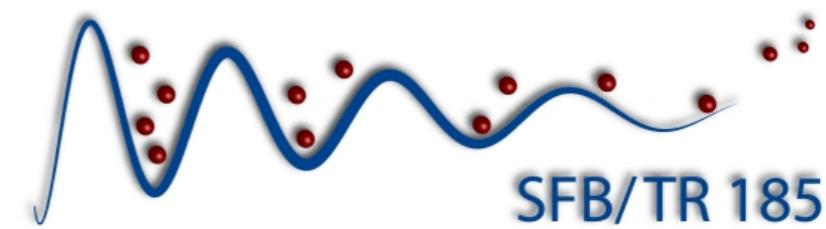


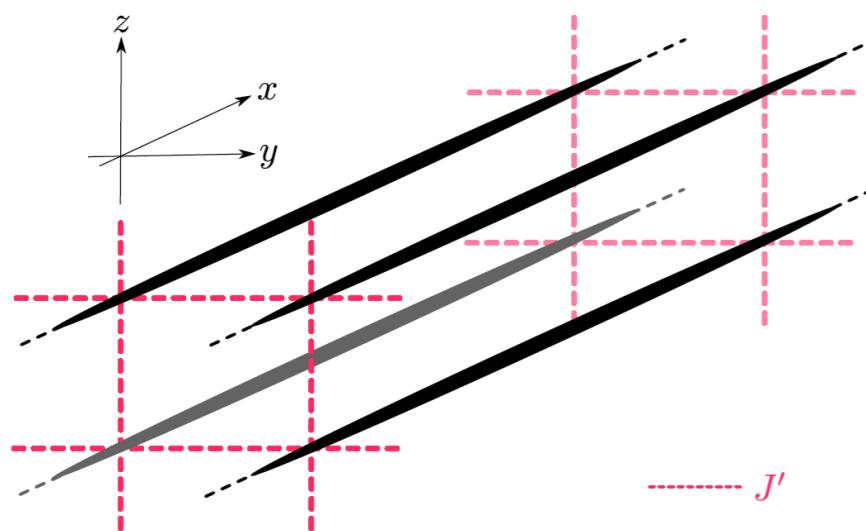
Dimensional crossover in a bosonic quantum gas

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I. Schneider, S. Eggert



DPG Spring Meeting, Rostock, 11.03.2019

The model



$$\hat{H} = \sum_{yz} \hat{h}_{yz} + \hat{H}_{hop}$$

\hat{h}_{yz} – single tube hamiltonian

$$\hat{H}_{hop} = -\frac{J'}{2} \sum_{jyz} \left(b_{jyz}^\dagger b_{jy+1z} + b_{jyz}^\dagger b_{jyz+1} + h.c. \right)$$

J' – NN hopping between the chains

If $J'=0$, $T_c=0$ – 1D system

Free bosons:

$$\hat{h}_{yz}^0 = -\frac{J}{2} \sum_j \left(b_{jyz}^\dagger b_{j+1yz} + h.c. \right) - \mu_{1D} \sum_j \hat{n}_{jyz}$$

