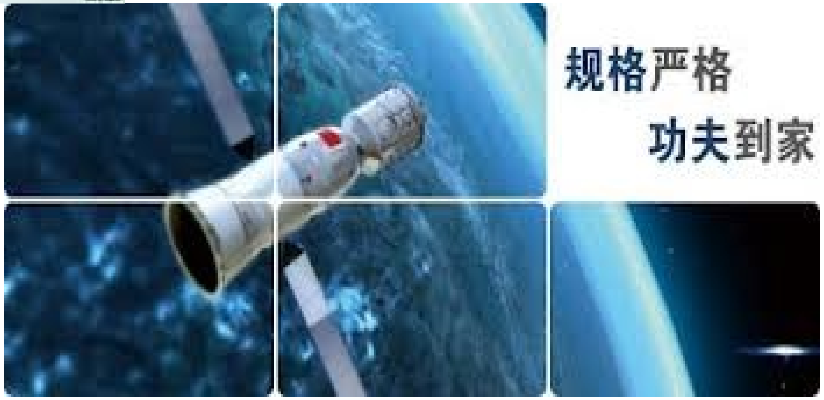
Harbin

6 million people, a middle size city in China

Harbin Institute of Techology



One of the top nine universities of China

Famous for robot and space craft

Therefore, the physicis department represents more the applied side of physics.

Harbin Robot Restaurant

Based on the robot technology of Harbin Institute of Technology

Artificially Tuned Optical Lattices

T. Wang, X.-F. Zhang, S. Eggert, and A. Pelster

05-09-2013

Institute of Physics,Belgrade

Outline

1. Superlattice System
   - .Introduction
   - .Model and GEPLT Method
   - .Quadratic Superlattice System
   - .Anisotropic Superfluidity in Kagome Superlattice
2. Periodically Driven System
   - .Introduction of AC Driven System
   - .Periodically Modulated S-Wave Scattering Length
   - .Floquet Theory and Effective Model
   - .Phase Boundary
   - .The Bose-Hubbard Like Effective Model
3. High order Perturbation Theory
   - .Basic Principle
   - .Result For Energy

.Superlattice System

.Introduction of Optical lattices

.Model and GEPLT Method

.Quadratic Superlattice System

.Anisotropic Superfluidity in Kagome Superlattice

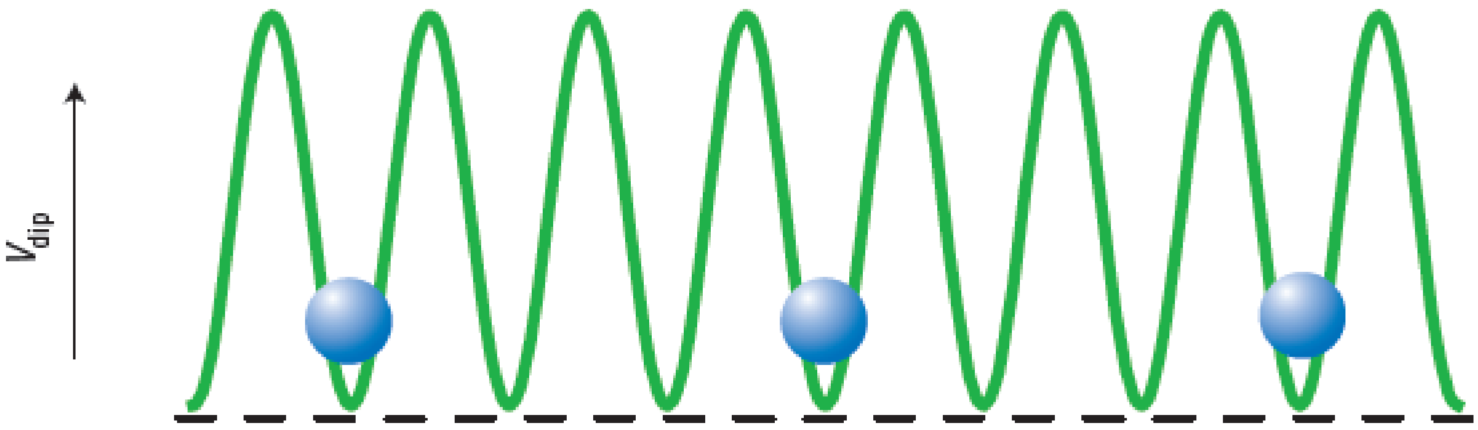
Introduction of Optical Lattice

An optical lattice is simply a standing laser wave

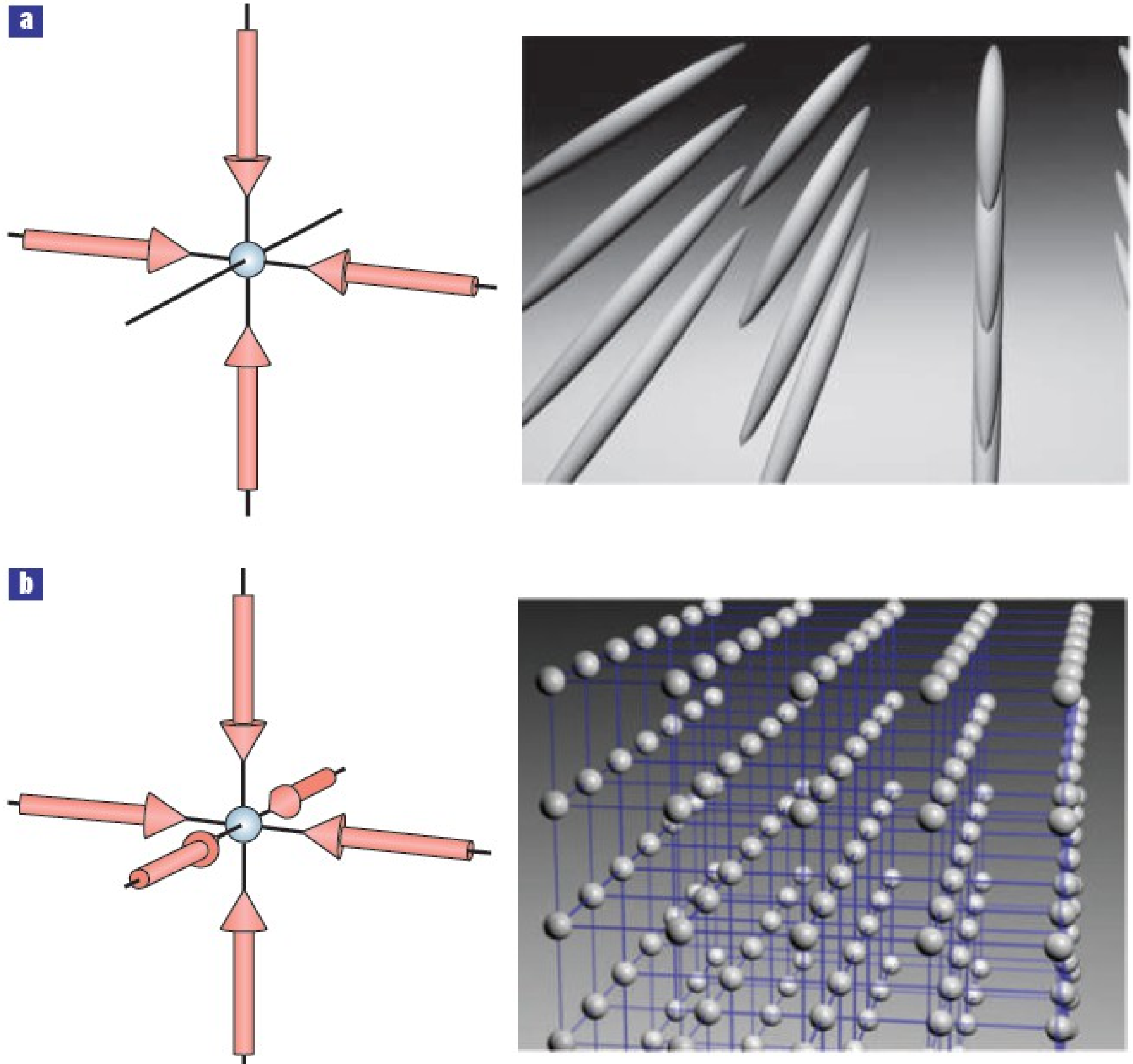
$$E = A\cos(kx - \omega t) + A\cos(-kx - \omega t)$$

