Primordial Models for Dissipative Bose-Einstein Condensates

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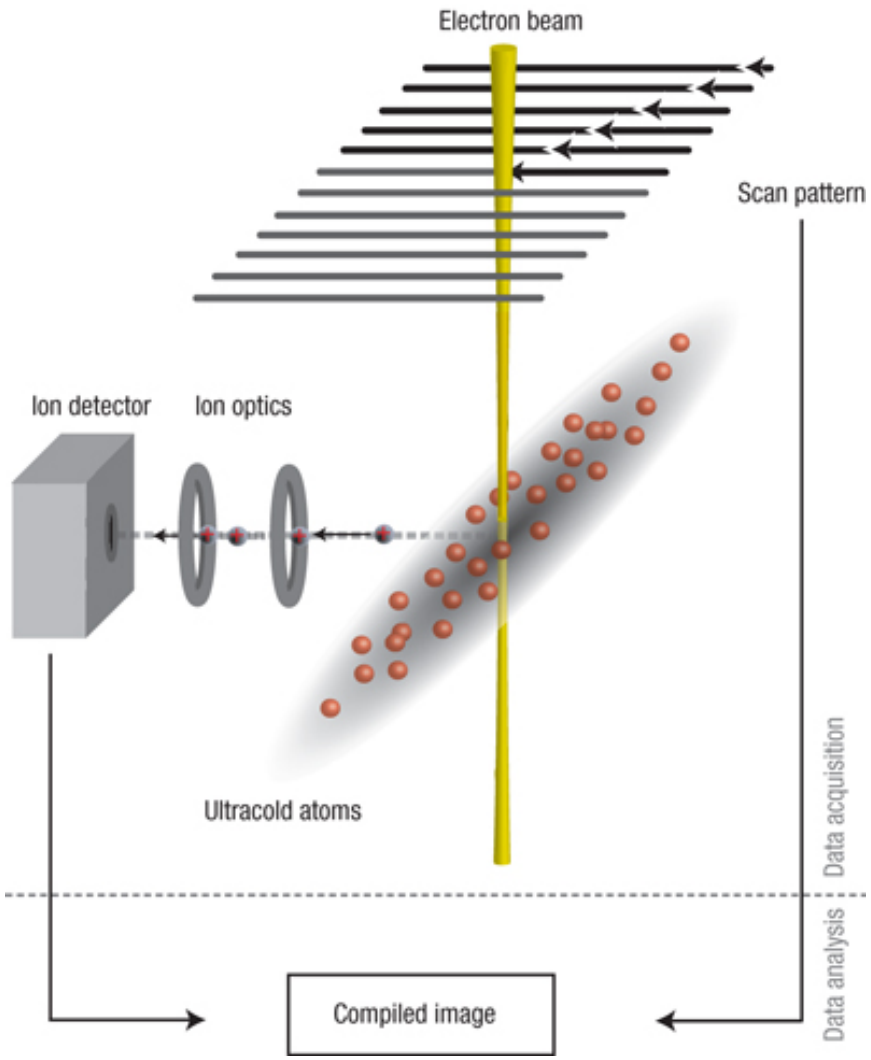
Experiment

- ^{87}Rb-BEC of 10^5 particles at $T = 80\ \mathrm{nK}$
- Anisotropic harmonic trap $\Omega_{||} = 2\pi \cdot 170\ \mathrm{Hz}$, $\Omega_{\perp} = 2\pi \cdot 13\ \mathrm{Hz}$
- Focussed electron beam of Gaussian shape with width $w = \frac{FWHM}{\sqrt{8 \ln 2}} = 100\ \mathrm{nm}$
- H. Ott et *al.*, Nature Phys. **4**, 949 (2008)



Complex potential

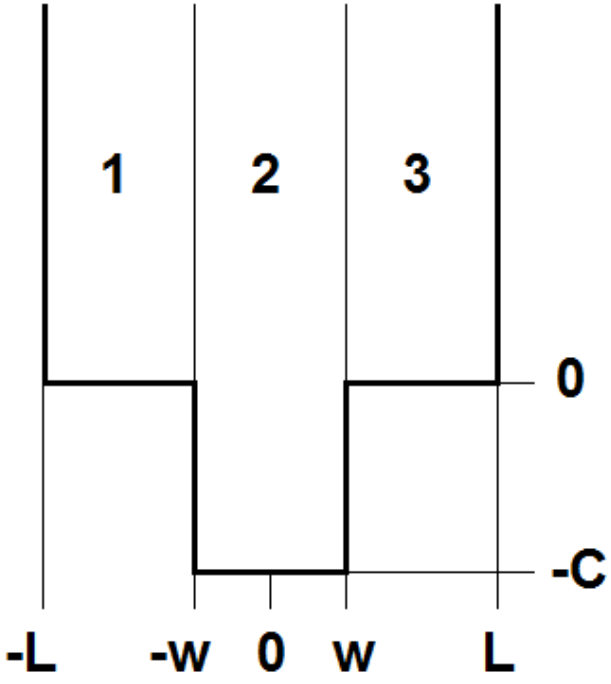
- Gross-Pitaevskii-equation with complex potential

