symmetry

Phase Classification: $n = n_0 + q + n_{\rm th}$

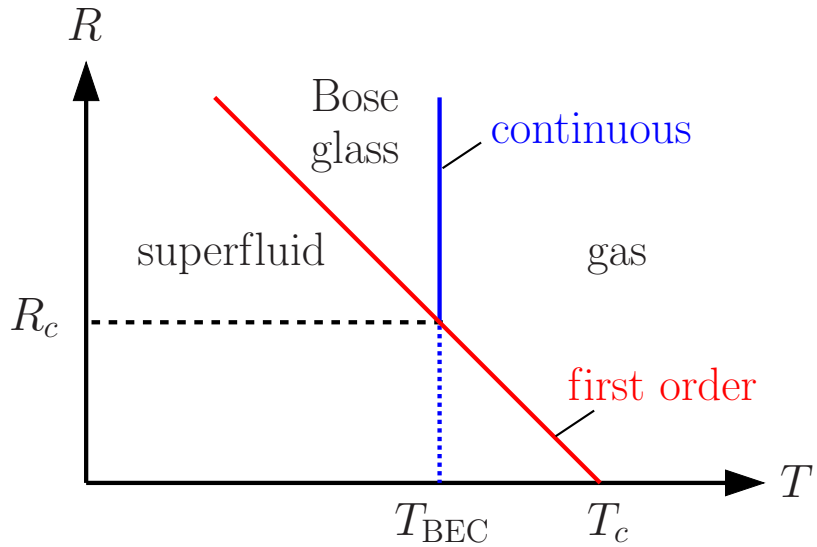
$$\lim_{|\mathbf{x}-\mathbf{x}'|\to\infty} \overline{\langle\psi(\mathbf{x},\tau)\psi^*(\mathbf{x}',\tau)\rangle} = n_0$$

$$\lim_{|\mathbf{x}-\mathbf{x}'|\to\infty} \overline{|\langle\psi(\mathbf{x},\tau)\psi^*(\mathbf{x}',\tau)\rangle|^2} = (n_0+q)^2$$

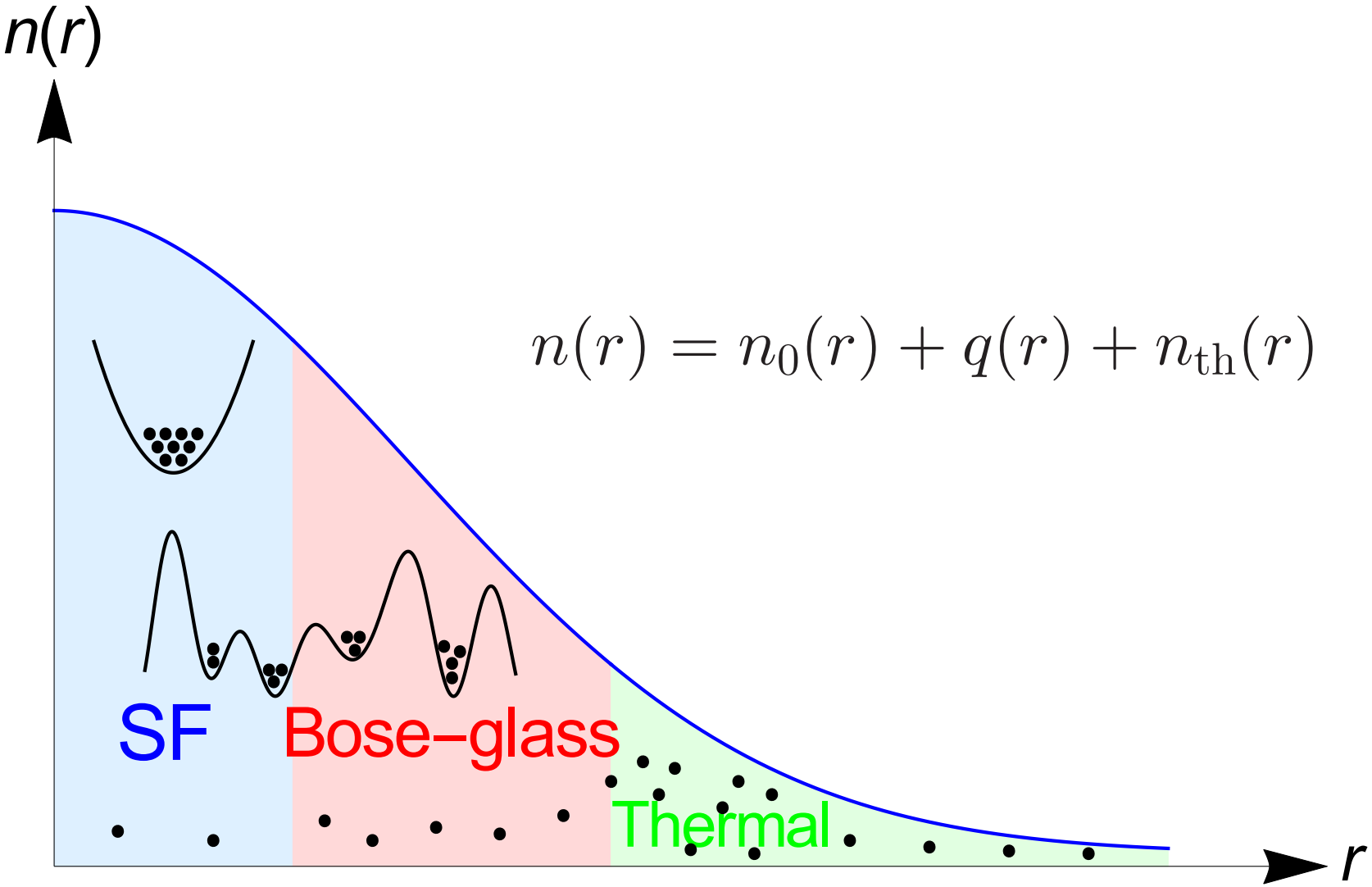
thermal gas	Bose-glass	superfluid
$q = n_0 = 0$	$q > 0, n_0 = 0$	$q > 0, n_0 > 0$