$$V \propto I \propto \int (E^2)\,dt \propto \cos^2(kx)$$

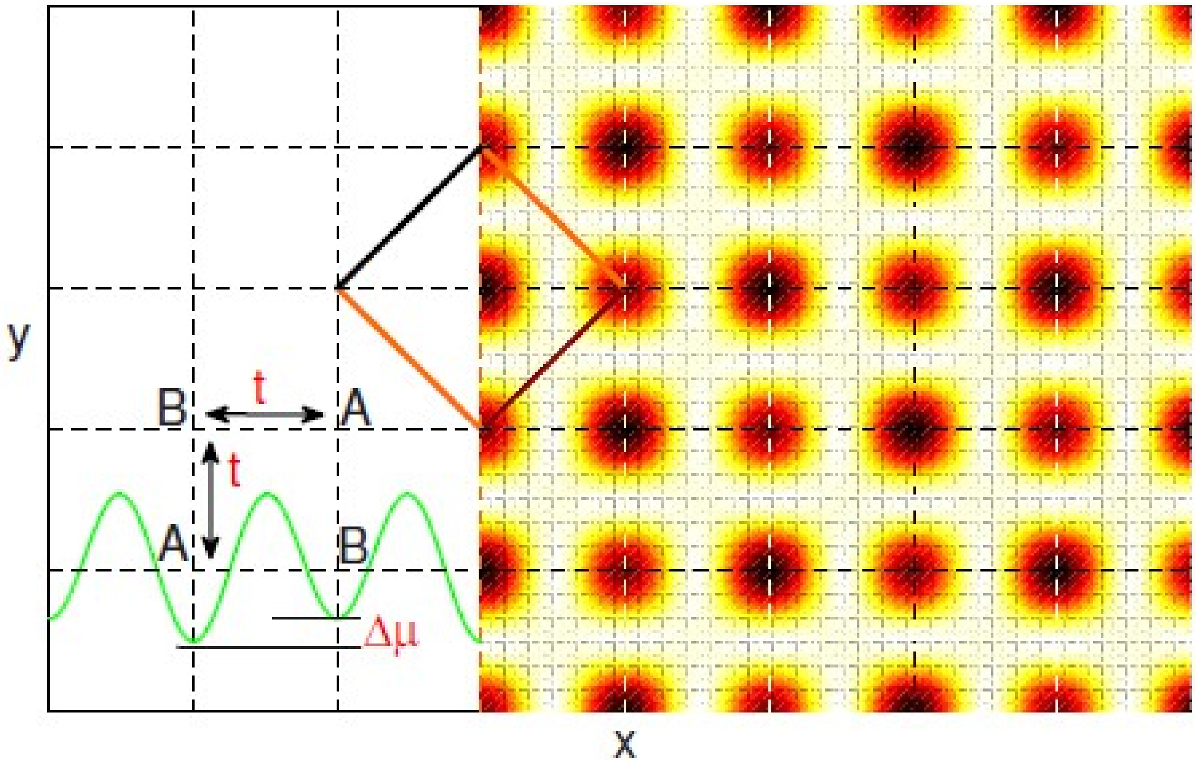
Red detuned: positive
Blue detuned: negative



I. Bloch, Nat. Phys. **1**, 23 (2005)

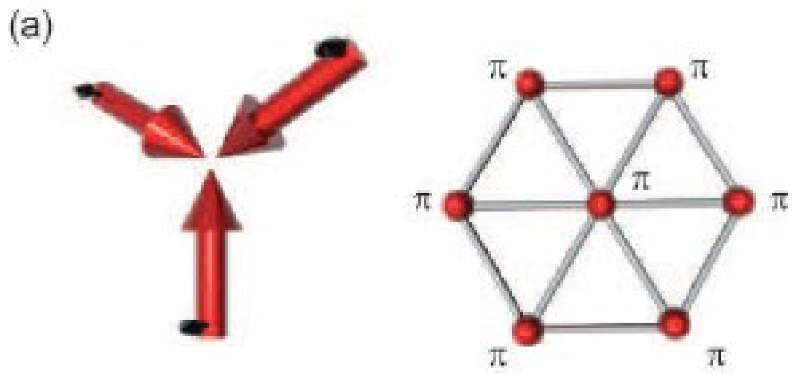

a
b

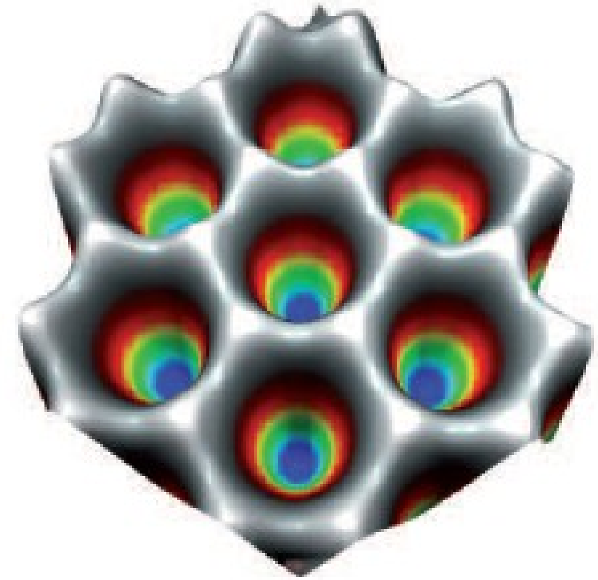
Optical Superlattice



1. use two short wave length lasers to form the optical lattice
2. use two long wave length lasers to enhance every second lattice

