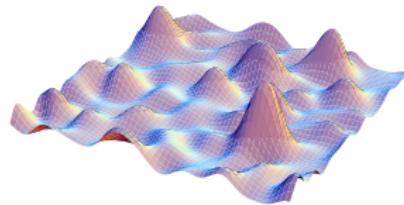
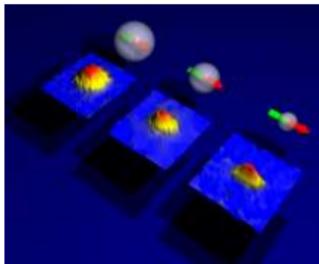


Dipolar Bose-Einstein Condensates with Weak Disorder

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Gross Pitaevskii theory

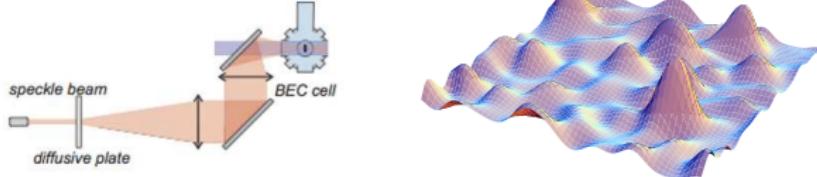
- ▶ neglecting thermal and quantum fluctuation

$$\left[-\frac{\hbar^2 \nabla^2}{2m} + U(\mathbf{x}) + \int d^3x' |\Psi(\mathbf{x}', t)|^2 V_{\text{int}}(\mathbf{x} - \mathbf{x}') \right] \Psi(\mathbf{x}, t) = i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{x}, t)$$

- ▶ continuity equation: $\frac{\partial}{\partial t} \rho + \nabla \cdot \mathbf{j} = 0$
- ▶ stationary state: $\Psi(\mathbf{x}, t) = \psi(\mathbf{x}) e^{-i\frac{\mu t}{\hbar}}$

$$\left[-\frac{\hbar^2 \nabla^2}{2m} - \mu + U(\mathbf{x}) + \int d^3x' |\psi(\mathbf{x}')|^2 V_{\text{int}}(\mathbf{x} - \mathbf{x}') \right] \psi(\mathbf{x}) = 0$$

Disorder



- ▶ strategy: express observables as functional of U and apply disorder ensemble average $\langle \cdot \rangle$
- ▶ homogeneity yields

$$\langle U(\mathbf{x}) \rangle = 0 , \quad \langle U(\mathbf{x})U(\mathbf{x}') \rangle = R(\mathbf{x} - \mathbf{x}')$$

Condensate

- ▶ assuming arbitrary interaction $V_{\text{int}}(\mathbf{x} - \mathbf{x}')$ and general disorder correlation $R(\mathbf{x} - \mathbf{x}')$
- ▶ expanding ground state with respect to disorder:

$$\psi(\mathbf{x}) = \psi_0 + \psi_1(\mathbf{x}) + \psi_2(\mathbf{x}) + \dots$$

- ▶ solve Gross Pitaevskii equation for different orders in U
- ▶ particle density: $n = \langle \psi(\mathbf{x})^2 \rangle$
- ▶ condensate density: $n_0 = \langle \psi(\mathbf{x}) \rangle^2$
- ▶ condensate depletion due to disorder in first order in R

$$n - n_0 = n \int \frac{d^3 k}{(2\pi)^3} \frac{R(\mathbf{k})}{\left[\frac{\hbar^2 \mathbf{k}^2}{2m} + 2n V_{\text{int}}(\mathbf{k}) \right]^2}$$

Superfluid

- ▶ assuming arbitrary interaction $V_{\text{int}}(\mathbf{x} - \mathbf{x}')$ and general disorder correlation $R(\mathbf{x} - \mathbf{x}')$
- ▶ apply boost \mathbf{k}_B to time-dependent Gross Pitaevskii equation and use for stationary case the ansatz $\Psi(\mathbf{x}) = \psi(\mathbf{x})e^{i\mathbf{k}_S \cdot \mathbf{x}}$

$$\left[-\frac{\hbar^2 \Delta}{2m} - i\frac{\hbar^2}{m} \mathbf{K} \nabla + U(\mathbf{x}) - \mu_{\text{eff}} + \int d^3x' V_{\text{int}}(\mathbf{x} - \mathbf{x}') |\psi(\mathbf{x}')|^2 \right] \psi(\mathbf{x}) = 0$$

with $\mathbf{K} = \mathbf{k}_S - \mathbf{k}_B$ and $\mu_{\text{eff}} = \mu - \frac{\hbar^2}{2m} \mathbf{k}_S^2 + \frac{\hbar^2}{m} \mathbf{k}_B \cdot \mathbf{k}_S$