Phase Diagram:

Graham and Pelster, IJBC **19**, 2745 (2009)



2.6 Trapped Dirty Bose-Einstein-Condensate



$$n(r) = n_0(r) + q(r) + n_{\mathrm{th}}(r)$$

Khellil, Balaž, and Pelster, `arXiv:1510.04985`

Khellil and Pelster, in preparation

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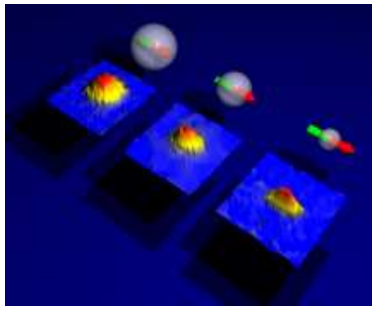
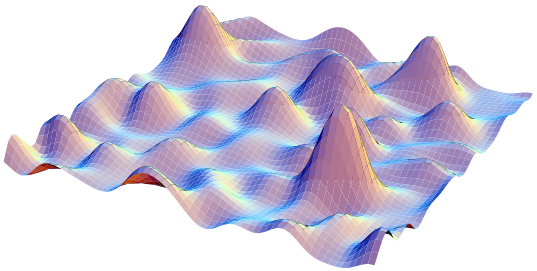




3.1 Superfluid Density as Tensor

- **Linear response theory:**

$$p_i = VM\left(n_{\mathrm{n}ij}v_{\mathrm{n}j} + n_{\mathrm{s}ij}v_{\mathrm{s}j}\right) + \ldots$$

M. Ueda, *Fundamentals and New Frontiers of Bose-Einstein Condensation* (2010)

- **Spin-orbit coupling:**
  $\Longrightarrow$ **Elliptic vortices**

  Devreese, Tempere, and Sá de Melo, PRL **113**, 165304 (2014)

- **Dipolar interaction at finite temperature:**
  $\Longrightarrow$ **Directional dependence of first and second sound velocity**

  Ghabour and Pelster, PRA **90**, 063636 (2014)

  Ghabour and Pelster, in preparation

- **Dipolar interaction and isotropic disorder at zero temperature:**

  Krumnow and Pelster, PRA **84**, 021608(R) (2011)