Triangular Lattice



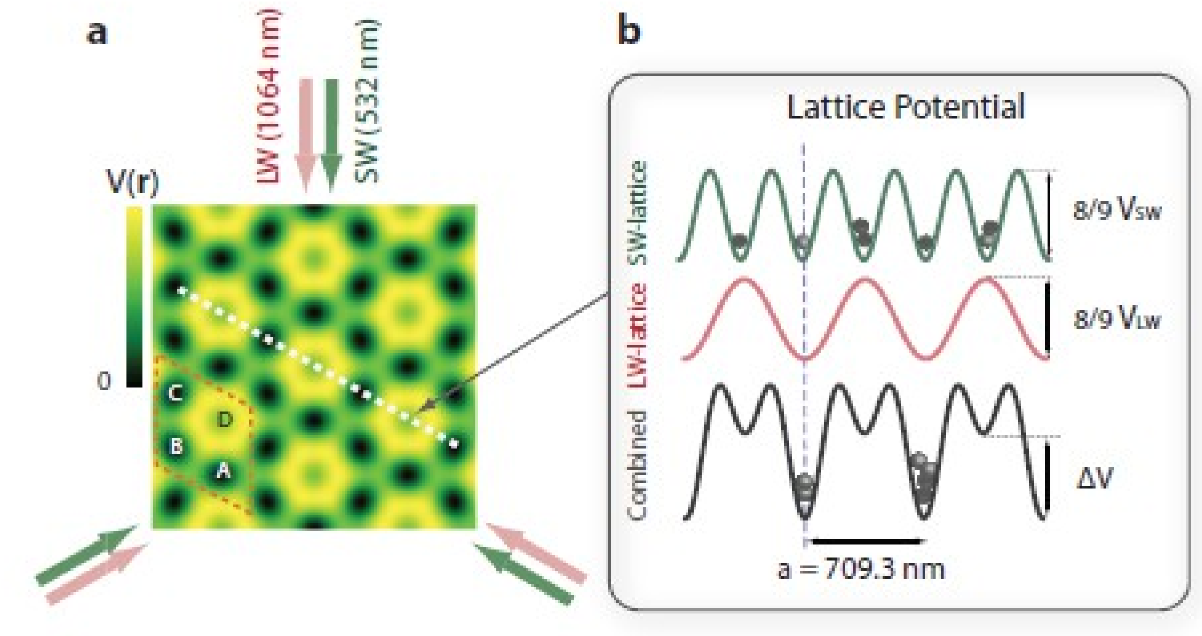


$$E = \Sigma_i A \cos\left(\vec{k}_i \bullet \vec{r} - \omega t\right)$$

$$k_1 = (0,1),\ k_2 = (\sqrt{3}/2, -1/2),\ k_3 = (-\sqrt{3}/2, -1/2)$$

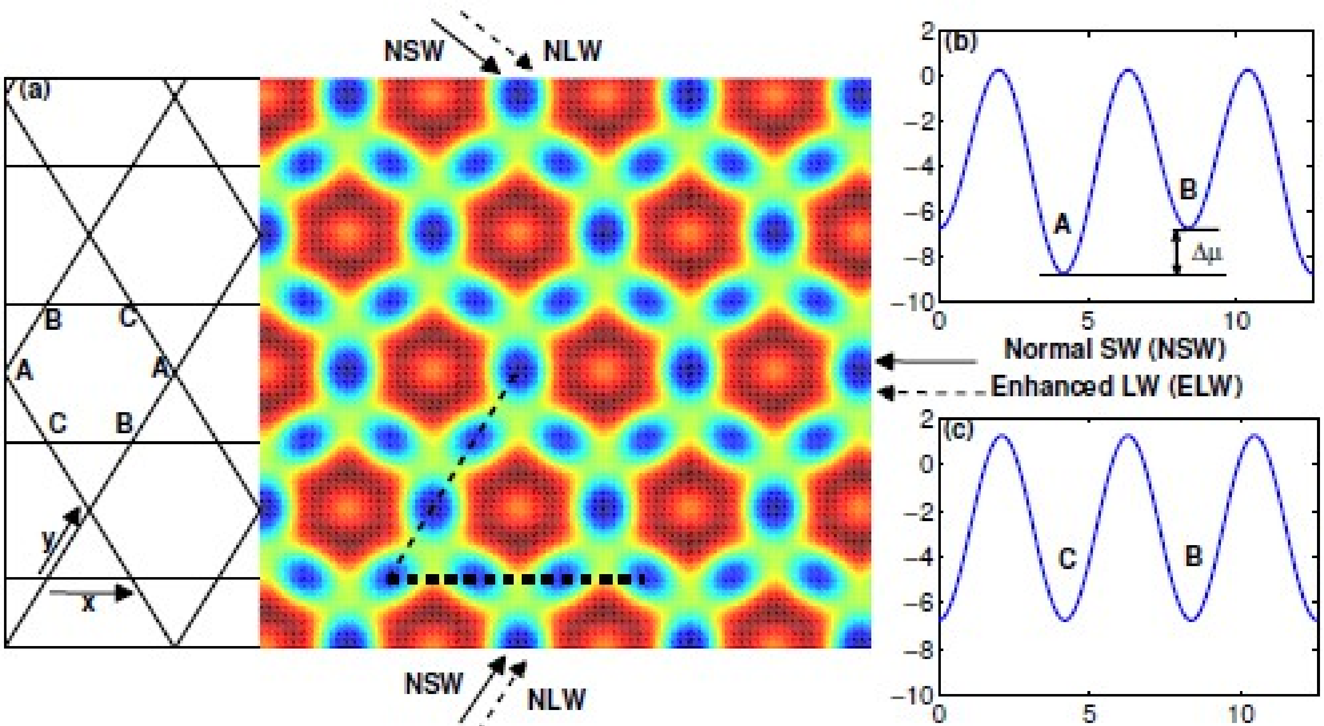
C. Becker, P. Soltan-Panahi, J. Kronjäger, S. Dörscher, K. Bongs and K. Sengstock, New J. Phys. **12,** 065025 (2010).

Kagome Optical Lattice



G.-B. Jo, J. Guzman, C. K. Thomas, P. Hosur, A. Vishwanath, and D. M. Stamper-Kurn, Phys. Rev. Lett.**108**, 045305 (2012).

Kagome Superlattice

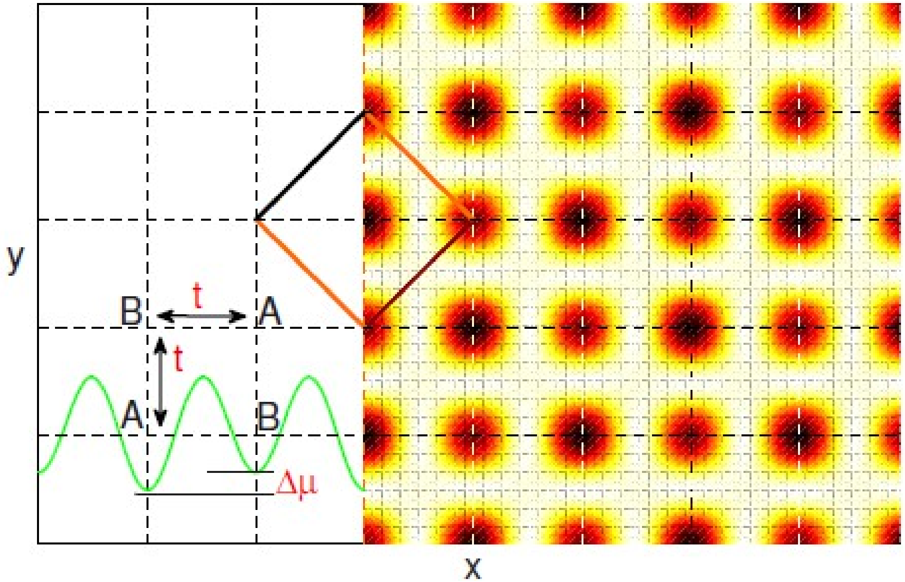


X-F. Zhang, T. Wang, A. Pelster and S. Eggert
Two face interaction blockade: anisotropic superfluidity of bosons in optical Kagome superlattice (in preparation)

Model

If the depth difference is small, the superlattice model can be described with:

$$\hat{H}_{\mathrm{SL}} = -t \sum_{\langle j\in A, j'\in B\rangle} (\hat{a}_j^\dagger \hat{a}_{j'} + \hat{a}_j \hat{a}_{j'}^\dagger) + \frac{U}{2} \sum_{j\in A,B} \hat{n}_j(\hat{n}_j - 1) - (\mu + \Delta\mu) \sum_{j\in A} \hat{n}_j - \mu \sum_{j\in B} \hat{n}_j$$



To deal with this system, we generalized the effective action theory to muti-component case.

Effective Action Method(1)

Effective action theory is based on Landau's phase transition theory and field theory.

