TOF for Trapped Dipolar Fermi Gases: From Collisionless to Hydrodynamic Regime

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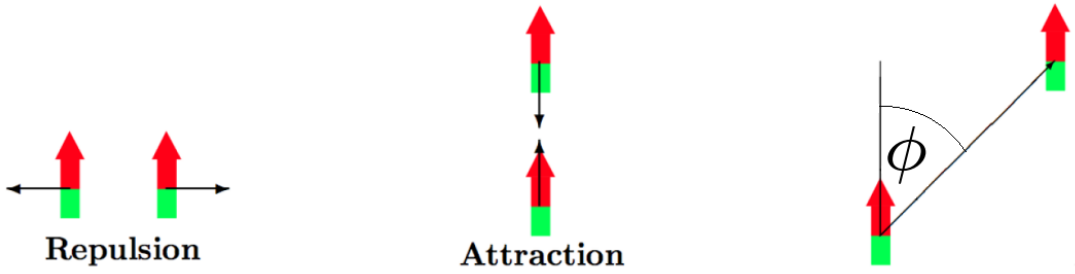
Dipole-dipole interaction (DDI)

- DDI potential $V_{\text{int}}(\mathbf{r}) = \frac{C_{\text{dd}}}{4\pi|\mathbf{r}|^3}\left(1 - 3\cos^2\phi\right)$



- $C_{\text{dd}} = \mu_0 m^2$ for magnetic dipole moment m
  Dipolar atoms: ^{53}Cr, ^{164}Dy, ^{167}Er
- $C_{\text{dd}} = d^2/\varepsilon_0$ for electric dipole moment d
  Dipolar molecules: ^{40}K^{87}Rb, ^{23}Na^{40}K
- Dipolar Fermi gas: Fermi ellipsoid

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Boltzmann-Vlasov equation

- Dynamics of the system:

$$\frac{\partial f(\mathbf{r},\mathbf{p},t)}{\partial t}+\frac{\mathbf{p}}{M}\nabla_{\mathbf{r}}f+\nabla_{\mathbf{p}}U(\mathbf{r},\mathbf{p},t)\nabla_{\mathbf{r}}f(\mathbf{r},\mathbf{p},t)-\nabla_{\mathbf{r}}U(\mathbf{r},\mathbf{p},t)\nabla_{\mathbf{p}}f(\mathbf{r},\mathbf{p},t)=I_{\mathrm{coll}}[f](\mathbf{r},\mathbf{p},t)$$

- Wigner function: $f(\mathbf{r},\mathbf{p},t) \rightarrow$ spatial density: $n(\mathbf{r},t)=\int \frac{d\mathbf{p}f(\mathbf{r},\mathbf{p},t)}{(2\pi\hbar)^3}$
- Hartree-Fock mean-field potential

$$U(\mathbf{r},\mathbf{p},t)=U_{\mathrm{ext}}(\mathbf{r})+\int d\mathbf{r}'V_{\mathrm{int}}(\mathbf{r}-\mathbf{r}')n(\mathbf{r}',t)-\int \frac{d\mathbf{p}'}{(2\pi\hbar)^3}\tilde{V}_{\mathrm{int}}(\mathbf{p}-\mathbf{p}')f(\mathbf{r},\mathbf{p}',t)$$

- Relaxation time approximation $I_{\mathrm{coll}}[f] = -\frac{f-f^{\mathrm{hy}}}{\tau}$

Scaling ansatz

- Ansatz for global equilibrium distribution function at $T = 0$

$$f^0(\mathbf{r},\mathbf{k}) = \Theta\left(1 - \sum_i \frac{r_i^2}{R_i^2} - \sum_i \frac{k_i^2}{K_i^2}\right),$$