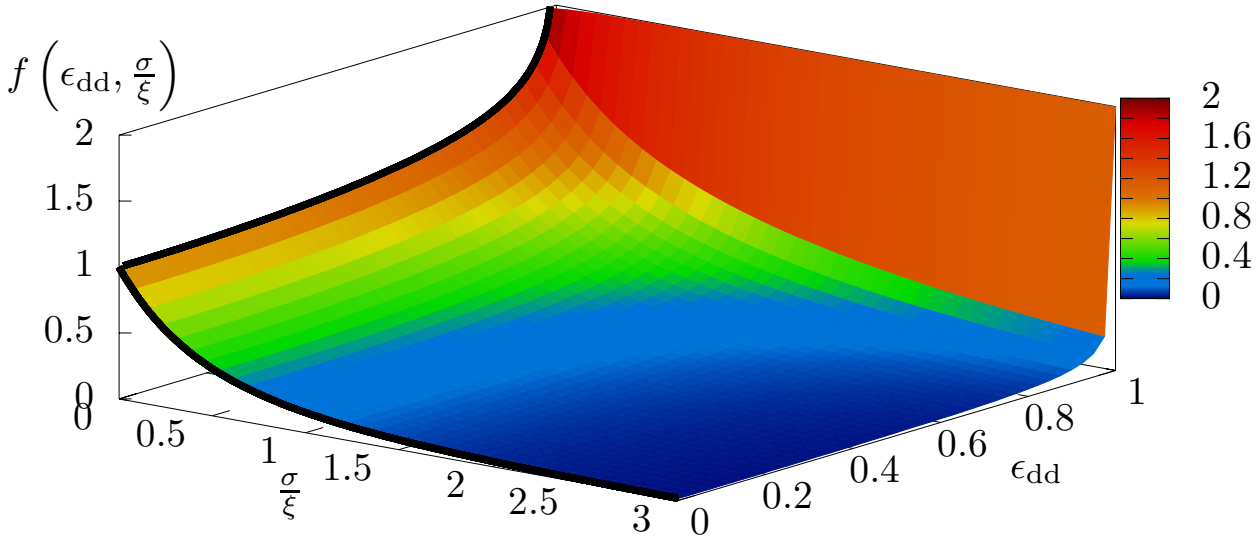
  Nikolić, Balaž, and Pelster, PRA **88**, 013624 (2013)

3.2 Condensate Depletion

$$n - n_0 = n_{\mathrm{HM}}\, f\left(\epsilon_{\mathrm{dd}}, \frac{\sigma}{\xi}\right)$$

Coherence length: $$\xi = \frac{1}{\sqrt{8\pi a n}}$$

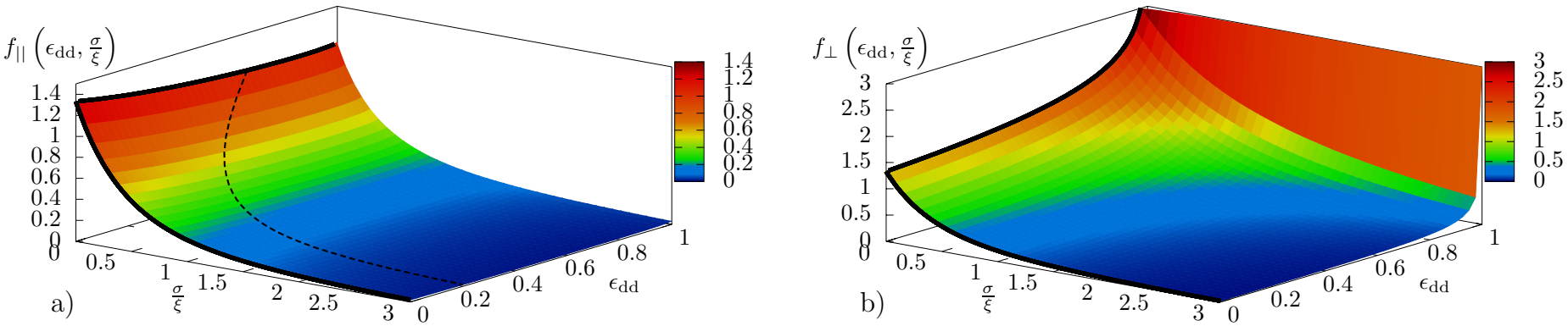
Huang-Meng depletion: $$n_{\mathrm{HM}} = \frac{M^2 R}{8\pi^{3/2}\hbar^4}\sqrt{\frac{n_0}{a}}$$



Krumnow and Pelster, PRA **84**, 021608(R) (2011)

3.3 Superfluid Depletion

$$n - n_{\mathsf{S}} = n_{\mathrm{HM}} \begin{pmatrix} f_{\perp}\left(\epsilon_{\mathrm{dd}}, \frac{\sigma}{\xi}\right) & 0 & 0 \\ 0 & f_{\perp}\left(\epsilon_{\mathrm{dd}}, \frac{\sigma}{\xi}\right) & 0 \\ 0 & 0 & f_{||}\left(\epsilon_{\mathrm{dd}}, \frac{\sigma}{\xi}\right) \end{pmatrix}$$



Krumnow and Pelster, PRA **84**, 021608(R) (2011)

$\Longrightarrow$ **Directional speed of sound: Bragg spectroscopy**

Graham and Pelster, IJBC **19**, 2745 (2009)