1. Add spatially constant source terms:

$$\hat{H}_{\mathrm{BH}}(J,J^*) = -t\sum_{i,j}\hat{a}_i^\dagger\hat{a}_j + U/2\sum_i \hat{n}_i(\hat{n}_i-1) - \mu\sum_i \hat{n}_i + \sum_i\left(J^*\hat{a}_i + J\hat{a}_i^\dagger\right)$$

2. Expand free energy in power series of the source:

$$F(J,J^*,t) = N_s\left(F_0(t) + \sum_{p=1}^{\infty} c_{2p}(t)\,|J|^{2p}\right)$$

Effective Action Method(2)

3. Perform Legendre transformation to get effective potential:

$$\Gamma(\psi, \psi^*, t) = F/N_s - \psi^* J - \psi J^*.$$

4. Expand effective potential in power series of order parameter:

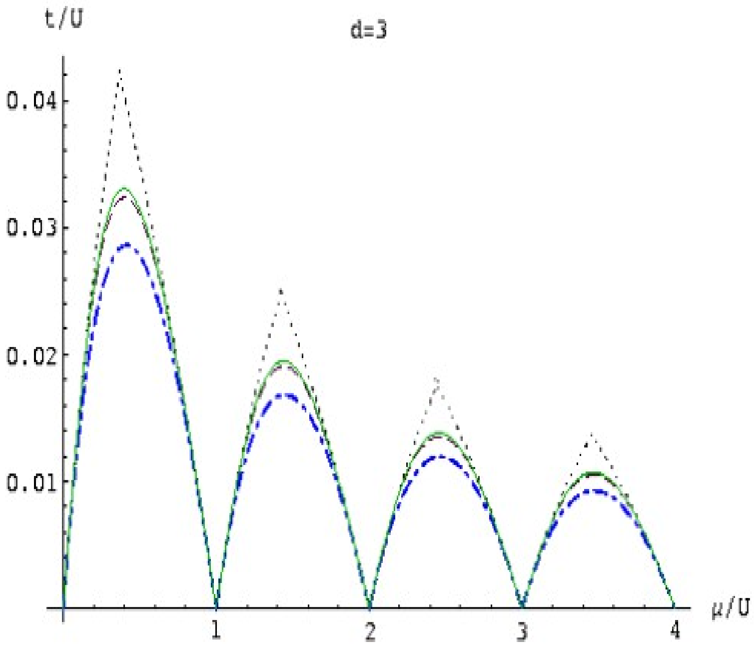
$$\Gamma(\psi, \psi^*, t) = F_0(t) - \frac{1}{c_2(t)}|\psi|^2 + \frac{c_4(t)}{c_2(t)^4}|\psi|^4 + \cdots$$

5. Perturbative calculation of second order coefficients:

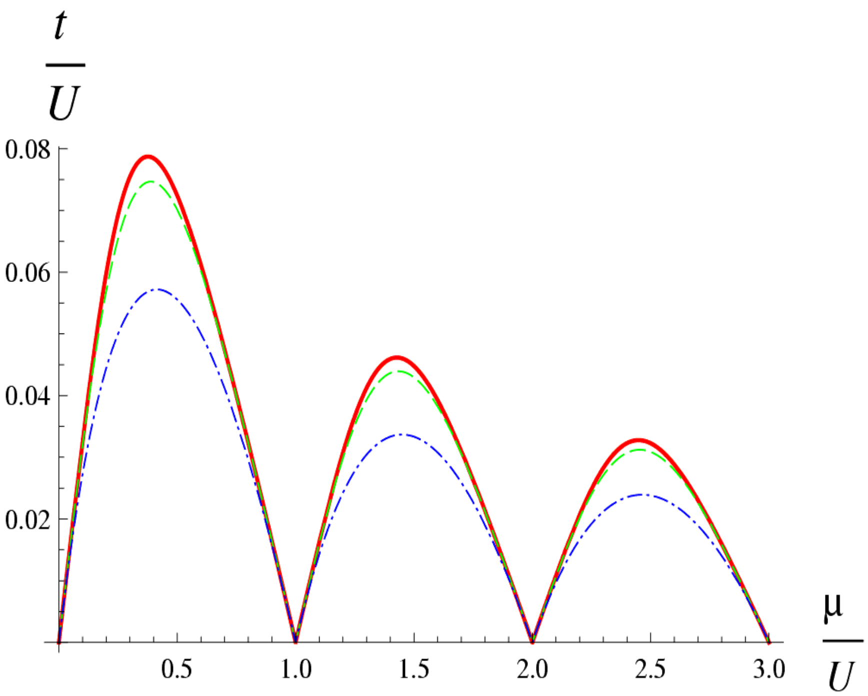
$$\frac{1}{c_2(t)} = \frac{1}{\alpha_2^{(0)}}\left(1 + \frac{\alpha_2^{(1)}}{\alpha_2^{(0)}}t + \left[\left(\frac{\alpha_2^{(1)}}{\alpha_2^{(0)}}\right)^2 - \frac{\alpha_2^{(2)}}{\alpha_2^{(0)}}\right]t^2 + \cdots\right)$$

Effective Action Method(3)

Effective action theory achieved great success both in quadratic and frustrated lattice system



Quadratic Lattice



Triangular lattice

1. F. E. A. dos Santos and A. Pelster, Phys. Rev. A **79**, 013614 (2009)
2. Z. Lin, J. Zhang and Y. Jiang, Phys. Rev. A **85**, 023619 (2012)

Multi-Components Effective Action Theory(1)

Multi-Components Effective Action Theory is in fact a more generalized form of effective action theory.

Purpose: to solve the second-order phase transition problem for a system that can be divided into several subsystems.

1. Add independent source terms for every subsystem:

$$\hat{H}_{\mathrm{BH}} = -\sum_{j,j'}\sum_{l,l'=1}^{m}[t_{j(l),j'(l')}\hat{a}^{\dagger}_{j(l)}\hat{a}_{j'(l')} + \mathrm{H.c.}] + \sum_{j}\sum_{l=1}^{m}\left[\frac{U_{(l)}}{2}\hat{n}_{j(l)}(\hat{n}_{j(l)} - 1) - \mu_{(l)}\hat{n}_{j(l)}\right]$$

Multi-Components Effective Action Theory(2)

2. Expand free energy in power series of source terms:

Define: $\vec{\mathbb{J}} = (J_1, \ldots, J_m)^{\mathrm{T}}$;

$$F(\mathbb{J}, t) = N_s \left(F_0(t) + \mathbb{J} C_2 \mathbb{J}^\dagger + \cdots\right)$$

3. Perform Legendre transformation to get effective potential:

Define: $\Psi = \frac{1}{N_s} \frac{\partial F}{\partial \mathbb{J}^\dagger} = \{\langle \hat{a}_1 \rangle, \ldots, \langle \hat{a}_m \rangle\}$

$$\Gamma(\Psi, t) = F/N_s - \Psi \mathbb{J}^\dagger - \mathbb{J} \Psi^\dagger$$

4. Expand effective potential in power series of order parameter:

$$\Gamma(\Psi, t) = \Gamma_0(t) + \Psi A_2 \Psi^\dagger + \cdots$$

Multi-Components Effective Action Theory(3)

5. Resulting relation between coefficients C and A:

$$-1 = A_2 \frac{d\Psi}{d\mathbb{J}} = A_2 \frac{1}{N_s} \frac{\partial^2 F}{\partial \mathbb{J}^\dagger \partial \mathbb{J}} = A_2 C_2$$

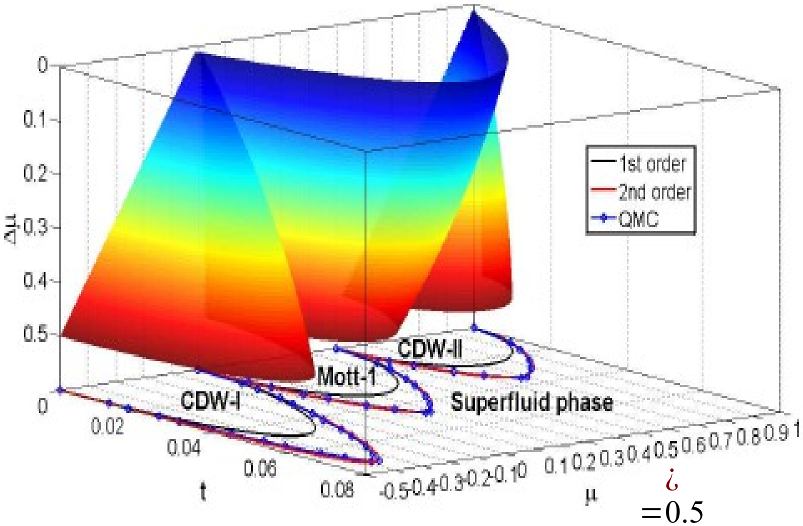
The second-order phase transition boundary condition is:

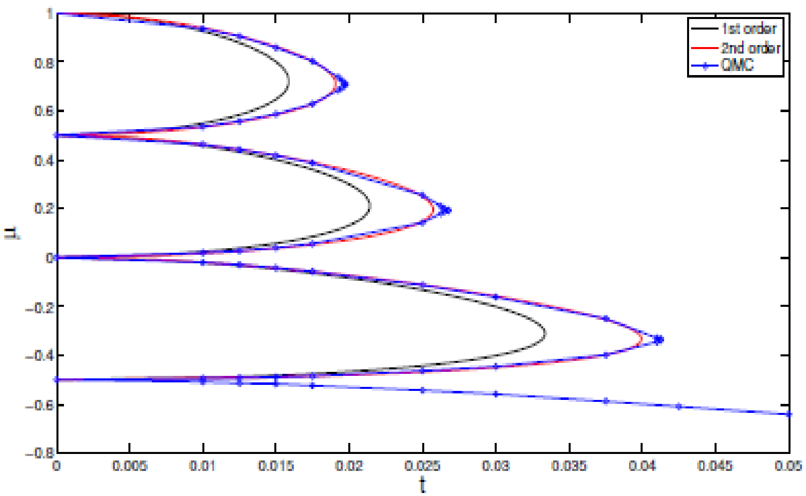
$$\frac{-1}{\begin{vmatrix} c_{2A11} & \cdots & c_{2A1m} \\ \cdots & \cdots & \cdots \\ c_{2Am1} & \cdots & c_{2Amm} \end{vmatrix}} = 0$$

or **The determinent of one of its submatrices goes to zero**

6. Perturbatively calculated second-order coefficient

Quadratic Superlattice Phase Boundary





2d system result

3d system result

Δμ=0.25

T. Wang, X.-F. Zhang, S. Eggert, and A. Pelster:
Generalized Effective Potential Landau Theory for Bosonic Superlattices;
Physical Review A **87**, 063615 (2013)

Advantages comparing to decoupled mean field theory

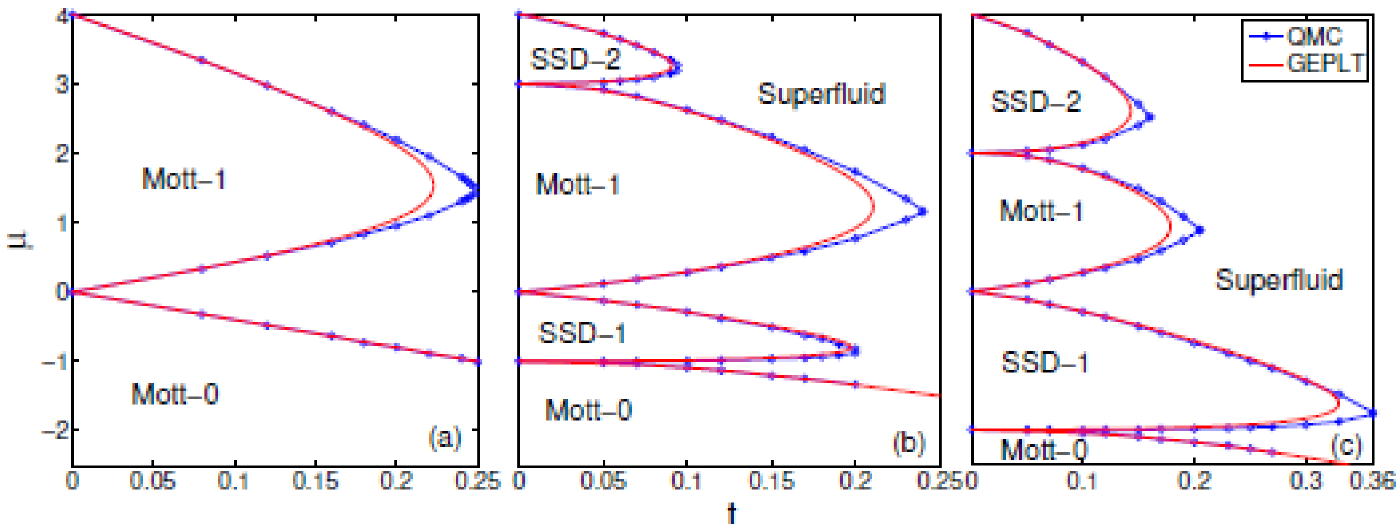
1. Our method has much higher accuracy:

method	DMF	MCEA (2nd order)
accuracy	About 30%	Less than 3%

2. Using DMF method the free energy is at the local maximal point in the Mott lobe, our method does not have this problem.

Kagome Superlattice

Phase boundary



Δμ=0

Δμ=1

Δμ=2

In all the cases, we set the on site interaction energy U=4

Anisotropic Superfluidity

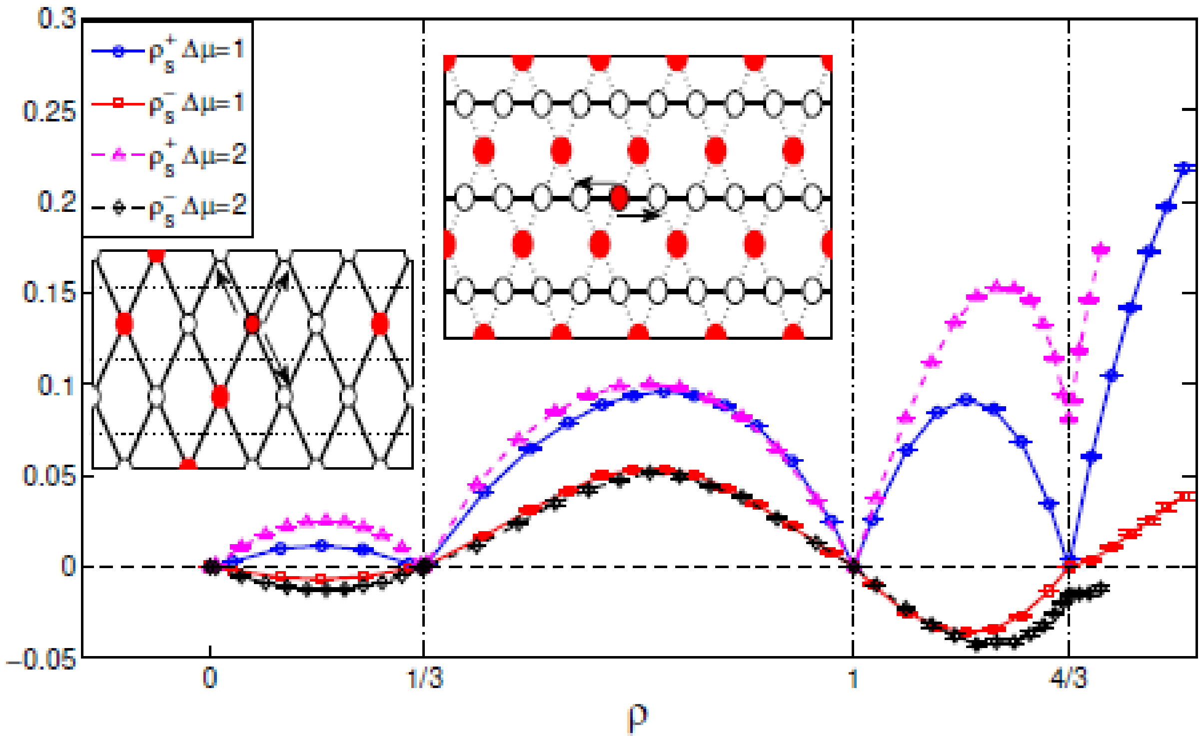


FIG. 4: The total superfluid density and the stagger superfluid density vs μ for $\beta = 300$ and $L = 9$ at $U = 4$ and $t = 0.1$. Inset: schematically showing the mechanism of the anisotropy of the superfluid density.