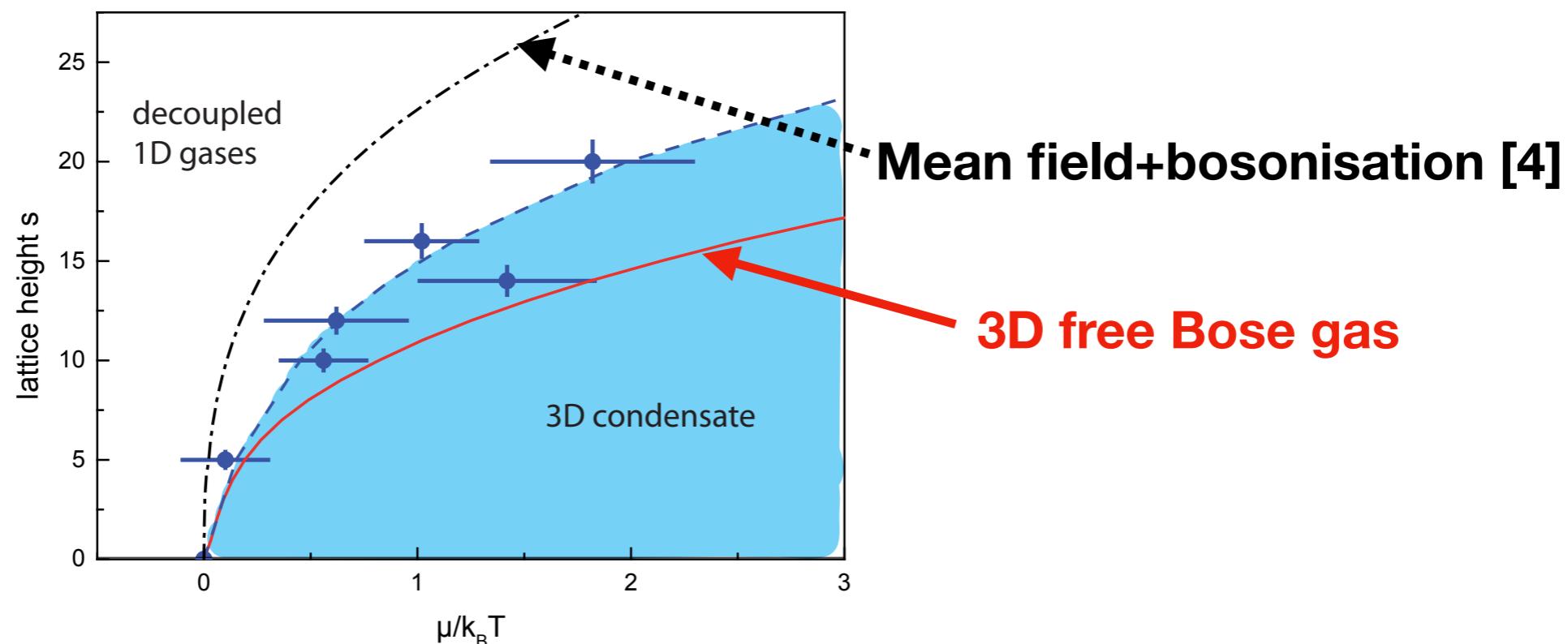
**Bosons
with on-site repulsion**

$$\hat{h}_{yz} = \int_0^L \left[\frac{\hbar^2}{2m} \partial_x \Psi^\dagger(x) \partial_x \Psi(x) + \frac{g}{2} \Psi^\dagger(x) \Psi^\dagger(x) \Psi(x) \Psi(x) - \mu_{1D} \Psi^\dagger(x) \Psi(x) \right] dx$$

the model is equivalent to the Heisenberg XXZ spin chain in continuum limit [1]

Problem: $T_c \propto (J')^\alpha$, $\alpha = \alpha(g)$

Experimental realisation with ^{87}Rb



[3] Andreas Vogler, Ralf Labouvie, Giovanni Barontini, Sebastian Eggert, Vera Guerrera, and Herwig Ott
Phys. Rev. Lett. **113**, 215301 (2014)