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[-\frac{\hbar^2}{2M}\Delta + V_{\mathrm{R}}(\mathbf{r}) + iV_{\mathrm{I}}(\mathbf{r}) + g\left|\Psi(\mathbf{r},t)\right|^2\right]\Psi(\mathbf{r},t)$$

- Real part: harmonic trapping $V_{\mathrm{R}}(\mathbf{r}) = \frac{1}{2}M\left[\Omega_\perp^2(x^2+y^2) + \Omega_{||}^2 z^2\right]$
- Imaginary part: Gaussian beam $V_{\mathrm{I}}(\mathbf{r}) = -C\exp\left(-\frac{x^2+y^2}{2w^2}\right)$
- $C = \frac{\hbar}{2}\frac{\sigma_{\mathrm{tot}}}{e}\frac{I}{2\pi w^2} = 1,8\cdot 10^{-30}\ \mathrm{J}\,, \quad \sigma_{\mathrm{tot}} = 100a_0\,, \quad I = 20\ \mathrm{nA}$
- Non-stationary probability density $\rho(x,t) = \exp\left[\frac{2}{\hbar}E_{\mathrm{I}}(t-t_0)\right]\rho(x,t_0)$

Complex potential square well

- Simplications for primordial model
- One-dimensional BEC, extension $L = R_\perp = 640\ \mu\mathrm{m}$
- No interaction $g = 0$ (Schrödinger equation)
- Square-well approximation of V_R and V_I
- Separation into area 1, 2 and 3



$$V_\mathrm{R}(x) = \begin{cases} 0 & , \ |x| < L \\ \infty & , \ |x| > L \end{cases} \qquad V_\mathrm{I}(x) = \begin{cases} -C & , \ |x| < w \\ 0 & , \ |x| > w \end{cases}$$

Quantization conditions

- Dimensionless variables (extension L normalized to $\frac{\pi}{2}$)

$$\varepsilon = \frac{E}{\frac{\hbar^2\pi^2}{2M(2L)^2}}, \qquad c = \frac{C}{\frac{\hbar^2\pi^2}{2M(2L)^2}}, \qquad \chi = \frac{\pi}{2}\frac{x}{L}, \qquad \omega = \frac{\pi}{2}\frac{w}{L}$$

- Experiment: $\omega = 0.0002\,, \qquad c = 80000$
- Quantization conditions for symmetric and antisymmetric states

$$0 = \sqrt{\varepsilon^s}\cot\left[\left(\omega - \frac{\pi}{2}\right)\sqrt{\varepsilon^s}\right] + \sqrt{\varepsilon^s + ic}\tan\left(\omega\sqrt{\varepsilon^s + ic}\right)$$

$$0 = \sqrt{\varepsilon^a}\cot\left[\left(\omega - \frac{\pi}{2}\right)\sqrt{\varepsilon^a}\right] - \sqrt{\varepsilon^a + ic}\cot\left(\omega\sqrt{\varepsilon^a + ic}\right)$$

- $\lim\limits_{c\to 0}\varepsilon = \lim\limits_{\omega\to 0}\varepsilon = m^2 \;; \qquad \lim\limits_{\omega\to\frac{\pi}{2}}\varepsilon = m^2 - ic \;, \qquad m \in \mathbb{N}$

General solutions of quantization conditions

- ε_{I} non-positive (damping)
- Correct limit $\varepsilon(c=0) = m^2$
- $\lim_{c\to\infty} \varepsilon_{\mathrm{I}} = 0 \to k$-states
  $\lim_{c\to\infty} \varepsilon_{\mathrm{I}} \overset{\text{linear}}{=} -\infty \to n$-states
- n-states most damped
- Corresponds to $\omega \to 0$, $\omega \to \frac{\pi}{2}$
- $m \in \mathbb{N}$ describes limit $c = 0$,
  $n, k \in \mathbb{N}$ for $c \to \infty$