$\Longrightarrow$ **Finite localization time**

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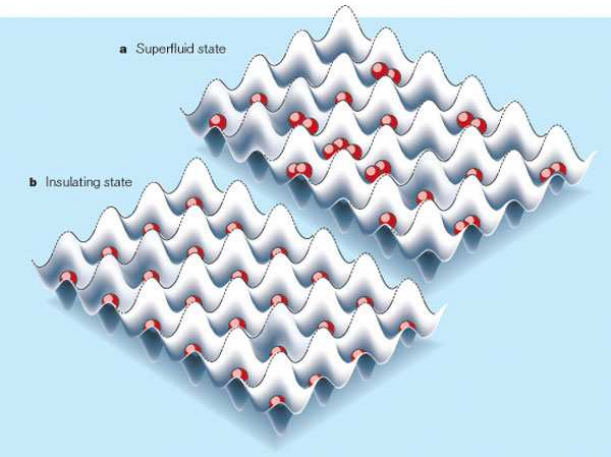
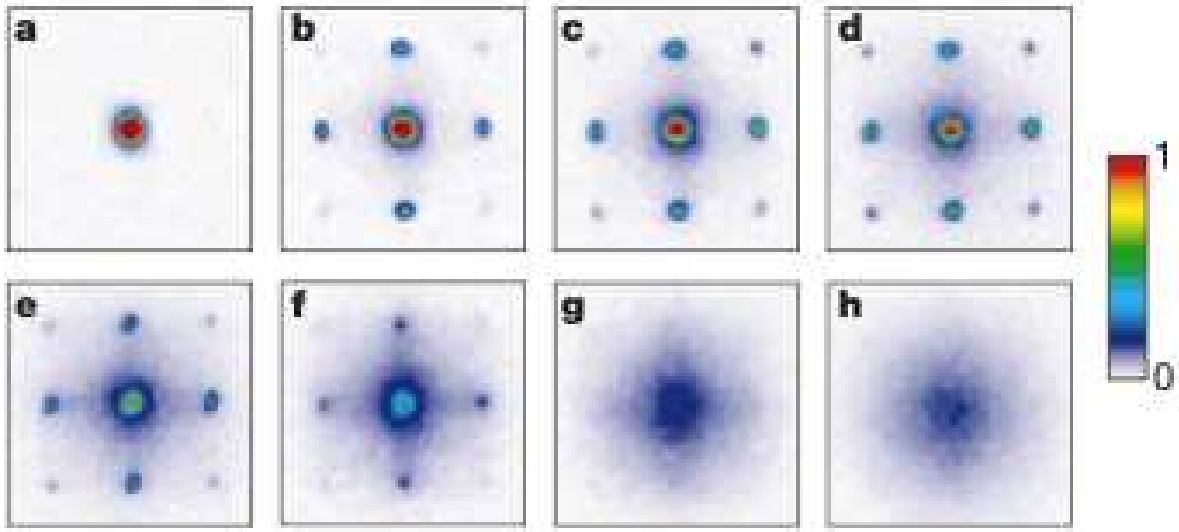