Tunneling amplitude between the chains

$$J'(s) = 4E_r s^{3/4} e^{-2\sqrt{s}}, \quad E_r = h^2 / 2\lambda^2 m_{Rb}$$

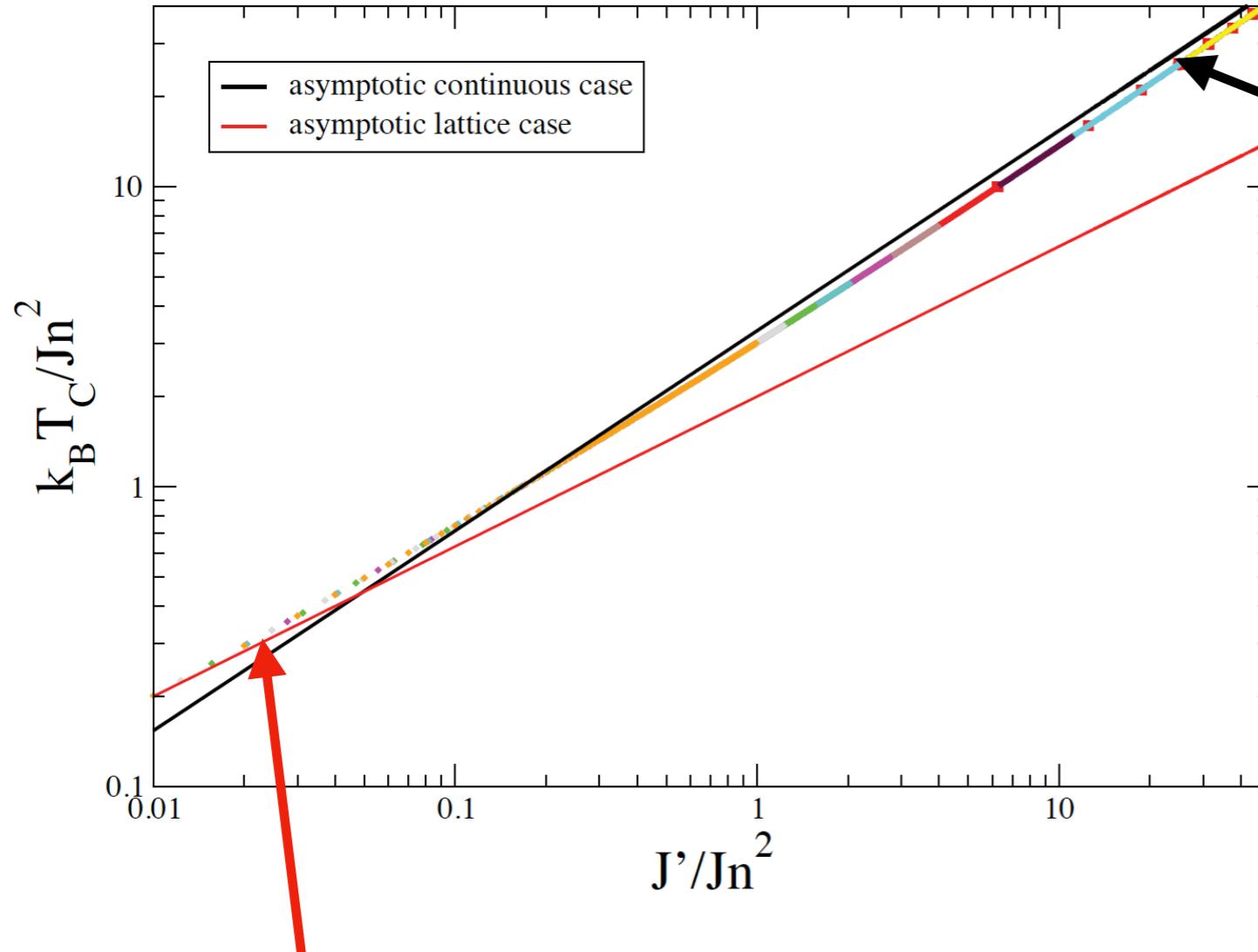
Recoil energy

[4] M. A. Cazalilla et al., New J. Phys. **8**, 158 (2006)

HFBP corrections:

[5] B.Irsigler and A.Pelster, Phys. Rev. A **95**, 043610 (2017)

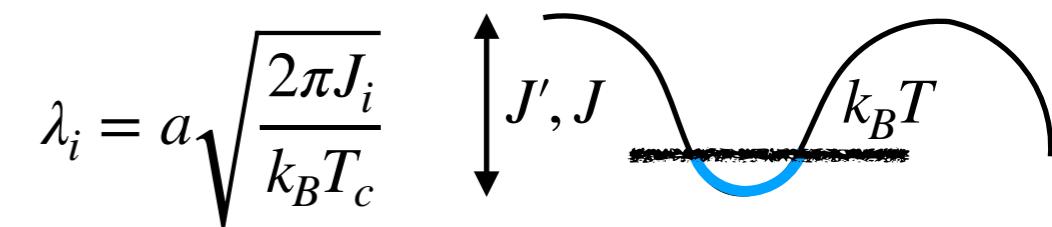
Crossover in free bosonic gas



3D, Continuum limit

$$k_B T_c / J = 2\pi \zeta \left(\frac{3}{2}\right)^{-2/3} n^{2/3} \left(\frac{J'}{J}\right)^{2/3}$$

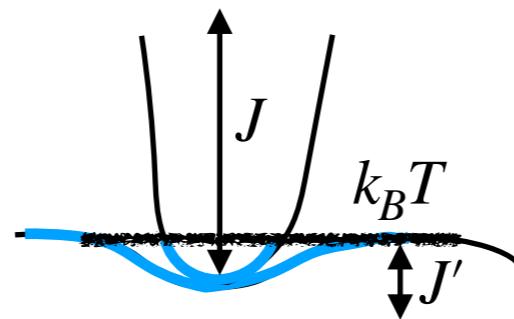
$$\lambda_{eff} = \sqrt[3]{\lambda_x \lambda_y \lambda_z} \sim n \cdot a$$



Continuum limit (quadratic spectrum) in the direction along the chains

$$n = \frac{a}{\lambda_c} \zeta_{\frac{1}{2}} (e^{-2\beta_c J'})$$

$$\frac{k_B T_c}{J} \simeq 1.56 n \sqrt{\frac{J'}{J}}$$



$$\frac{k_B T_c^{MF}}{J} = 2n \sqrt{\frac{J'}{J}}$$

Mean-field result

Crossover in interacting system

Chain Mean-field ansatz:

$$b_{jyz} = \langle b_{jyz} \rangle + \delta b_{jyz}$$

Mean-field Hamiltonian:

$$H_{MF} = H_0 - 2J' \sum_{jyz} \left(-\langle b^\dagger \rangle \langle b \rangle + \langle b^\dagger \rangle b_{jyz} + \langle b \rangle b_{jyz}^\dagger \right)$$