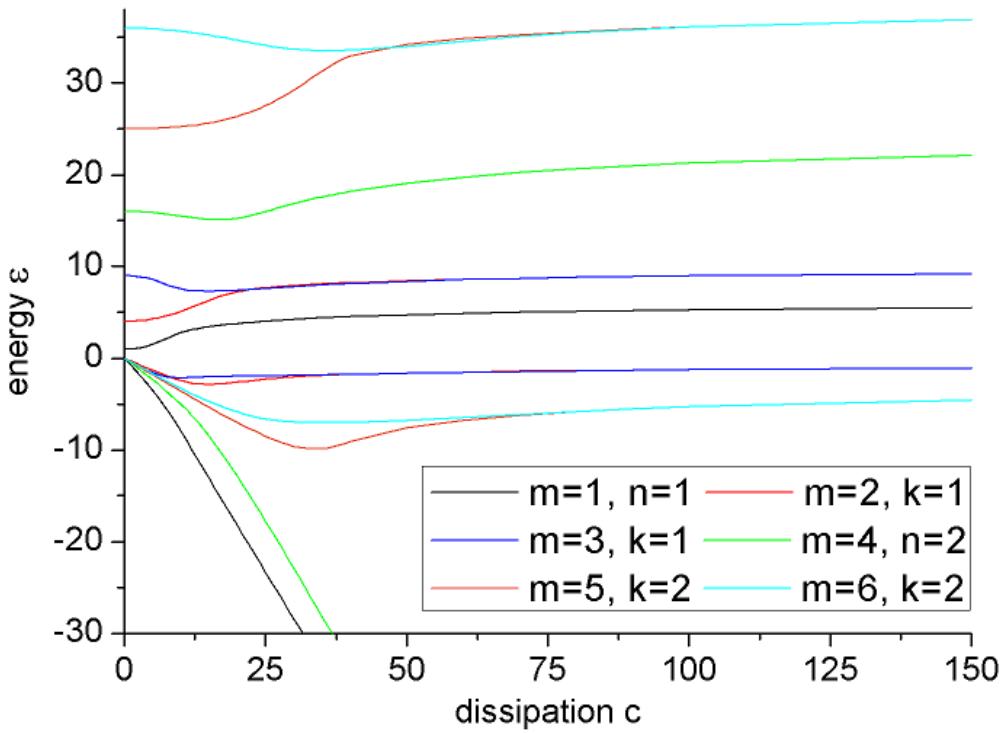


Figure: $\omega = 0.6$

- 2 adjoining k-states (different parity) fuse to one state (same k)