2. Periodically Driven System

.Introduction of Driven System

.Periodically Modulated S-Wave Scattering Length

.Floquet Theory and Effective Model

.Phase Boundary

.The Bose-Hubbard Like Effective Model

BEC of Time- Periodically Driven System(1)

Is driven system an equibrium system?

To answer this question, first we should introduce the concept of the Floquet state.

Floquet theory

for $\hat{H}(t)=\hat{H}(t+T)$

$$i\hbar\partial_t|\Psi(t)\rangle=\hat{H}(t)|\Psi(t)\rangle$$

Has the Floquet state solution

$$|\Psi_n(t)\rangle=|u_n(t)\rangle\exp(-i\varepsilon_n t/\hbar)$$

The wave function of the system can be written as

$$|\psi\rangle=\sum_n c_n|u_n(t)\rangle\exp(-i\varepsilon_n t/\hbar)$$

BEC of Time- Periodically Driven System(2)

So the occupation number of the Floquet state does not change during the driving. In this point of view, the driven system can be seen as an equibrium system.

Floquet condensate

If you redo the Bose statistics for Floquet state, then:

$$\sum_i n_i = N$$

$$\sum_i \bar{E}_i = E_{tot}$$

With these constrains we get:

$$n_i / g_i = \frac{1}{\exp(-\gamma \bar{E} + \alpha) - 1} \qquad \gamma = \frac{-1}{k\Theta}$$

So the particles will condensate into the Floquet state with the lowest average energy when the driven relevant temperature is very low.

Periodically Modulated S-Wave Scattering Length

The on-site interaction is determined by s wave scattering length, if we generate it with a small driving near the Feshbach:

$$a(B)=a_{BG}\left(1-\frac{\Delta}{B-B_{\infty}}\right)$$

If the driving amplitude is small

$$\hat{H}(t)=-\sum_{ij}J_{ij}\hat{a}_i^{\dagger}\hat{a}_j+\sum_i\left[\frac{U}{2}\hat{n}_i\left(\hat{n}_i-1\right)-\mu\hat{n}_i\right.$$
$$\left.+Ag_i(\hat{n}_i)cos\left(\omega t\right)\right].$$

with

$$g_i(\hat{n}_i)=\frac{1}{2}\left(\hat{n}_i^2-\hat{n}_i\right)$$

Time-Independent Hamiltonian

The Floquet state in the representation of occupation number:

$$|n_i, m(t)\rangle = exp(im\omega t)\prod_i exp\left(-\frac{iAg_i(n_i)}{\hbar\omega} sin\omega t\right)|n_i\rangle$$

In the extended Fock space, time is regarded as a coordinate

$$\langle\langle\bullet|\bullet\rangle\rangle = \frac{1}{T}\int_0^T dt\langle\bullet|\bullet\rangle$$

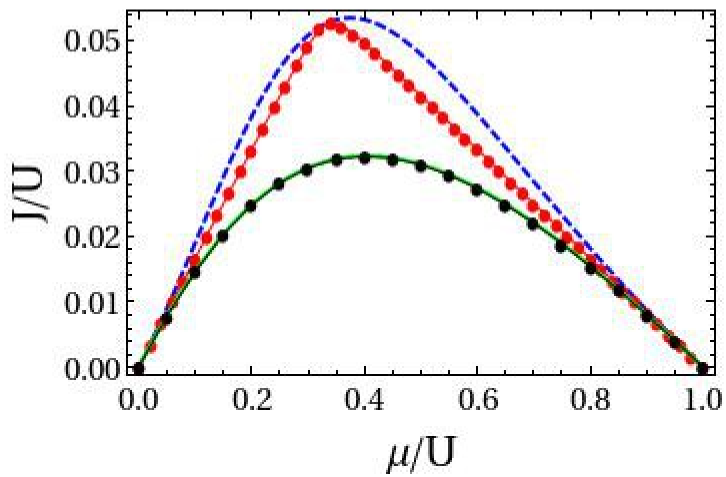
Then we can get the effective Hamiltonian

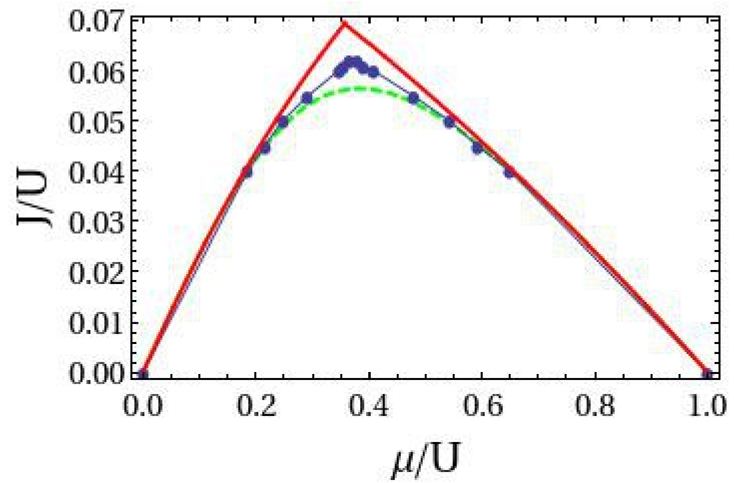
$$\hat{H}_{eff} = -\sum_{ij} J_{ij}\hat{a}_i^{\dagger} J_0\left(\frac{A}{\hbar\omega}(\hat{n}_j - \hat{n}_i)\right)\hat{a}_j + \sum_i \frac{U}{2}\hat{n}_i(\hat{n}_i - 1) - \sum_i \mu\hat{n}_i$$

condition

$U < \hbar\omega < \Delta$

Validity of EPLT Method and Phase Diagram



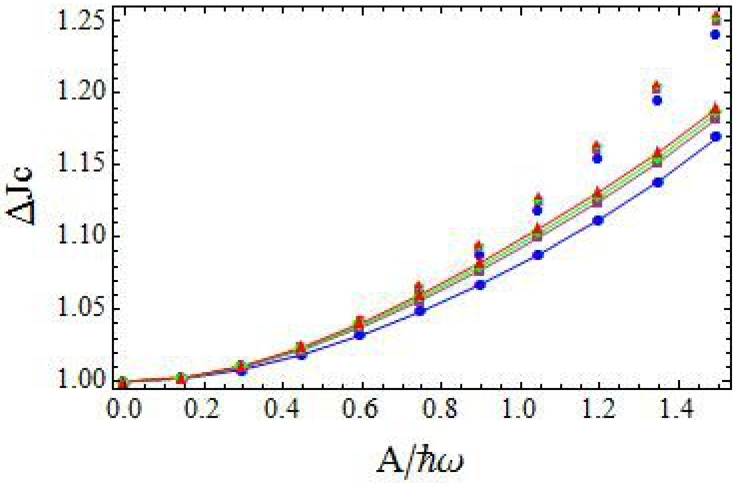


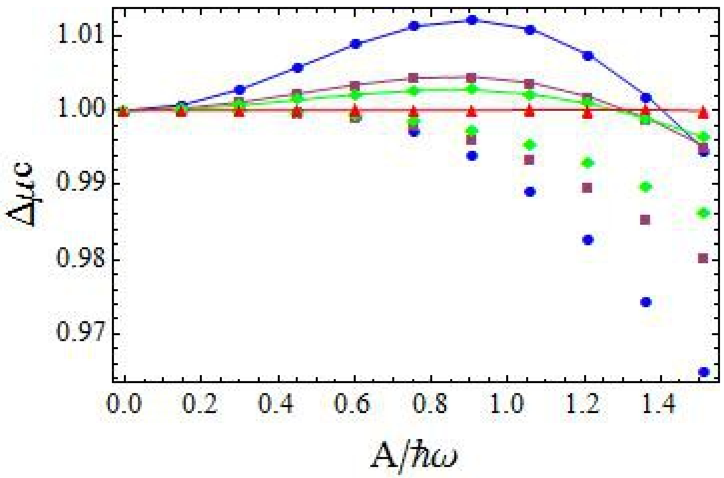
From the comparison between phase diagrams of the first order EPLT method and GMF for 2d driven system, we can see that when driving is small, EPLT method is believable.