Cumulant expansion (exact):

$$F = F_{MF} - \frac{1}{\beta} \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \int_0^\beta d\tau_1 \dots \int_0^\beta d\tau_n T_\tau \left\langle V_{MF}(\tau_1) \dots V_{MF}(\tau_n) \right\rangle_{MF,c}$$

Mean-field free energy

Quantum fluctuations: $V_{MF} = H - H_{MF}$

Expansion in $\Psi = \langle b_{jyz} \rangle_{MF}$ gives **Landau free energy**

$$F = a_0 + a_2 \Psi^\dagger \Psi + a_4 (\Psi^\dagger \Psi)^2 + \dots$$

From $a_2 = 0$ one gets **critical temperature**

Interacting bosons: mean-field level

$$1 = 2J' \int_{-\infty}^{\infty} dx \int_0^{\beta} G(x, \tau, T) d\tau$$

Where $G(x, \tau, T) = \langle b^\dagger(x, \tau) b(0, 0) \rangle$ — correlation function of 1D tube [6]

$$\langle b^\dagger(x, \tau) b(0, 0) \rangle = A_{00}^{-+} \left(\frac{\pi T}{av} \right)^{\frac{1}{2K}} \left[\frac{1}{\sinh \left[\left(\frac{x}{v} + i\tau + i\epsilon \right) \pi T \right]} \frac{1}{\sinh \left[\left(\frac{x}{v} - i\tau - i\epsilon \right) \pi T \right]} \right]^{\frac{1}{4K}} + \text{subleading}$$

Luttinger Parameter $K(\mu_{1D}, g)$ and $A_{00}^{+-}(\mu_{1D}, g), v(\mu_{1D}, g)$

$$\frac{k_B T_c}{J} = \beta_0^{MF} \left(\frac{J'}{J} \right)^\alpha, \alpha = \frac{1}{2 - \frac{1}{2K}}$$

$$\beta_0^{MF} = \frac{1}{\pi} (v)^{1-\alpha} \left(A_{00}^{-+} \sin \frac{\pi}{4K} 2^{\frac{1}{2K}-1} B^2 \left(\frac{1}{8K}, 1 - \frac{1}{4K} \right) \right)^\alpha$$

Free boson limit $K \rightarrow \infty, \alpha \rightarrow 1/2$

[6] M. Dugave et al. J. Stat. Mech 1404, P04012 (2014)

[7] M. A. Cazalilla et al., New J. Phys. 8 158 (2006)

Non-universal exponent! [7]

Interacting bosons: quantum corrections to Tc

We calculate the quantum correction to free energy (first order cumulant vanishes)

$$F^{(2)} = -\frac{1}{2\beta} \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \langle T_\tau V_{MF}(\tau_1) V_{MF}(\tau_2) \rangle_{MF}, V_{MF} = H - H_{MF}$$

And evaluate the quantum correction to α_2

Quantum corrections do not change the critical exponent, but only renormalise the prefactor

$$\frac{k_B T_c}{J} = \beta_0^{MF} \left(\frac{J'}{J} \right)^\alpha \rightarrow \beta_0 \left(\frac{J'}{J} \right)^\alpha$$