Superfluid

- ▶ expand total momentum $\langle \mathbf{P} \rangle = - \left\langle \int d^3x \Psi^*(x) i\hbar \nabla \Psi(x) \right\rangle$ for small \mathbf{k}_S and \mathbf{k}_B

$$\langle \mathbf{P} \rangle = V n_S \hbar \mathbf{k}_S + V n_B \hbar \mathbf{k}_B + \dots$$

with requirement: $n = n_S + n_B$

- ▶ superfluid depletion due to disorder in first order in R

$$n - n_S = 4n \int \frac{d^3k}{(2\pi)^3} \frac{R(\mathbf{k}) \begin{pmatrix} k_x^2 & k_x k_y & k_x k_z \\ k_x k_y & k_y^2 & k_y k_z \\ k_x k_z & k_y k_z & k_z^2 \end{pmatrix}}{\mathbf{k}^2 \left[\frac{\hbar^2 \mathbf{k}^2}{2m} + 2n V_{\text{int}}(\mathbf{k}) \right]^2}$$

- ▶ isotropic interaction $V_{\text{int}}(\mathbf{k}) = V_{\text{int}}(|\mathbf{k}|)$ and correlation $R(\mathbf{k}) = R(|\mathbf{k}|)$ yields in general

$$n - n_S = \frac{4}{3}(n - n_0)$$

Dipole-Dipole interaction

- ▶ assuming contact and dipole-dipole-interaction of dipoles polarized along the z -axis yields

$$V_{\text{int}}(\mathbf{k}) = g[1 + \epsilon_{\text{dd}}(3x^2 - 1)]$$

where $x = \hat{\mathbf{k}}\hat{\mathbf{e}}_z = \cos \vartheta$, g controls strength of contact interaction, and $\epsilon_{\text{dd}} = \frac{C_{\text{dd}}}{3g}$ with

$$C_{\text{dd}} = \begin{cases} \mu_0 \mathbf{m}^2 & \text{magnetic dipoles} \\ 4\pi \mathbf{d}^2 & \text{electric dipoles} \end{cases}$$

- ▶ mean field results without disorder contain additional angular parameter ϑ from $V_{\text{int}}(\mathbf{k} = \mathbf{0})$

Dipole-Dipole interaction: Condensate

- ▶ example: Gauss correlated disorder with $R(\mathbf{k}) = R e^{-\frac{1}{2}\sigma^2 \mathbf{k}^2}$ and dipole-dipole interaction yields

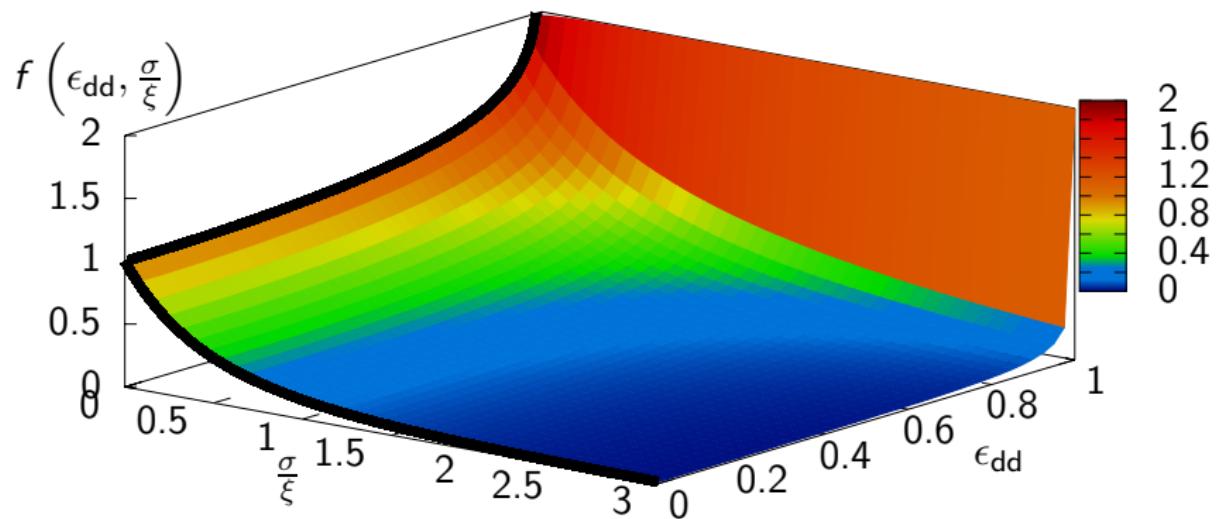
$$n - n_0 = n_{\text{HM}} f \left(\epsilon_{\text{dd}}, \frac{\sigma}{\xi} \right)$$

with the coherence length $\xi = \sqrt{\frac{\hbar^2}{4mng}}$ and

$$\text{Huang-Meng depletion } n_{\text{HM}} = \left(\frac{m}{2\pi\hbar^2} \right)^{\frac{3}{2}} \sqrt{\frac{\pi}{2g}} \sqrt{n} R$$