So we get the condition for the driven system:

$$\frac{A}{\hbar\omega} < 1.52$$

Critical Points





The change of the critical point of the phase diagram. All the lobes increase with almost the same percentage.

It means that the scaling property for Bose-Hubbard model still exists:

$$\widetilde{\left(\frac{J}{U}\right)}_c = \sqrt{g(g+1)}\left(\frac{J}{U}\right)_c$$

Bose -Hubbard Like Effective Hamiltonian

This means that we can guess a new effective Hamiltonian

$$\hat{H} = -J\lambda\left[\frac{A}{\hbar\omega}\right]\sum_{ij}\hat{a}_i^{\dagger}\hat{a}_j + \sum_i \frac{U}{2}\hat{n}_i(\hat{n}_i - 1) - \sum_i \mu\hat{n}_i$$

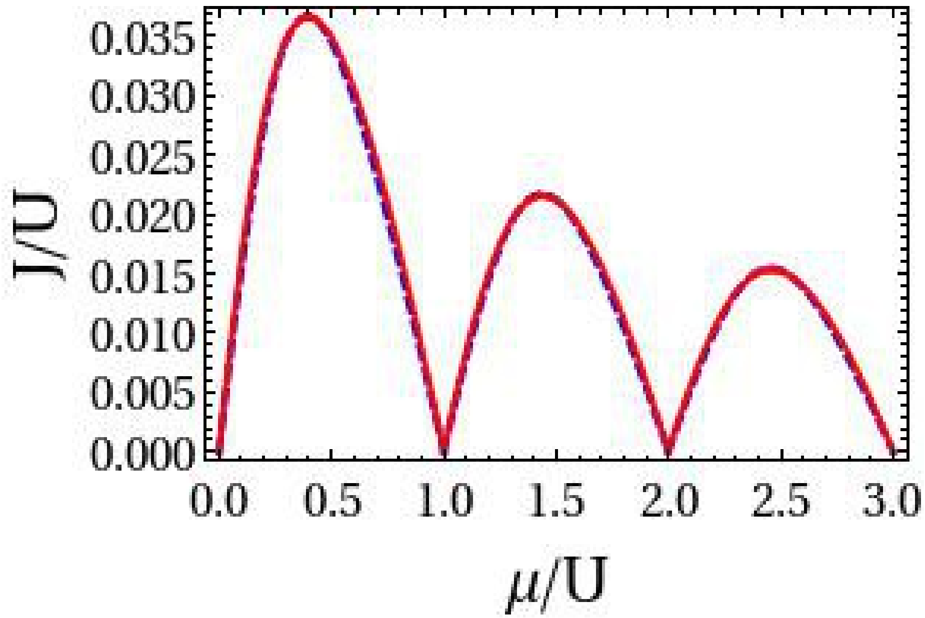
In which:

$$\lambda\left[x\right] = 1 + ax + bx^2 + cx^3 + dx^4$$

	a	b	c	d
2d	-0.0018	0.1212	0.0561	0.0178
3d	-0.0045	0.1356	0.0366	0.0129

New Hamiltonian has the same critical point as original model

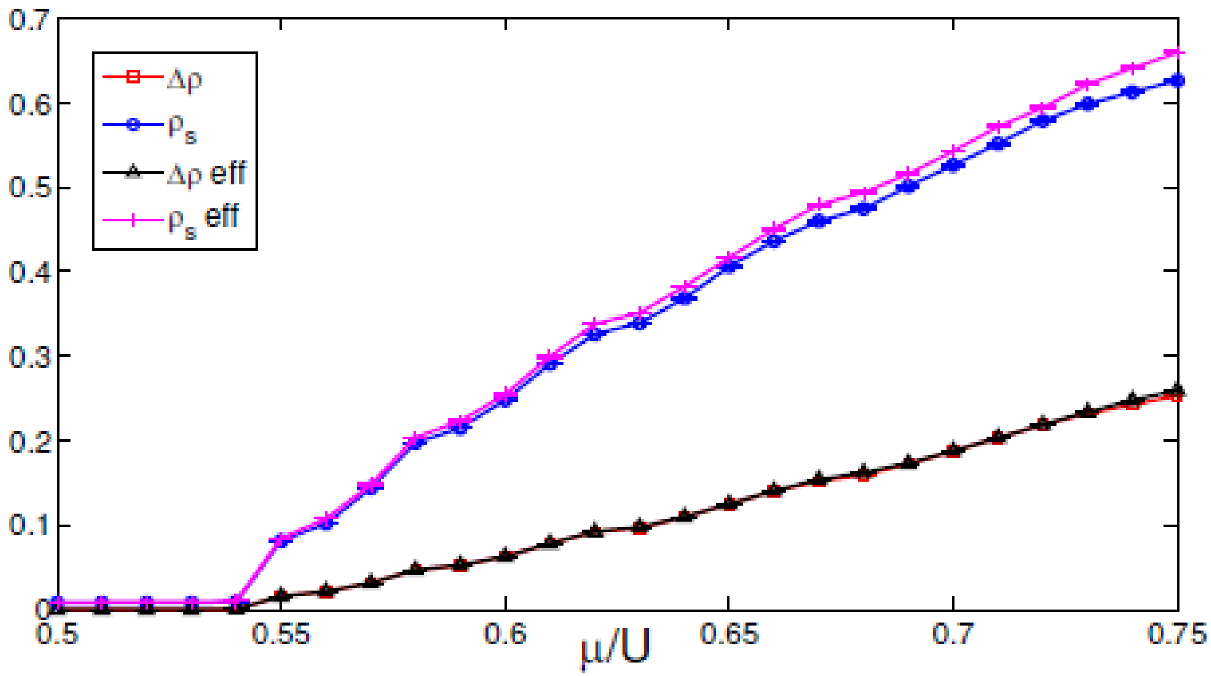
Comparing with Phase Diagram



Comparing with the phase diagram between the new effective model and original model with driven parameter equals 1.5.