Prefactor satisfies the equation

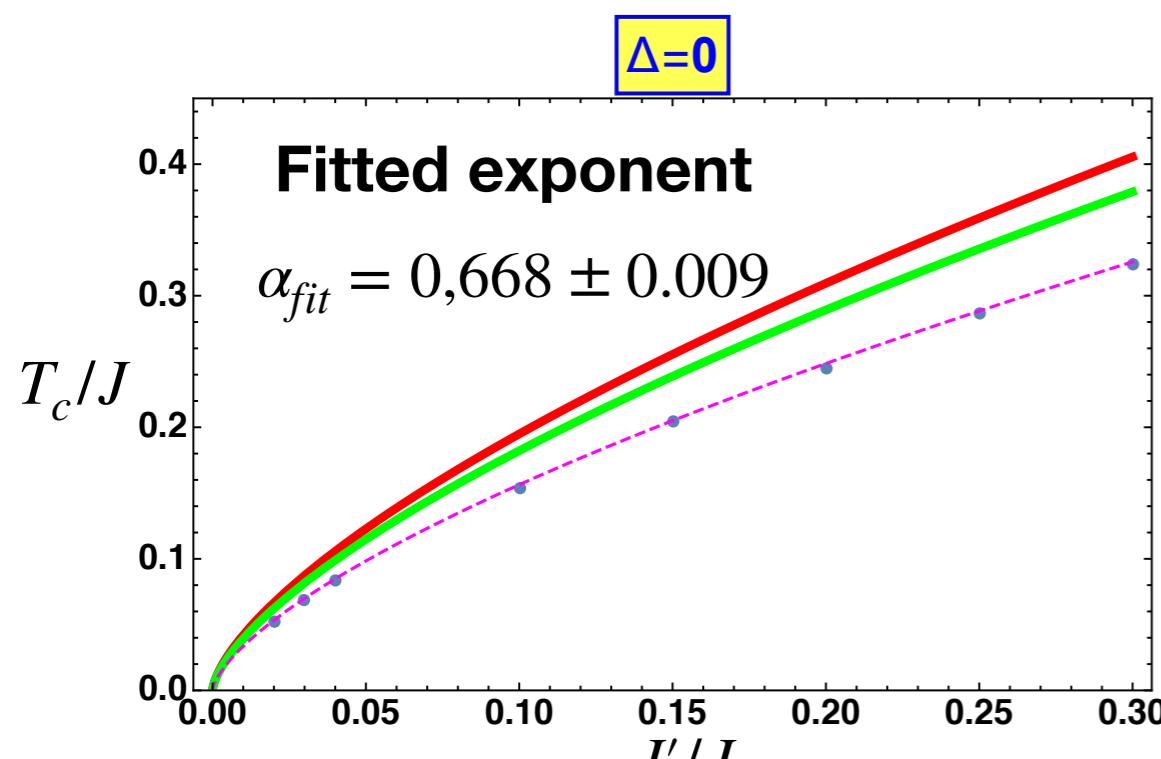
$$2 + C_1 \beta_0^{-1/\alpha} + \underline{C_2 \beta_0^{-3/\alpha}} = 0$$

Originates from quantum corrections

$$C_1 = -\frac{3A_{00}^{-+}}{\nu} \left(\frac{\pi}{\nu} \right)^{\frac{1}{2K}} \sin \frac{\pi}{4K} 2^{1/2K} \left(\frac{\nu}{2\pi} \right)^2 \mathbf{B}^2 \left(\frac{1}{8K}, 1 - \frac{1}{4K} \right), C_2 = -2 \left(\frac{\nu A_{00}^{-+}}{\pi^2} \right)^3 \left(\frac{\pi}{\nu} \right)^{\frac{3}{2K}} 2^{\frac{1}{2K}-2} \sin \frac{\pi}{4K} \mathbf{B}^2 \left(\frac{1}{8K}, 1 - \frac{1}{4K} \right) \mathcal{J}[K]$$

$$\mathcal{J}[K] = \int_0^\pi d\tau \int_0^\pi d\tau_2 \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dx_2 \left[\frac{1}{\sinh[x - i\tau] \sinh[x + i\tau]} \right]^{\frac{1}{4K}} \left[\frac{1}{\sinh[(x - x_2) - i(\tau - \tau_2)] \sinh[(x - x_2) + i(\tau - \tau_2)]} \right]^{\frac{1}{4K}}$$