R_i, K_i: Thomas-Fermi radii, momenta

- Scaling ansatz:

$$f(\mathbf{x},\mathbf{q},t) \to \Gamma(t) f^0(\mathbf{r}(t),\mathbf{k}(t)),$$

rescaled variables: $r_i(t) = \frac{x_i}{b_i(t)}, \qquad k_i(t) = \frac{1}{\sqrt{\theta_i(t)}}\left[q_i - \frac{M\dot{b}_i(t)}{\hbar b_i(t)}\right]$

- Normalization factor

$$\Gamma(t)^{-1} = \prod_i b_i(t)\sqrt{\theta_i(t)}$$

Phys. Rev. A **68**, 043608 (2003)

Equations

- Equations of motion for scaling parameters:

$$\ddot{b}_i+\omega_i^2 b_i-\frac{\hbar^2 K_i^2\theta_i}{M^2 b_i R_i^2}+\frac{48Nc_0}{Mb_iR_i^2\prod_j b_jR_j}\left[F\left(\frac{b_xR_x}{b_zR_z},\frac{b_yR_y}{b_zR_z}\right)-b_iR_i\frac{\partial}{\partial b_iR_i}F\left(\frac{b_xR_x}{b_zR_z},\frac{b_yR_y}{b_zR_z}\right)\right]$$

$$-\frac{48Nc_0}{Mb_iR_i^2\prod_j b_jR_j}\left[F\left(\frac{\sqrt{\theta_z}K_z}{\sqrt{\theta_x}K_x},\frac{\sqrt{\theta_z}K_z}{\sqrt{\theta_y}K_y}\right)+\sqrt{\theta_i}K_i\frac{\partial}{\partial\sqrt{\theta_i}K_i}F\left(\frac{\sqrt{\theta_z}K_z}{\sqrt{\theta_x}K_x},\frac{\sqrt{\theta_z}K_z}{\sqrt{\theta_y}K_y}\right)\right]=0,$$

$$\dot{\theta}_i+2\frac{\dot{b}_i}{b_i}\theta_i=-\frac{1}{\tau}(\theta_i-\theta_i^{\mathrm{hy}})$$

- Strength of dipolar interaction

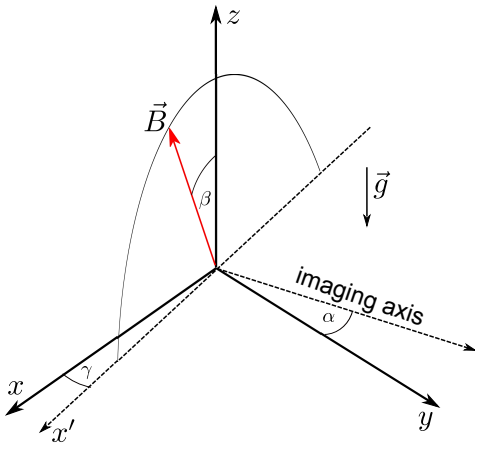
$$c_0=\frac{2^{10}C_{\mathrm{dd}}}{3^4\cdot 5\cdot 7\cdot \pi^3}$$

- Collisionless regime: $\tau\to\infty$
- Hydrodynamic regime: $\tau\to 0$, definition: $\prod_i b_i^{\mathrm{hy}}(t)\sqrt{\theta_i^{\mathrm{hy}}(t)}=1$

arXiv:1311.5100 (2013)

Experimental setup

- Parameters:
  $^{167}\mathrm{Er}$, $N = 7 \cdot 10^4$
  $(\omega_x, \omega_y, \omega_z) = 2\pi(579, 91, 611)$ Hz
  $\alpha = 28^\circ$, $\gamma = 14^\circ$



- Aspect ratios of the cloud (Fermi surface) in imaging plane:

$$A_{\mathrm{R}}(t) = \sqrt{\frac{\langle r_z^2 \rangle}{\langle r_x^2 \rangle \cos^2\alpha + \langle r_y^2 \rangle \sin^2\alpha}}, \quad A_{\mathrm{K}}(t) = \sqrt{\frac{\langle k_z^2 \rangle}{\langle k_x^2 \rangle \cos^2\alpha + \langle k_y^2 \rangle \sin^2\alpha}}$$