4.1 Time-of-Flight Absorption Pictures

- **Superfluid phase:**
  delocalization in space, localization in Fourier space

- **Mott phase:**
  localization in space, delocalization in Fourier space

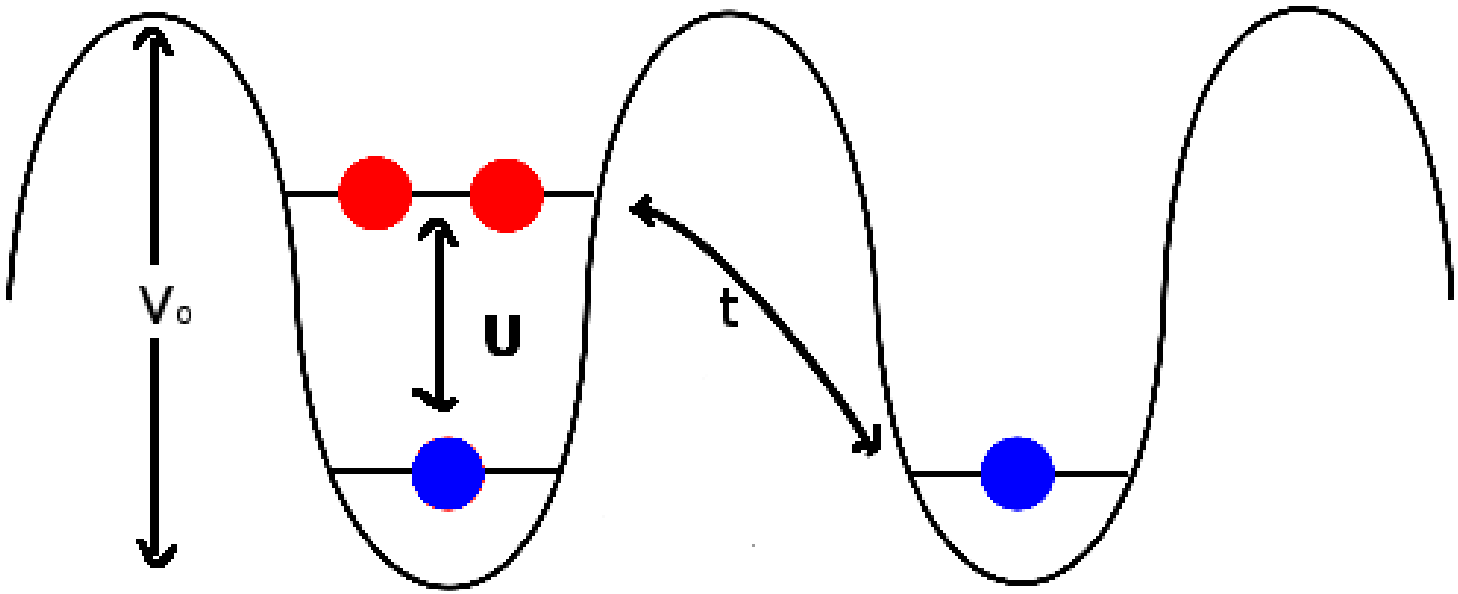


Greiner, Mandel, Esslinger, Hänsch, and Bloch, Nature **415**, 39 (2002)

4.2 Theoretical Description

Bose-Hubbard Hamiltonian:

$$\hat{H}_{\mathrm{BH}} = -t \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \left[\frac{U}{2} \hat{n}_i (\hat{n}_i - 1) - \mu \hat{n}_i \right], \qquad \hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$$



4.3 Landau Theory

Bose-Hubbard Hamiltonian with Current:

$$\hat{H}_{\mathrm{BH}}(J^*, J) = \hat{H}_{\mathrm{BH}} + \sum_i \left(J^* \hat{a}_i + J \hat{a}_i^\dagger \right)$$

Grand-Canonical Free Energy: $$F = -\frac{1}{\beta} \ln \mathrm{Tr} \left[e^{-\beta \hat{H}_{\mathrm{BF}}(J^*, J)} \right]$$

$$\psi = \langle \hat{a}_i \rangle = \frac{1}{N_{\mathrm{s}}} \frac{\partial F(J^*, J)}{\partial J^*} \quad ; \quad \psi^* = \langle \hat{a}_i^\dagger \rangle = \frac{1}{N_{\mathrm{s}}} \frac{\partial F(J^*, J)}{\partial J}$$

Legendre Transformation: $$\Gamma(\psi^*, \psi) = \psi^* J + \psi J^* - F/N_{\mathrm{s}}$$

$$\frac{\partial \Gamma}{\partial \psi^*} = J \quad ; \quad \frac{\partial \Gamma}{\partial \psi} = J^*$$

$\Longrightarrow$ **Physical limit of vanishing current**

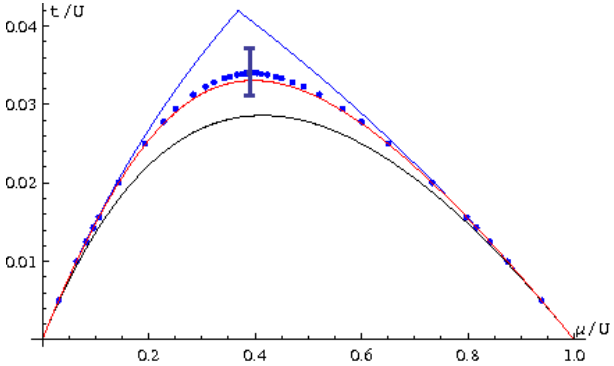
Landau expansion: $$\Gamma = a_0 + a_2 |\psi|^2 + a_4 |\psi|^4 + \cdots$$

$\Longrightarrow$ **Landau coefficients in tunneling expansion**

dos Santos and Pelster, PRA **79**, 013614 (2009)

4.4 Quantum Phase Diagram

Zero temperature:



Error bar: Extrapolated strong-coupling series

Black line: Mean-field

Blue line: 3rd strong-coupling order

Red line: Landau theory

Blue dots: Monte-Carlo data

dos Santos and Pelster, PRA **79**, 013614 (2009)

Extension to higher orders:

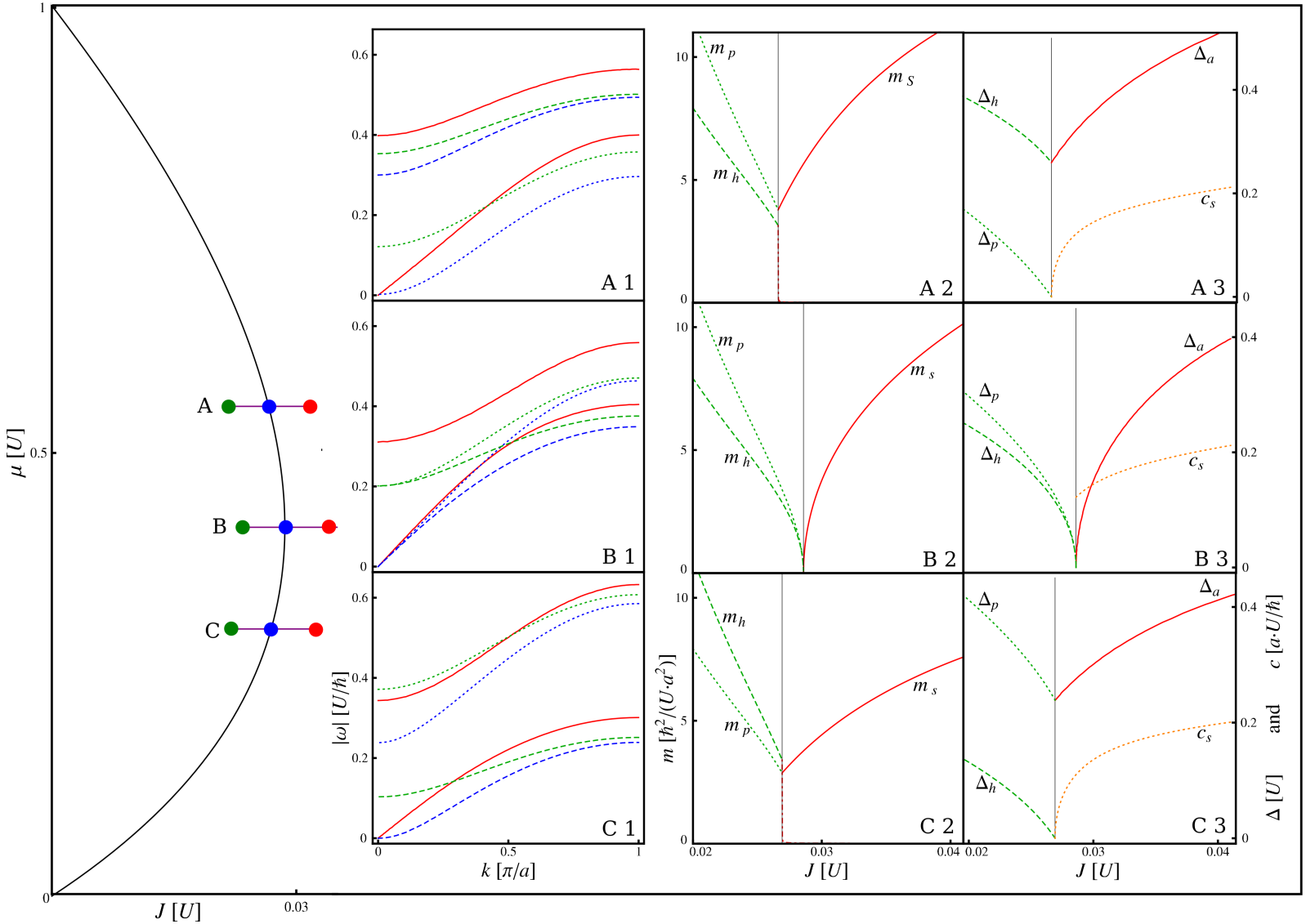
Teichmann, Hinrichs, Holthaus, and Eckardt, PRB **79**, 100503(R) (2009)

Hinrichs, Pelster, and Holthaus, APB **113**, 57 (2013)

Extension to superlattices:

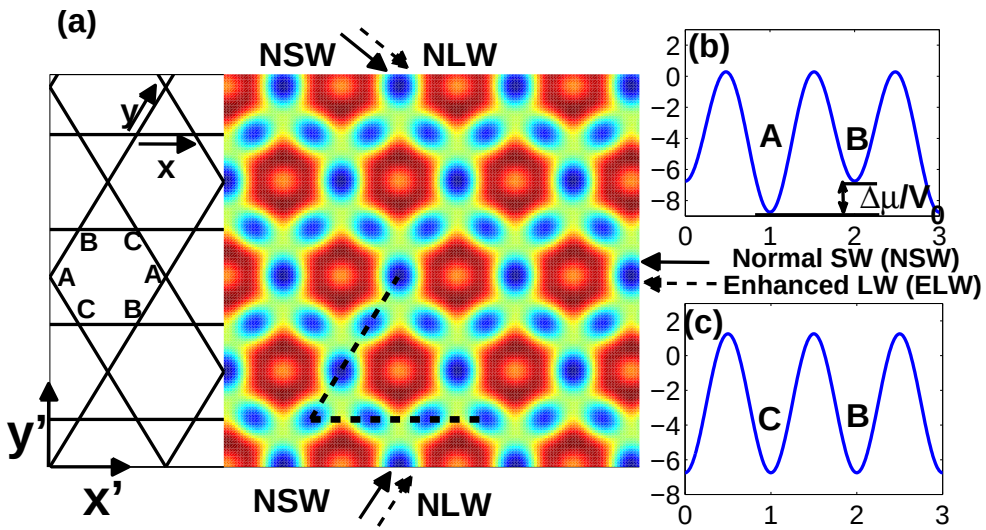
Wang, Zhang, Eggert, and Pelster, PRA **87**, 063615 (2013)

4.5 Ginzburg-Landau Theory: Excitation Spectra



Graß, Santos, and Pelster, PRA **84**, 013613 (2011)

4.6 Proposed Kagome Superlattice

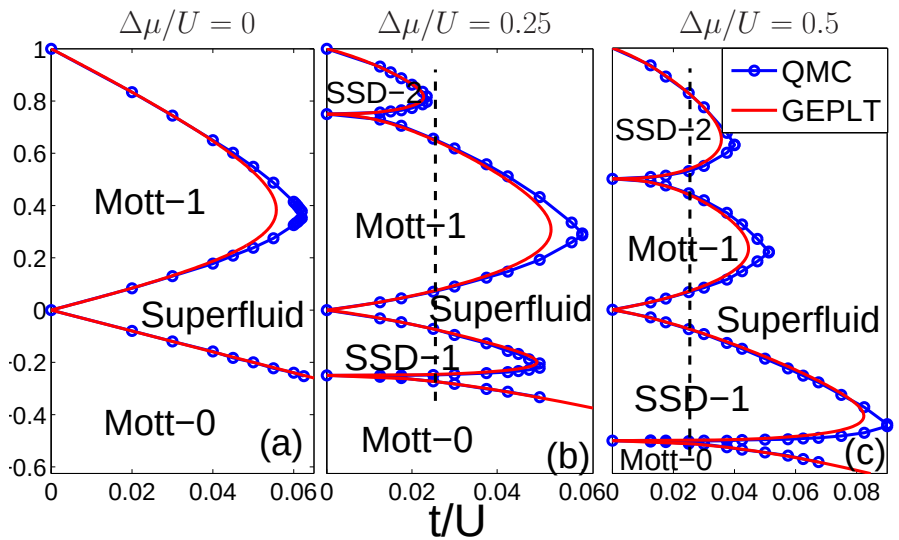


$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{a}_i^\dagger \hat{a}_j + \hat{a}_i \hat{a}_j^\dagger) + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) - \mu \sum_i \hat{n}_i - \Delta\mu \sum_{i \in A} \hat{n}_i$$

Zhang, Wang,

Eggert, and Pelster

PRB **92**, 014512 (2015)