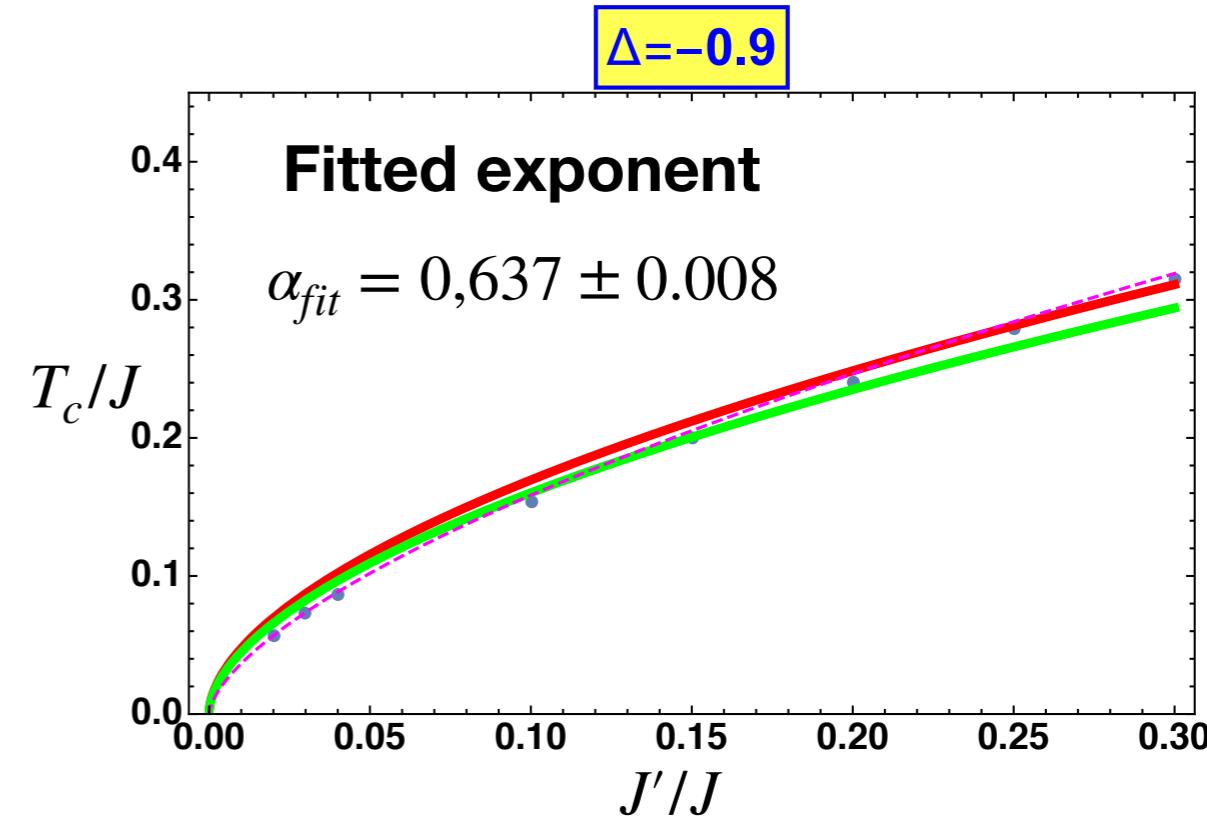
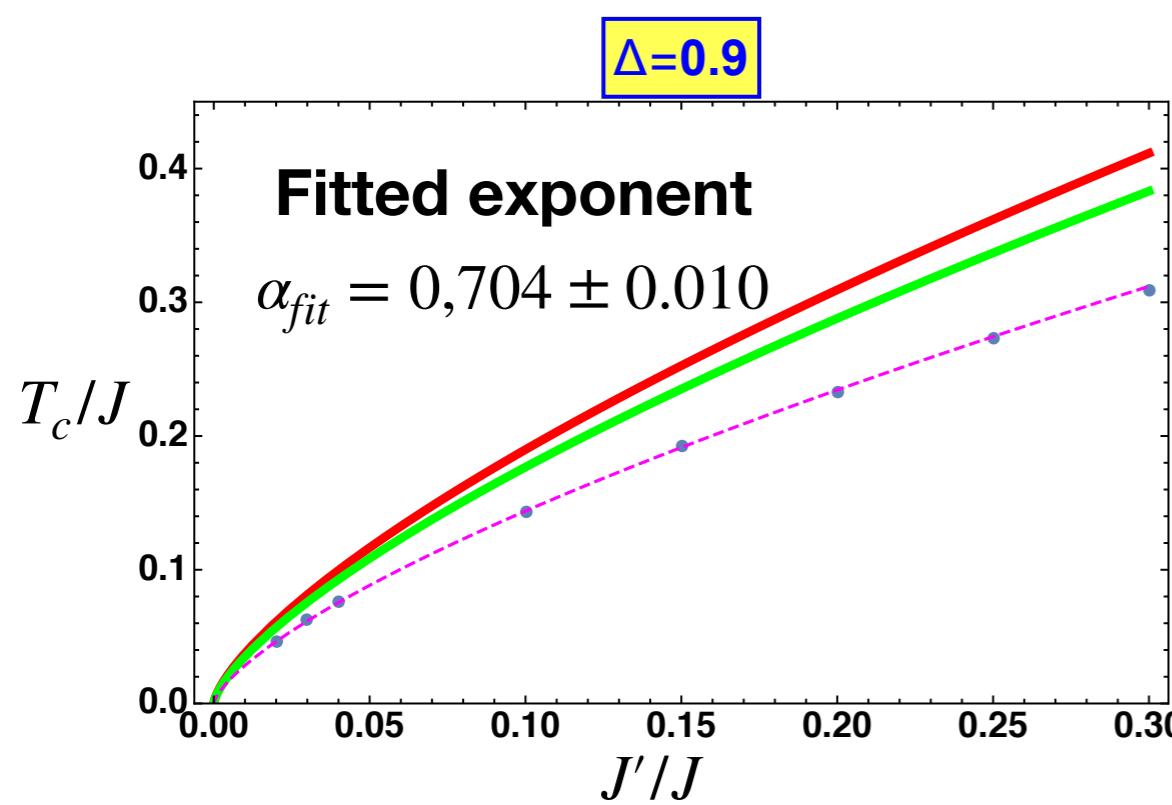
Critical temperature: QMC results, $n = 0.15$