Limit of small and big waist

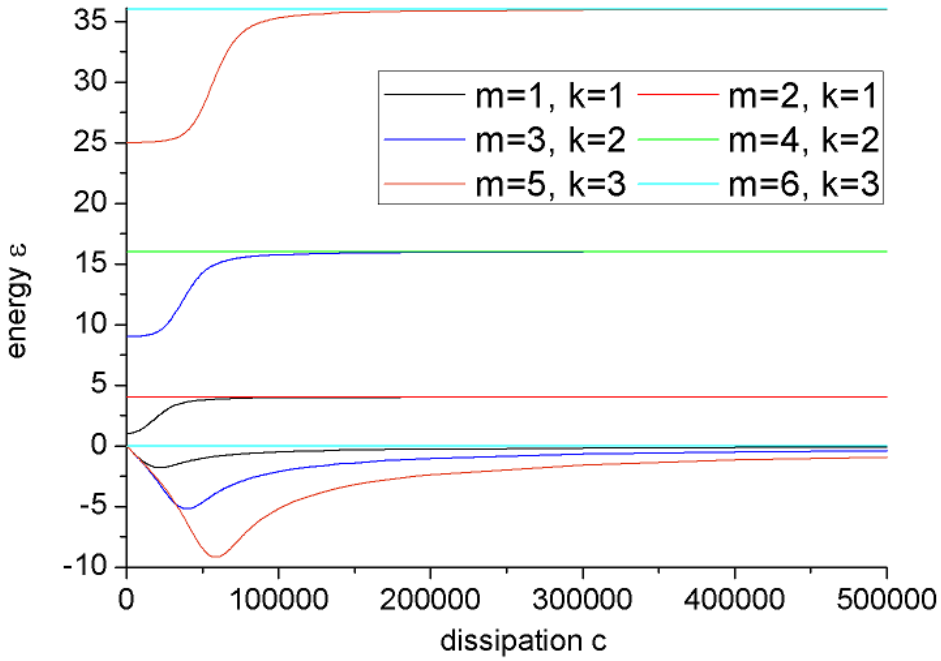


Figure: $\omega = 0.0002 \approx 0$

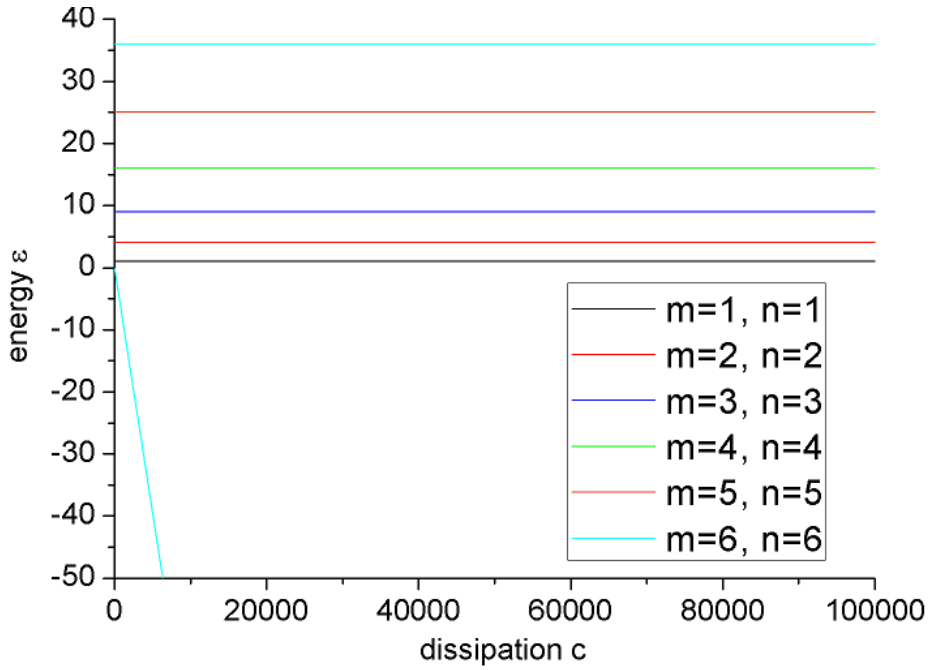


Figure: $\omega = 1.57 \approx \frac{\pi}{2}$

- $\omega \approx 0$: only k-states; antisymmetric states $\varepsilon^a \approx m^2$ for all c; ε^s tends to ε^a; $\varepsilon^s_{\mathrm{I}}$ reveals deep minimum
- $\omega \approx \frac{\pi}{2}$: only n-states; all states $\varepsilon \approx m^2 - ic$ for all c