- ▶ Huang, Meng, PRL **69**, 644, 1992: $R(\mathbf{k}) = R$, $V_{\text{int}}(\mathbf{k}) = g$

Dipole-Dipole interaction: Condensate



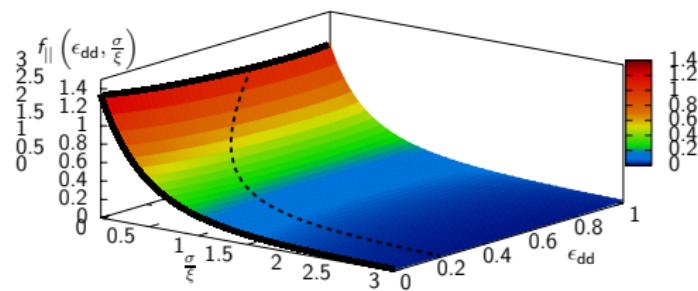
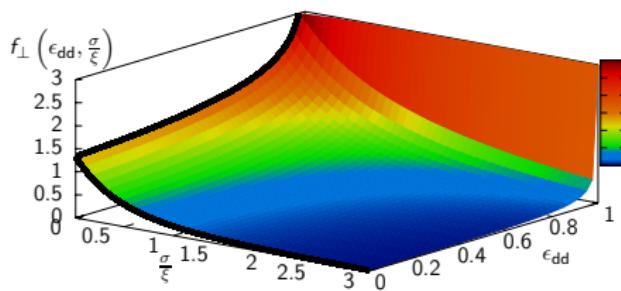
Dipole-Dipole interaction: Superfluid

- ▶ example: Gauss correlated disorder with $R(\mathbf{k}) = R e^{-\frac{1}{2}\sigma^2 \mathbf{k}^2}$ and dipole-dipole interaction yields

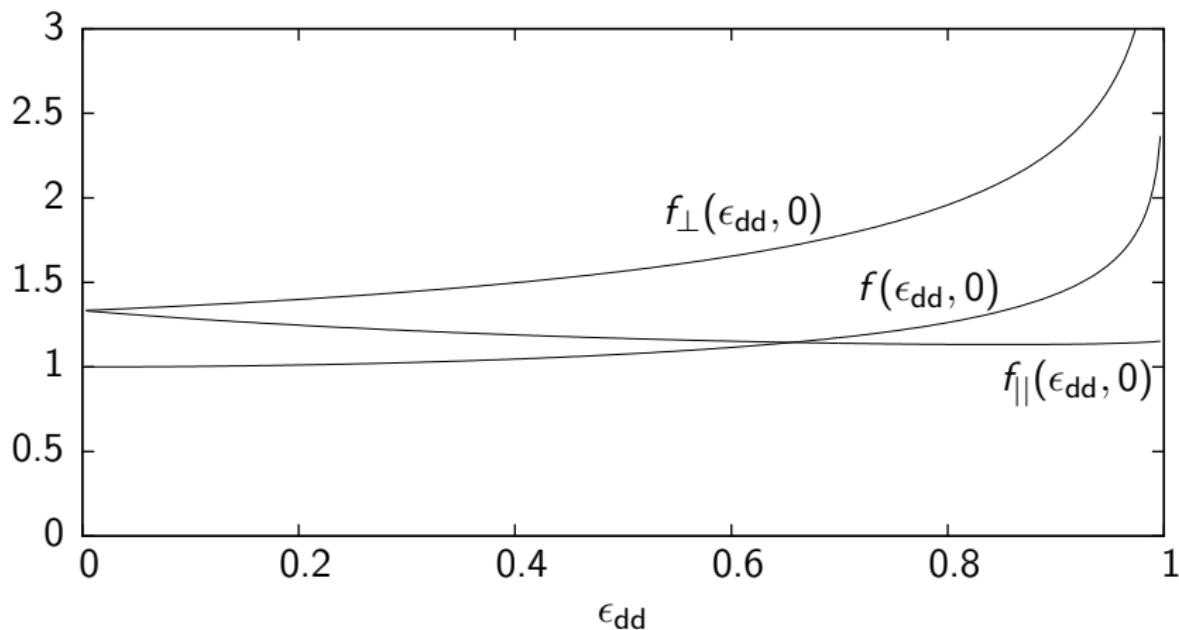
$$n - n_S = n_{HM} \begin{pmatrix} f_{\perp} \left(\epsilon_{dd}, \frac{\sigma}{\xi} \right) & 0 & 0 \\ 0 & f_{\perp} \left(\epsilon_{dd}, \frac{\sigma}{\xi} \right) & 0 \\ 0 & 0 & f_{||} \left(\epsilon_{dd}, \frac{\sigma}{\xi} \right) \end{pmatrix}$$

where $\xi = \sqrt{\frac{\hbar^2}{4mng}}$ and $n_{HM} = \left(\frac{m}{2\pi\hbar^2} \right)^{\frac{3}{2}} \sqrt{\frac{\pi}{2g}} \sqrt{n}R$