$$g = -J \frac{1 + \Delta}{\Delta}$$

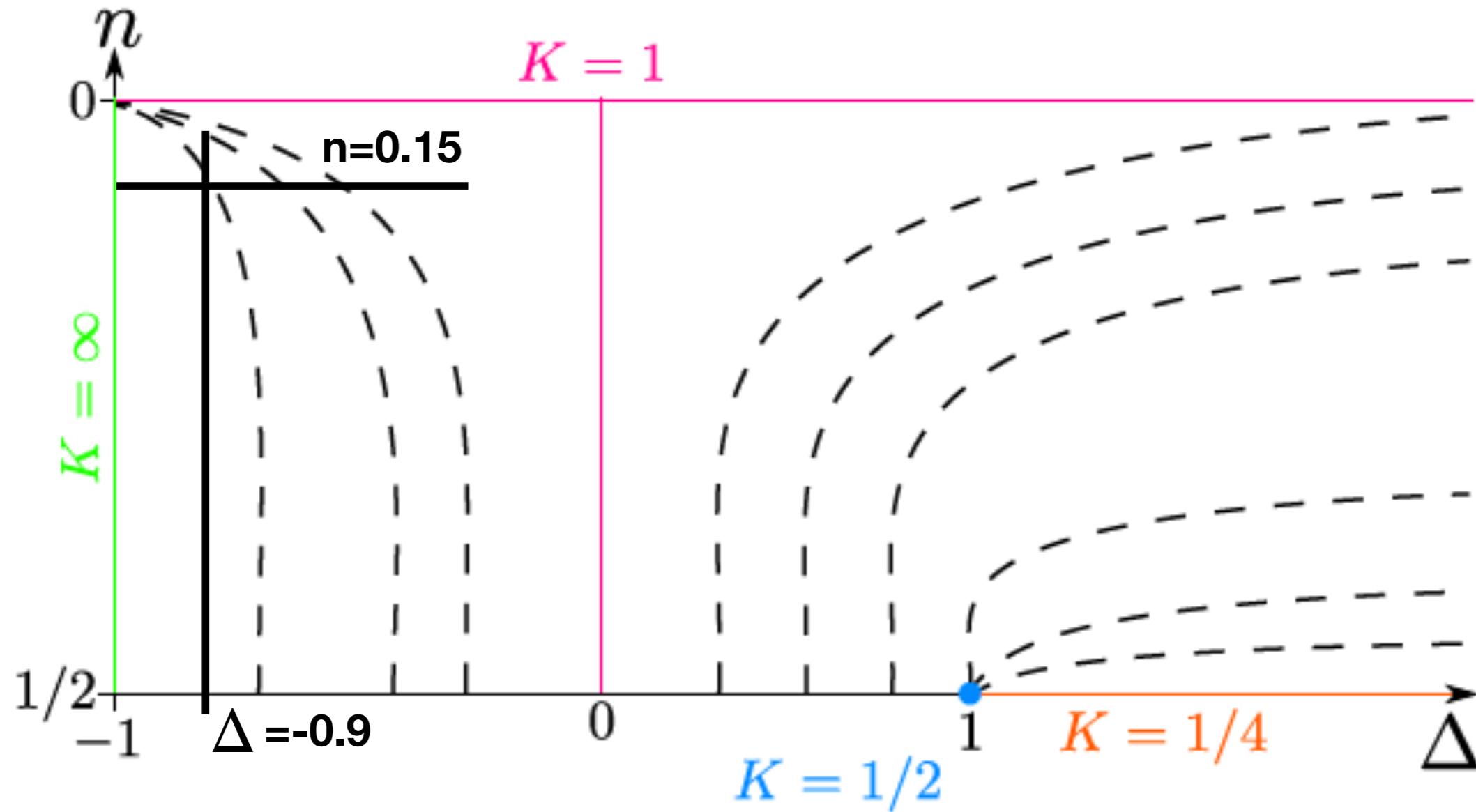
Δ	α	β_0/β_0^{MF}
0	2/3	0,945
0.9	0,704	0,930
-0.9	0,552	0,934
-1	1/2	0,950