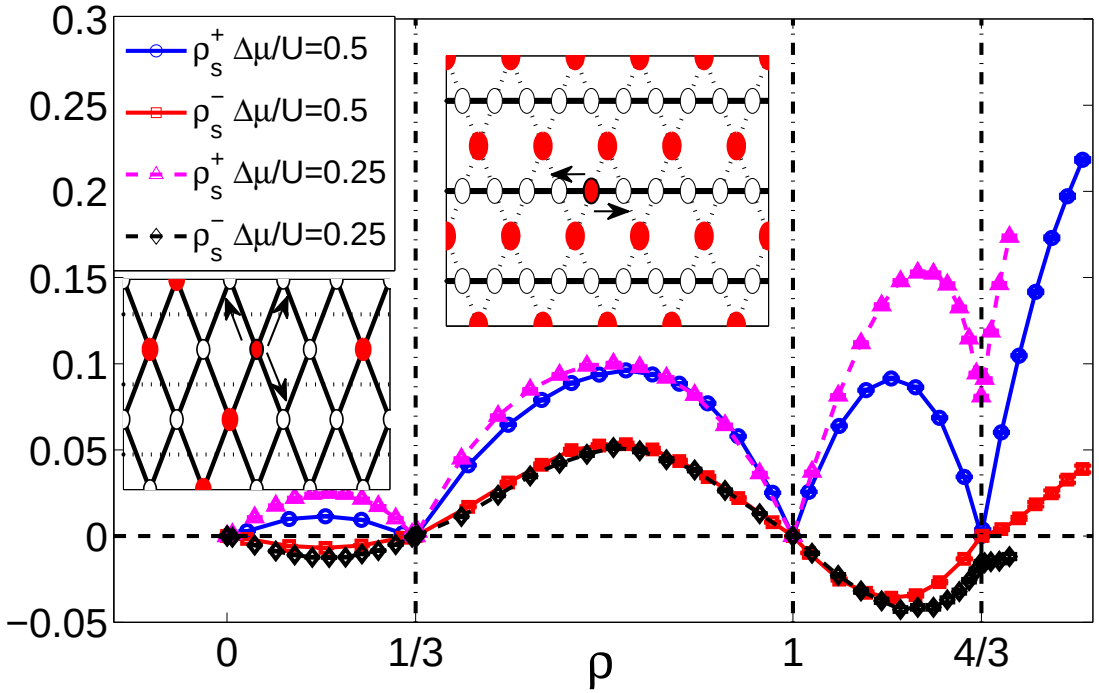


4.7 Tunable Anisotropic Superfluidity

- **Superfluid density via winding number**
  $\rho_s^{x/y} = \langle W_{x/y}^2 / 4\beta t \rangle$
  Pollock and Ceperley, PRB **36**, 8343 (1987)

- **Total superfluid density:**
  $\rho_s^+ = (\rho_s^x + \rho_s^y)/2$

- **Superfluid density difference:**
  $\rho_s^- = \rho_s^x - \rho_s^y$



- $\rho_s^x < \rho_s^y$
- A preferred
- Effective square lattice

- $\rho_s^x > \rho_s^y$
- A full, B/C preferrred
- No supersolid due to artificial symmetry-breaking

Zhang, Wang, Eggert, and Pelster, PRB **92**, 014512 (2015)

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5.1 Summary and Outlook

- **Dipolar Bose-Einstein condensates:**
  quantum fluctuations relevant for larger dipole-dipole interaction
  $\Longrightarrow$ **extension for finite temperatures**
- **On the dirty boson problem:**
  local condensates in minima + global condensate + thermally excited
  $\Longrightarrow$ **phase diagram yet unknown for strong disorder**
  Navez, Pelster, and Graham, APB **86**, 395 (2007)
- **Anisotropic superfluidity:**
  interplay between anisotropic disorder and dipolar interaction:
  $\Longrightarrow$ **necessitates anisotropic 3-fluid model**
- **Tunability of quantum phase transition:**
  - **Spin 1 bosons:**
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  - **Periodic modulation of s-wave scattering:**
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5.2 Research Projects Related to TU Kaiserslautern

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- **Dimensional phase transition:** Vogler et al., PRL **113**, 215301 (2014)
  $\Longrightarrow$ **Morath, Straßel, Eggert**
- **Near-field interferometric coherence mapping**
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- **Periodically driven impurity:** Thuberg, Reyes, and Eggert, `arXiv:1509.00035`
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- **Cs impurity in Rb BEC:** Akram and Pelster, `arXiv:1510.07138`
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- **Photon BEC:** Kopylov, Radonjić, Brandes, Balaž, and Pelster, `arXiv:1507.01811`
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ICRANet
International Center for Relativistic Astrophysics Network

- **Mapping of quantum field theories via space-time transformations:**
  $\Longrightarrow$ **Wamba, Anglin**

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5.4 Announcement

616th Wilhelm and Else Heraeus Seminar

Ultracold Quantum Gases -

Current Trends and Future Perspectives

organized by Carlos Sá de Melo and Axel Pelster

Bad Honnef (Germany); May 9 – 13, 2016

Invited Speakers: Nigel Cooper (UK), Eugene Demler (USA), Rembert Duine (Netherthelands), Tilman Esslinger (Switzerland), Michael Fleischhauer (Germany), Thierry Giamarchi (Switzerland), Rudi Grimm (Austria), Johannes Hecker-Denschlag (Germany), Andreas Hemmerich (Germany), Jason Ho (USA), Walter Hofstetter (Germany), Randy Hulet (USA), Massimo Inguscio (Italy), Corinna Kollath (Germany), Stefan Kuhr (UK), Kazimierz Rzazewski (Poland), Anna Sanpera (Spain), Luis Santos (Germany), Jörg Schmiedmayer (Austria), Dan Stamper-Kurn (USA), Sandro Stringari (Italy), Leticia Tarruell (Spain), Jacques Tempere (Belgium), Matthias Weidemüller (Germany), Eugene Zaremba (Canada), Peter Zoller (Austria)

`http://www-user.rhrk.uni-kl.de/~apelster/Heraeus4/index.html`