This means they have almost the same phase diagram

Density and Superfluid Density



: A comparison of QMC calculation of density and superfluid density between new effective model and original model shows that they have the same critical properties

T. Wang, X-F. Zhang, S. Eggert and A. Pelster. Quantum Phase Diagram of Bosons with Modulated Scattering Length in higher dimensional Optical Lattice(in preparation)

High order Perturbation Theory

.Basic Principle

.Result For Energy

Kato Representation(1)

For Hamiltonian

$$H_{\mathrm{BH}} = H_0 + H_{\mathrm{tun}}$$

The nth order correction of the energy:

$$E_{\mathbf{m}}^{(n)} = \mathrm{tr}\left[\sum_{\{\alpha_\ell\}} S^{\alpha_1} V S^{\alpha_2} V S^{\alpha_3} \ldots S^{\alpha_n} V S^{\alpha_{n+1}} \right]$$

In which

$$S^{\alpha} = \begin{cases} -|\mathbf{m}\rangle\langle\mathbf{m}| & \text{for } \alpha = 0 \\ \displaystyle\sum_{i \neq \mathbf{m}} \frac{|i\rangle\langle i|}{(E_{\mathbf{m}}^{(0)} - E_i^{(0)})^{\alpha}} & \text{for } \alpha > 0 \end{cases}$$

With the condition

$$\sum_{\ell=1}^{n+1} \alpha_\ell = n - 1$$

Kato Representation(2)

$$\hat{S}^{\alpha}\hat{S}^{\alpha'} = \begin{cases} -\hat{S}^0 & \text{for} \quad \alpha = 0 \text{ and } \alpha' = 0 \\ 0 & \text{for} \quad \alpha = 0 \text{ and } \alpha' \neq 0 \\ & \quad \text{or } \alpha \neq 0 \text{ and } \alpha' = 0 \\ \hat{S}^{\alpha+\alpha'} & \text{for} \quad \alpha \neq 0 \text{ and } \alpha' \neq 0 \end{cases}$$

So energy correction can always be expressed as an expectation value

$$E_g^{(\nu)} = \sum_{(\nu-1)} G_{\{\alpha_k\}} \langle g|\hat{V}\hat{S}^{\alpha_{\nu-1}}\hat{V}\cdots\hat{S}^{\alpha_2}\hat{V}\hat{S}^{\alpha_1}\hat{V}|g\rangle$$

can be simply written as:

katolist

$$E_g^{(\nu)} = \sum_{(\nu-1)}{}' (\alpha_{\nu-1}, \ldots, \alpha_2, \alpha_1)$$

The first step is to use computer to generate all the katolist

Expectation Value

To calculate the expectation value of A and B

$$\hat{H}_{AB} = \hat{H}_0 + \lambda\hat{V} + x\hat{A} + y\hat{B}$$

The energy correction can be written as

$$E_{G_{AB}} = \sum_{\nu,m,k} E_g^{(\nu,m,k)} \lambda^\nu x^m y^k$$

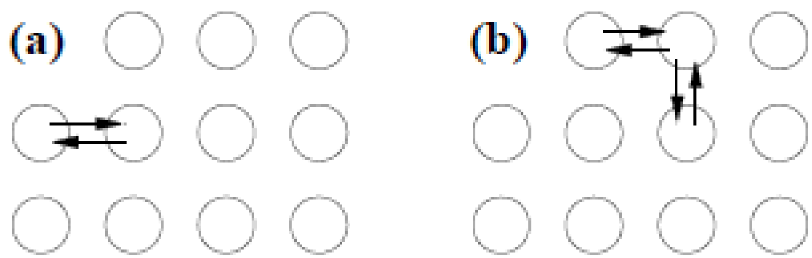
Let x and y go to zero

$$\chi_{AB} = \sum_{\nu} \lambda^\nu E_g^{(\nu,1,1)}$$

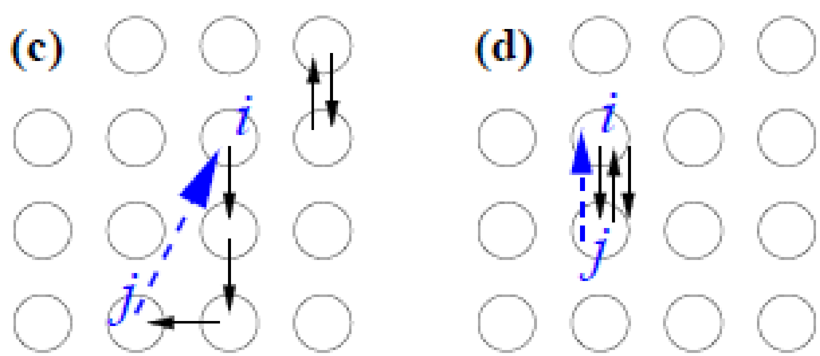
$$\chi_{AA} = \frac{1}{2} \sum_{\nu} \lambda^\nu E_g^{(\nu,2,0)}$$

Lattice system

For the lattice diagram, the energy correction can be expressed diagrammatically as follows:



Energy correction diagram(diagonal)



Non diagonal physical quantity diagram

The second step is to generate all the diagrams in computer

Calculation

After having obtained all diagrams, we can calculate the correction of each diagram, because all the diagrams are independent, it is very easy to parallelize.

Calcualtion of diagram

1. get all the process-chain of the diagram

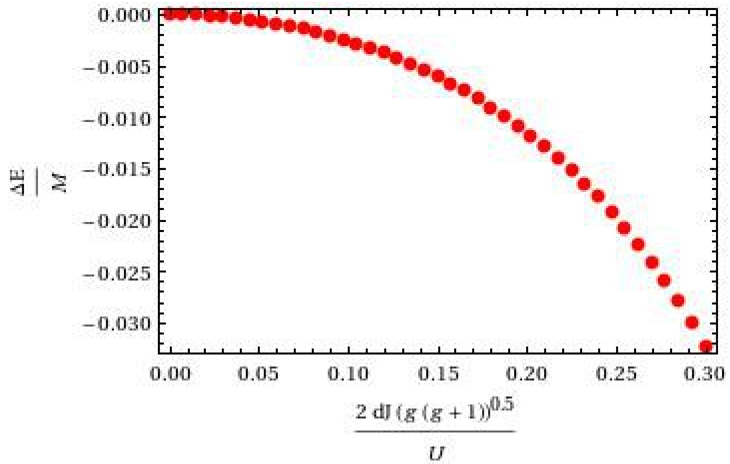
   . all possible sequence of the operators.

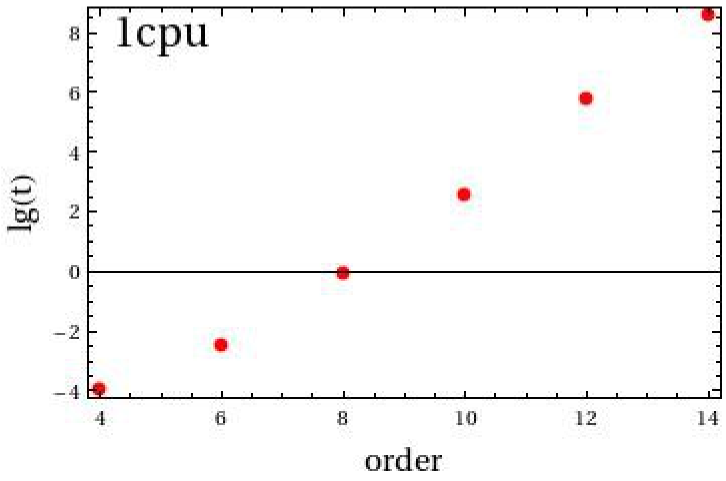
2. calculate the process-chain

   .over the path judge ground or excited $|g\rangle \rightarrow |e_1\rangle \rightarrow |e_2\rangle \rightarrow \cdots$
   . looking for appropriate katolist
   .calculating over every approriate katolist
   .add all the values to get the value of process-chain value

3. add all process-chain value

Result for Ground State Energy





The result for ground state energy of 2d Bose-Hubbard model, for g=1,2,10, the value fall on top of each other.

The calculating time of the energy correction is like this, when we increase two orders, the calculation time increase 1000 times.

Conclusion

.we generalized the EPLT method to multi-component case and used it on superlattice system. This opens the possibility to deal with other complicated systems in the future.

.we propose a very simple way to fulfill Kagome superlattice system in the experiment, found that in this system, due to two face of interaction blockade, the bias of anisotropy is alternating between two directions while detuning the particle numbers.

.we get a new Hubbard-like effective Hamiltonian for s-wave scattering system near the phase-transition, and point out that the driven system has the same critical phenomena with the non-driven system when the driving is not large.

.we use process-chain method to calculate the ground-state energy for Bose-Hubbard system.

Thank You!