Dipole-Dipole interaction: Superfluid



Dipole-Dipole interaction: Condensate and Superfluid



Speed of Sound

- ▶ starting from the hydrodynamic equations

continuity eq. $\frac{\partial}{\partial t}n + \nabla \cdot \mathbf{j} = 0$

Euler eq. $m \frac{\partial}{\partial t} \mathbf{v}_S + \nabla \left(\mu + \frac{1}{2} m v_S^2 \right) = 0$

where $\mathbf{j} = \mathbf{v}_S n_S + \mathbf{v}_N n_N$ and $n = n_S + n_N$

- ▶ linearising around equilibrium by $\mathbf{v}_S = \delta \mathbf{v}_S(\mathbf{x}, t)$, $\mathbf{v}_N = \mathbf{0}$,
 $n = n_{\text{eq}} + \delta n(\mathbf{x}, t)$, $n_S = n_{S,\text{eq}} + \delta n_S(\mathbf{x}, t)$, $\mu = \mu(n + \delta n(\mathbf{x}, t))$

$$\frac{\partial^2}{\partial t^2} \delta n(\mathbf{x}, t) - \frac{1}{m} \left. \frac{\partial \mu}{\partial n} \right|_{\text{eq}} \nabla \left[n_{S,\text{eq}}(\mathbf{x}) \nabla \delta n(\mathbf{x}, t) \right] = 0$$

Speed of Sound

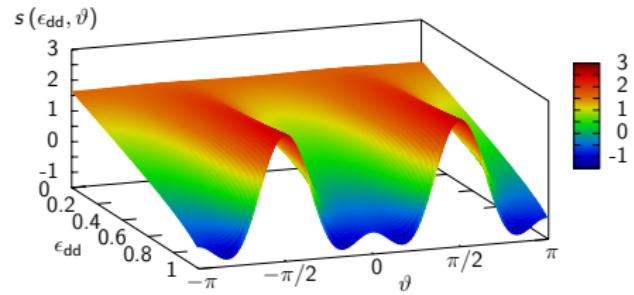
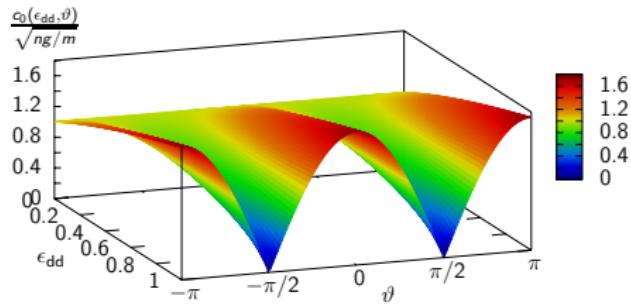
- ▶ example: delta correlated disorder with $R(\mathbf{k}) = R$ and dipole-dipole interaction yields

$$c^2(\epsilon_{dd}, \vartheta) = \frac{ng}{m} [3\epsilon_{dd} \cos^2(\vartheta) + 1 - \epsilon_{dd}] + \frac{n_{HM}g}{m} s(\epsilon_{dd}, \vartheta)$$

where $n_{HM} = \left(\frac{m}{2\pi\hbar^2}\right)^{\frac{3}{2}} \sqrt{\frac{\pi}{2g}} \sqrt{n}R$

- ▶ disorder correction in case of contact interaction: $s(0, \vartheta) = \frac{5}{3}$
- ▶ Giorgini, Pitaevskii, Stringari, PRB **49**, 12938, 1994:
 $R(\mathbf{k}) = R$, $V_{int}(\mathbf{k}) = g$

Speed of Sound



Summary and Outlook

- ▶ Anisotropic superfluidity at zero temperature due to delicate interplay between isotropic disorder and dipolar interaction:
 - characteristic direction dependent sound velocity
 - possible experimental detection: Bragg spectroscopy
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- ▶ Lorentzian instead of Gaussian correlated disorder:
 - all calculations analytically possible
 - interplay between anisotropies of disorder and dipolar interaction
- ▶ Taking into account quantum and thermal fluctuations necessitates to work out 3-fluid model:
superfluid + normalfluid + local BECs