Free bosons



Discrepancy for $\Delta = -0.9$

Grand canonical measurements in QMC (For more details see D.Straßel, PhD thesis (2017))



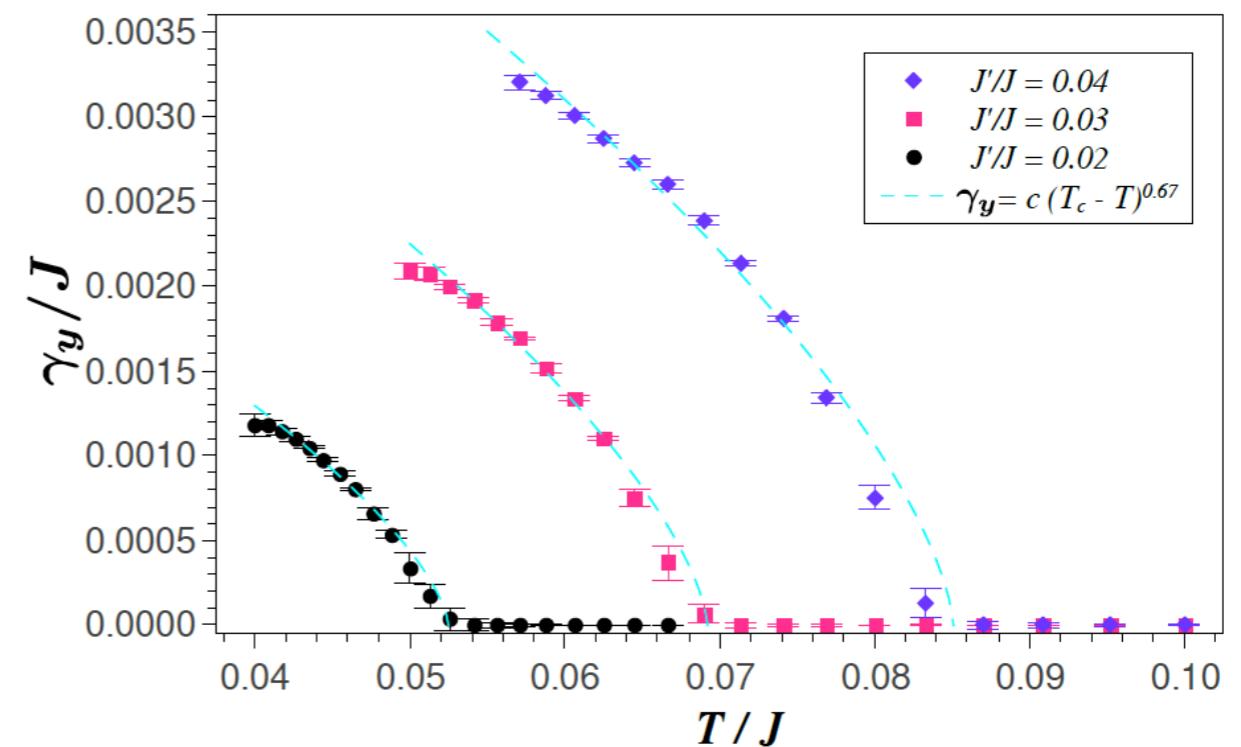
Conclusions

1. Non-universal critical exponents, quantitative description by mean-field
2. Quantum corrections of the lowest order do not modify the exponent
3. Strong quantum corrections of higher orders to the critical temperature

Further questions

Superfluid density

Universality class of O(2) symmetric
 ϕ^4 theory in 3D?



Non-homogeneous order parameter: Bogoliubov spectrum

Bad Honnef Physics School on Methods of Path Integration in Modern Physics

organized by Stefan Kirchner and Axel Pelster

Bad Honnef (Germany); August 25 – 31, 2019

Speakers and Topics:

Lawrence Schulman (Potsdam, USA): *Quantum Mechanics*

Andreas Wipf (Jena, Germany): *Statistical Field Theory*

Carlos Sa de Melo (Atlanta, USA): *Many-body Theory, BEC-BCS Crossover*

Jean Zinn-Justin (Paris, France): *Quantum Field Theory, Large-N Technique*

Victor Dotsenko (Paris, France): *Random Matrix Theory, Replica Trick*

Steve Simon (Oxford, UK): *Wilson Loops Spin, Topology, Holonomy Group*

Wolfhard Janke (Leipzig, Germany): *Quantum Monte Carlo*

Hagen Kleinert (Berlin, Germany): *Vortices and GIMPs*