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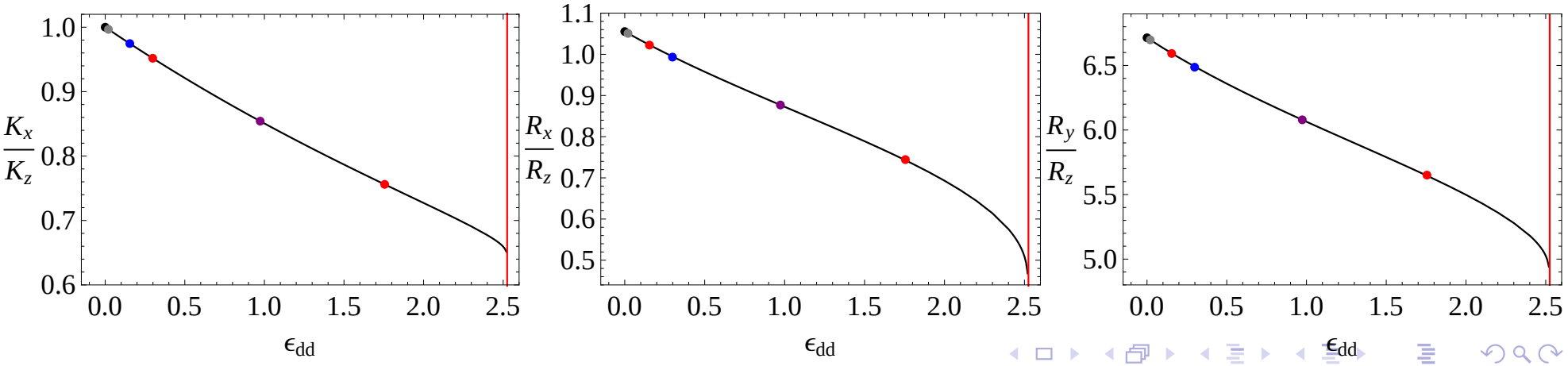
Fermi surface deformation ($\beta = 0$)

- Relative interaction strength:

$$\epsilon_{\rm dd} = \frac{C_{\rm dd}}{4\pi}\sqrt{\frac{M^3\omega}{\hbar^5}}N^{1/6}, \qquad \omega = (\omega_x\omega_y\omega_z)^{1/3}$$