Interchange of states

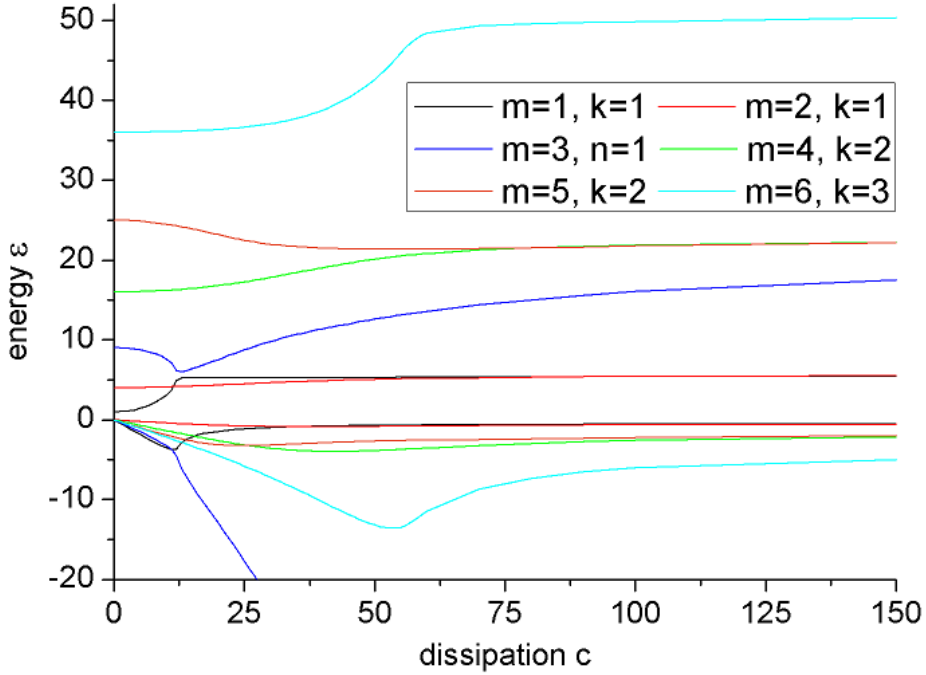


Figure: $\omega = 0.3$

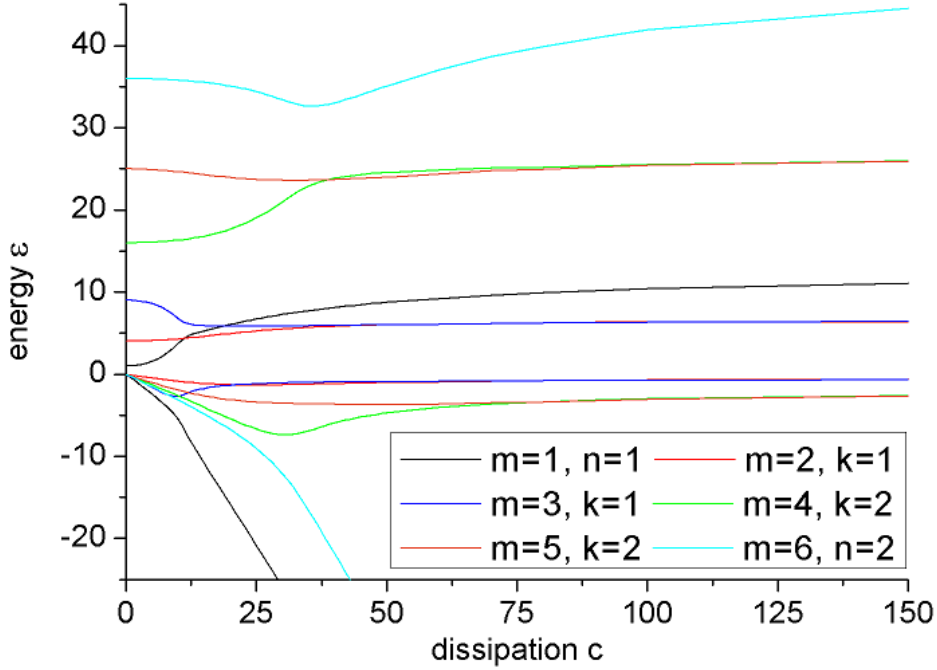
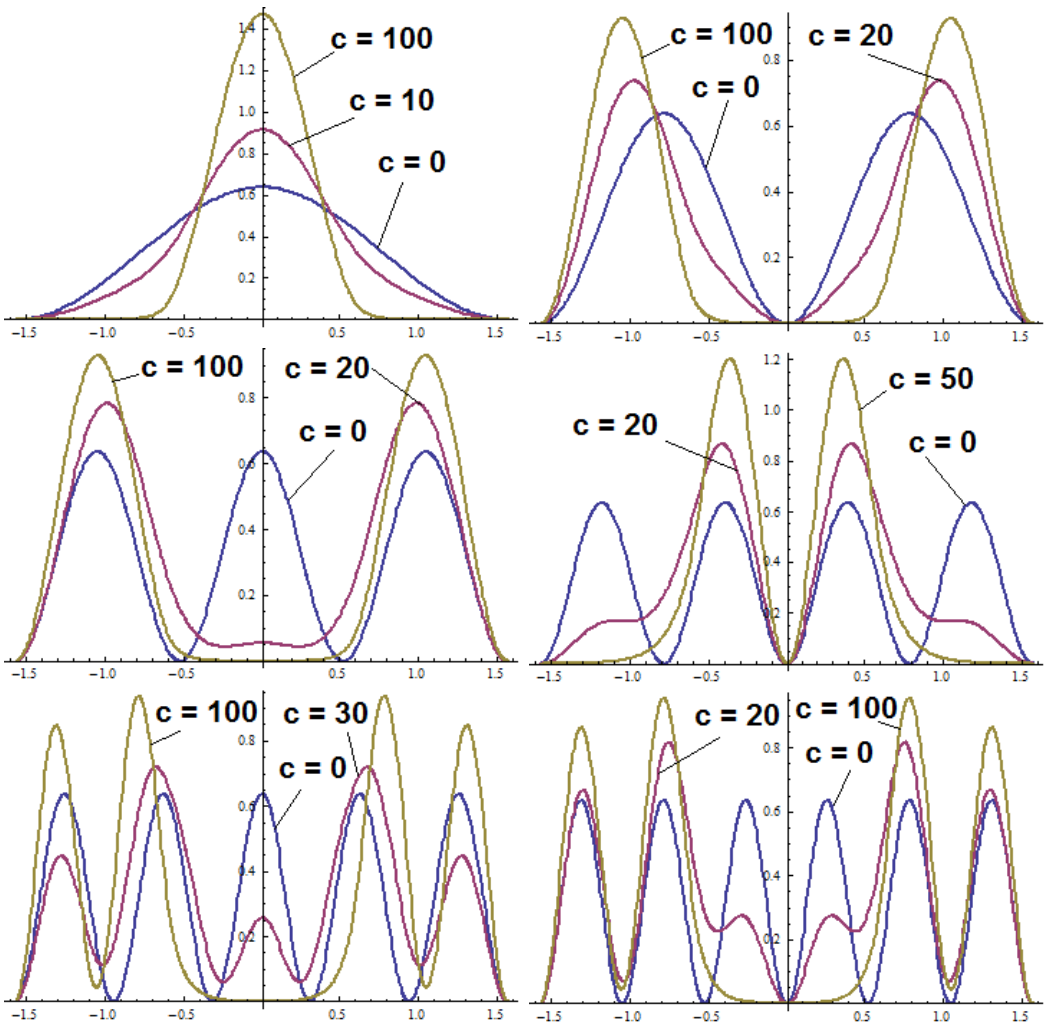


Figure: $\omega = 0.4$

- $$\left.\begin{aligned} \omega = 0.3 : m = 1 \to n = 1; m = 3 \to k = 1 \\ \omega = 0.4 : m = 1 \to k = 1; m = 3 \to n = 1 \end{aligned}\right\} \omega_{\text{crit}}$$
- Continuous transfer of k- and n-states between $\omega = 0$ and $\omega = \frac{\pi}{2}$

Densities



- Lowest energy levels for $\omega = 0.6$
- n-states tend to inside (area 2), k-states to outside (area 1, 3)
- Corresponds to $\omega = 0, \omega = \frac{\pi}{2}$ (areas vanish: states disappear)
- k-states: no central maximum
  $\rightarrow$ no symmetric states
- Three independent wells develop
- Area 1 and 3 equivalent
  $\rightarrow$ always 2 k-states fuse

Limit of large dissipation

- $\left.\begin{array}{l} n\text{-states: tend to center, strongly damped} \\ k\text{-states: tend to borders, less damped} \end{array}\right\}$ plausible results
- Ansatz: $\varepsilon_{\mathrm{I}}^{k} = 0$ for k-states and $\varepsilon_{\mathrm{I}}^{n} = -c$ for n-states for $c \to \infty$
- Limit for real part via quantization conditions (in original variables)

$$E_{\mathrm{R}}^{n} = \frac{\hbar^2 \pi^2}{2M(2w)^2} n^2 \ , \qquad E_{\mathrm{R}}^{k} = \frac{\hbar^2 \pi^2}{2M(L-w)^2} k^2$$

- Energies of real square well potentials with the width of each area
- Two k-states have same energy (area 1 and 3 have equal width)

Possible improvements

- Taking more dimensions, only two areas ($|\mathbf{r}| < w$ and $|\mathbf{r}| > w$)
- Considering interaction $g \neq 0$, derive Gross-Pitaevskii-Equation with complex potential via variational ansatz
- Harmonic trap $V_{\mathrm{R}} = \frac{1}{2} M \Omega_{\perp}^2 x^2$ instead of square well
- Next order Taylor approximation of V_{I}, harmonic approximation

$$V_{\mathrm{I}}(x) = C \left(1 - \frac{x^2}{w^2} \right)$$

- Actually no upper border for ω
- Nearly same results for harmonic approximation