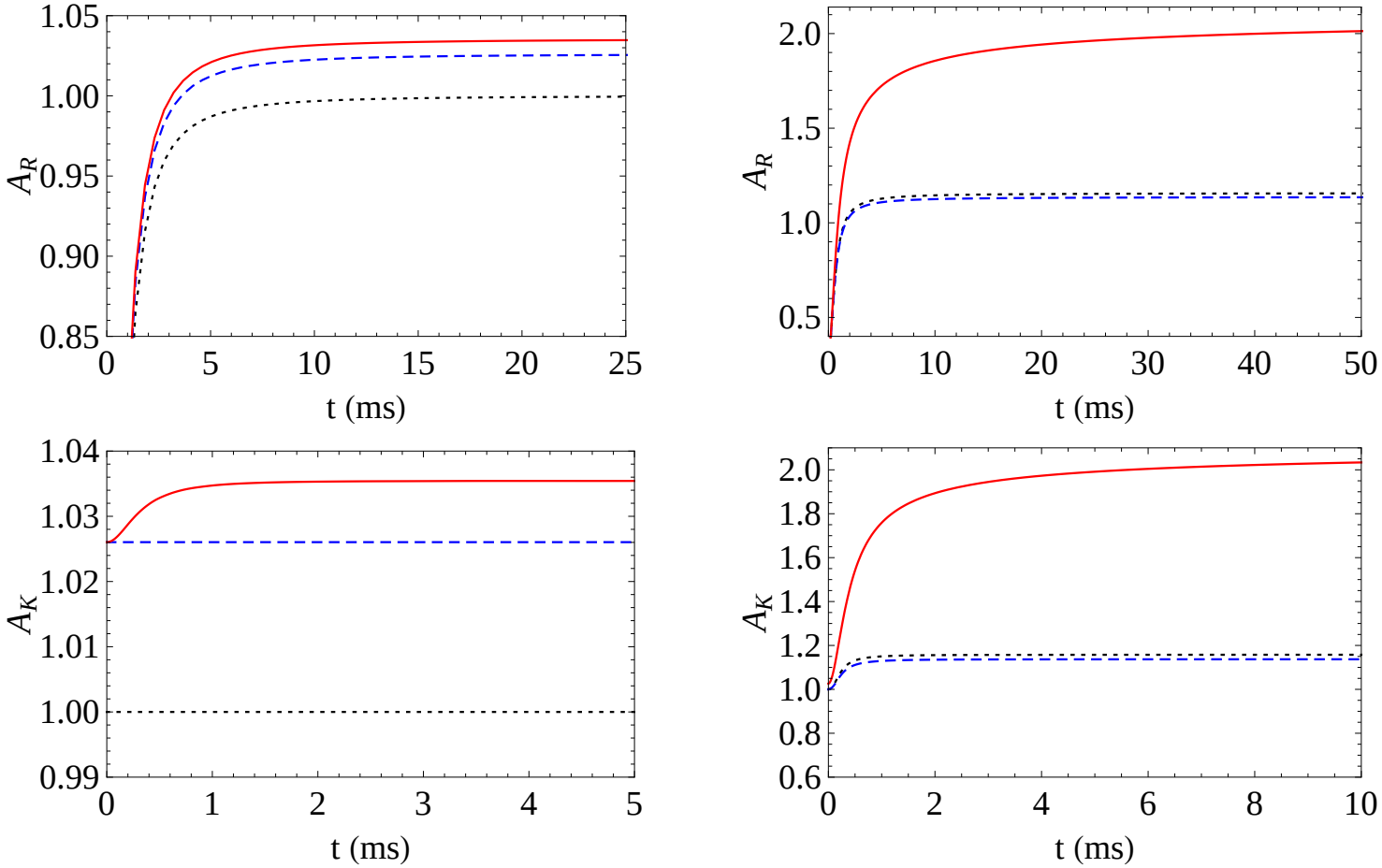
- Dipolar Fermi gases in typical cold-atom experiments:

gas	^{53}Cr	^{167}Er	^{161}Dy	^{40}K^{87}Rb	^{167}Er^{168}Er	^{23}Na^{40}K
m/d	6 $\mu_{\rm B}$	7 $\mu_{\rm B}$	10 $\mu_{\rm B}$	0.2 D	14 $\mu_{\rm B}$	0.8 D
$\epsilon_{\rm dd}$	0.02	0.15	0.30	0.97	1.76	5.44



Time-of-flight expansion ($\beta = 0$)

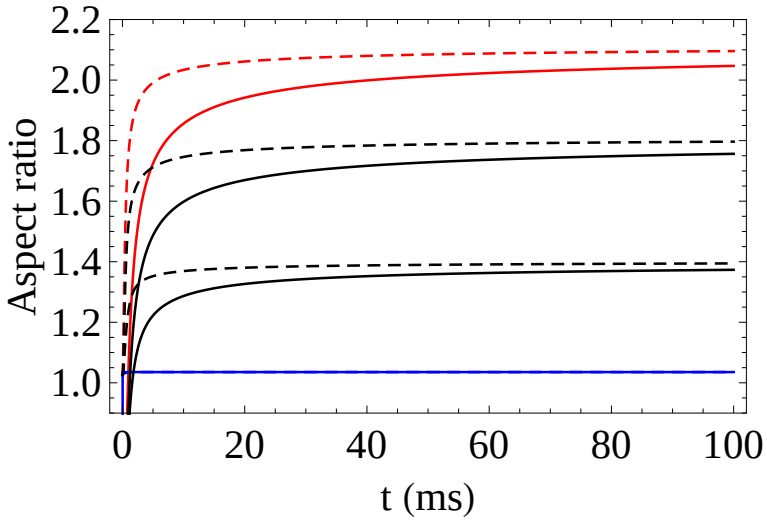
Collisionless regime | Hydrodynamic regime