Harmonic potential

- Nearly same results for harmonic approximation

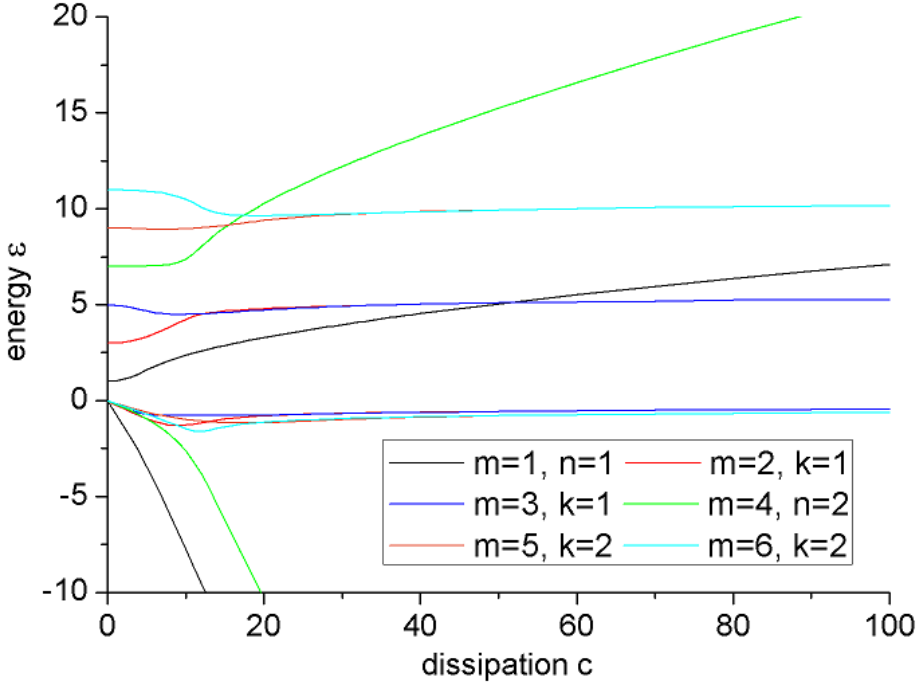
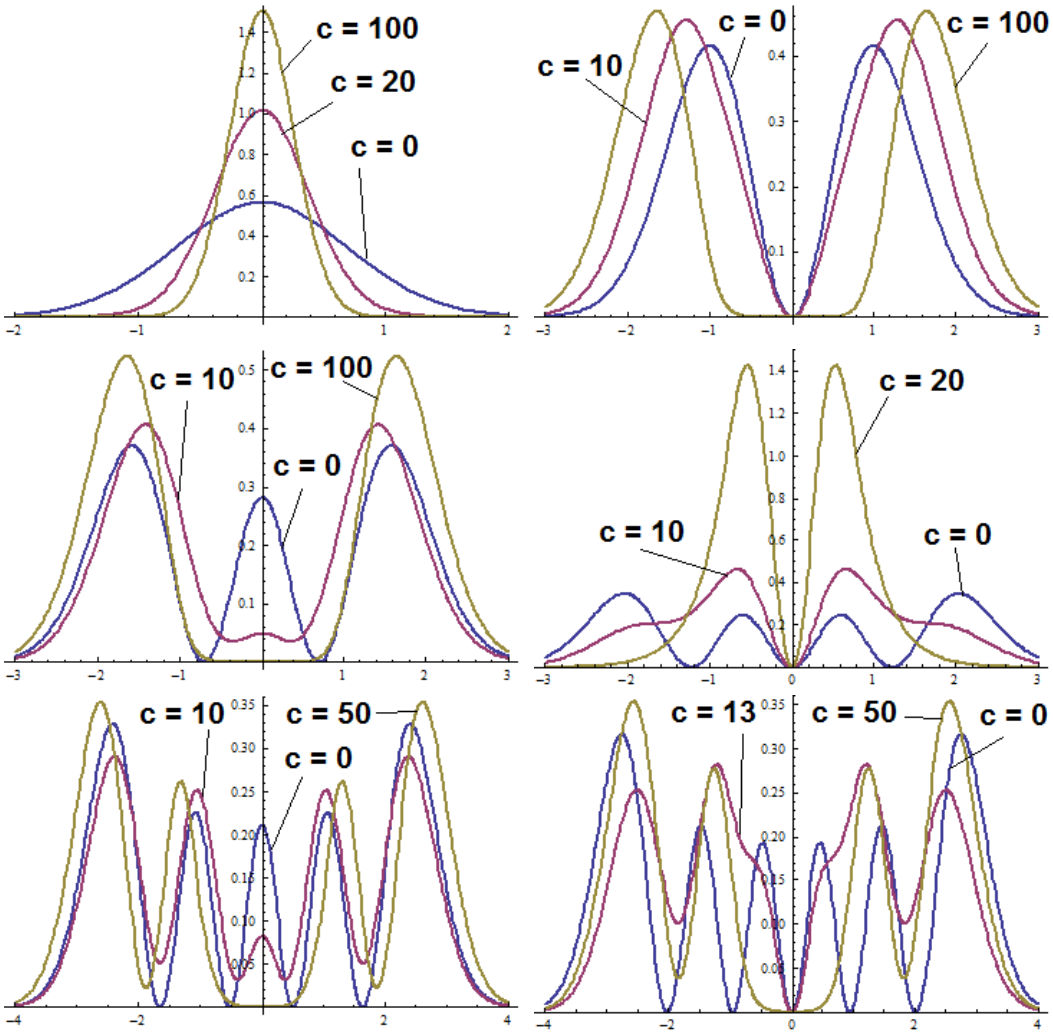
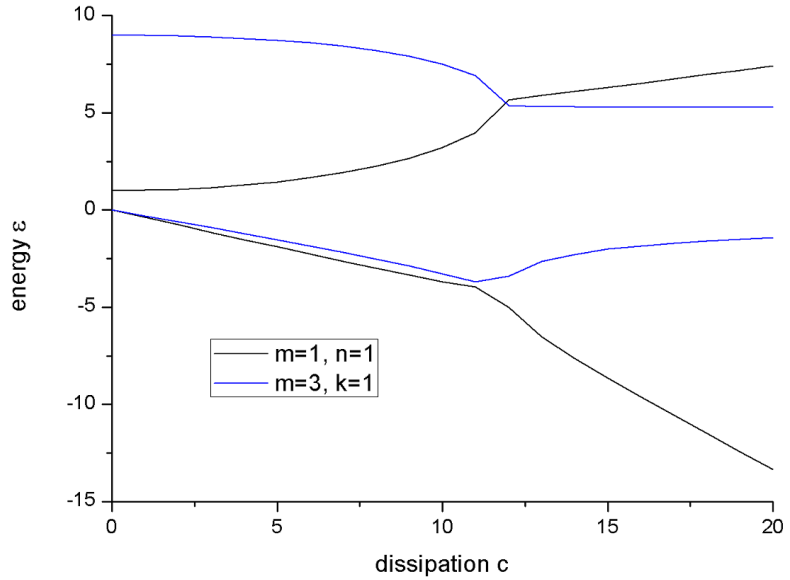
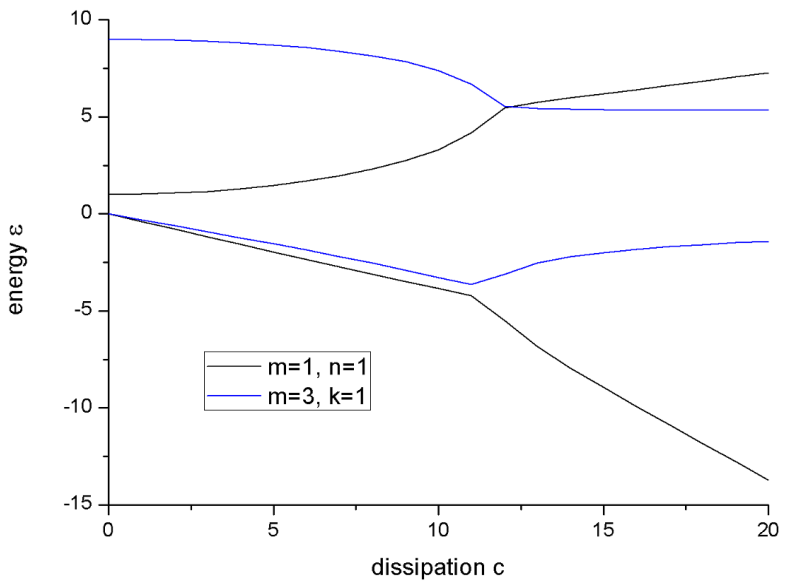


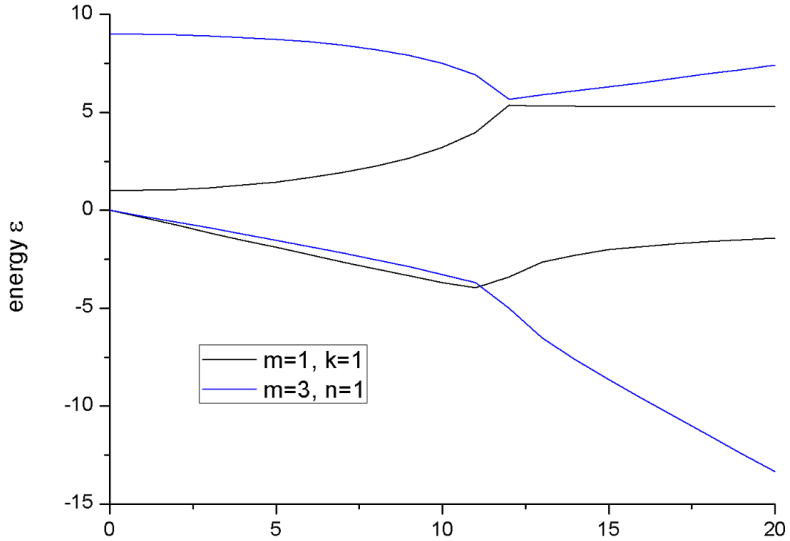
Figure: $\omega = 1.0$

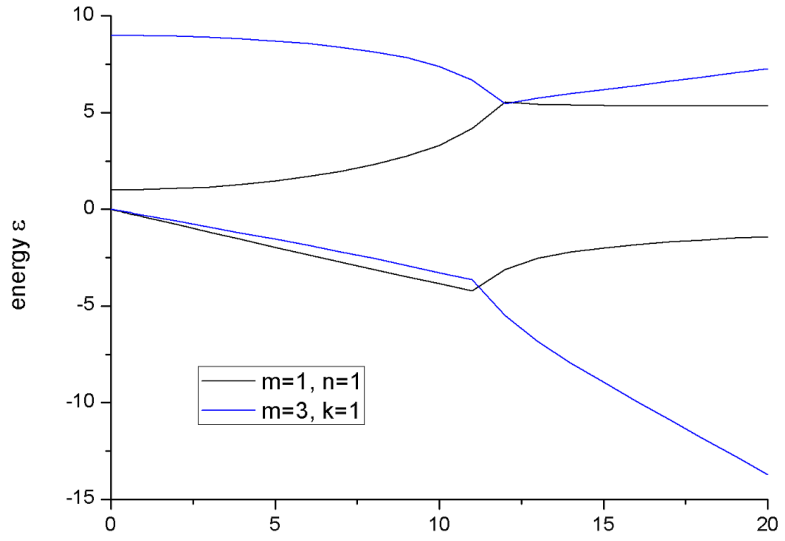


Possible assignments of numerical solutions









Non-hermitian dynamics

- Non-hermitian Hamilton operator $H = -\frac{\hbar^2}{2M}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + V_\mathrm{R}(x) + iV_\mathrm{I}(x)$
- Complex energy eigenvalues $E = E_\mathrm{R} + iE_\mathrm{I}$
- Time evolution exhibits non-stationary probability density

$$\rho(x,t) = \exp\left[\frac{2}{\hbar}E_\mathrm{I}(t-t_0)\right]\rho(x,t_0)$$

- Continuity equation includes drain of probability since $V_\mathrm{I}(x) \leq 0$

$$\frac{\partial}{\partial t}\rho(x,t) + \frac{\partial}{\partial x}j(x,t) = \frac{2}{\hbar}V_\mathrm{I}(x)\rho(x,t)$$