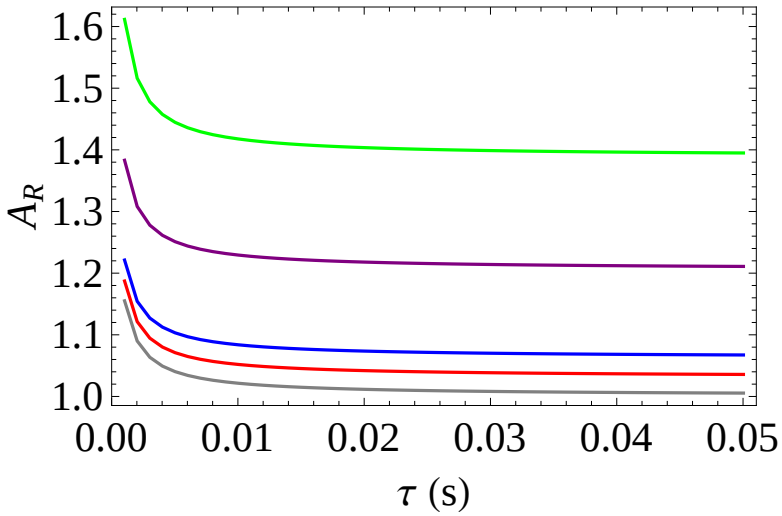


Non-ballistic expansion, Ballistic expansion, Non-interacting Fermi gas

Time-of-flight expansion ($\beta = 0$, τ finite)

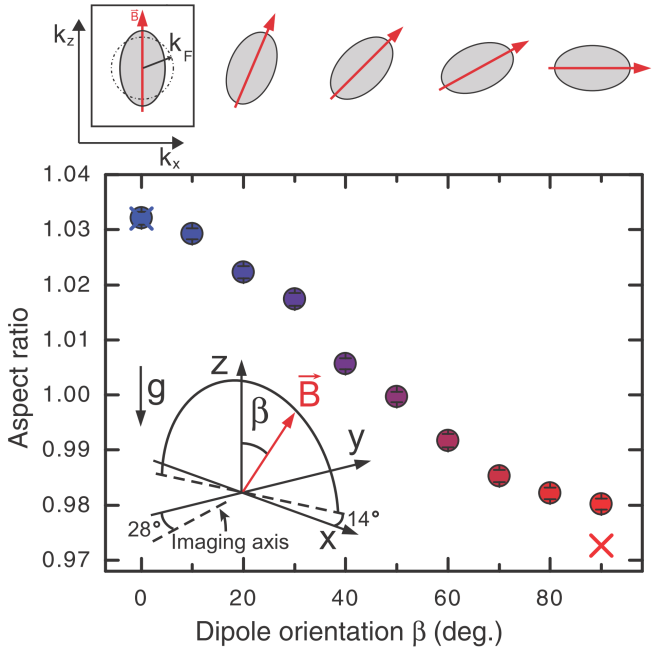
- Non-ballistic expansion: hydrodynamic, collisional for $\omega\tau = 1$ and $\omega\tau = 5$, and collisonless regime





Cloud aspect ratios after $t = 10$ ms as function of relaxation time for: ^{161}Dy, ^{167}Er, ^{53}Cr, ^{40}K^{87}Rb, ^{167}Er^{168}Er

Time-of-flight expansion (β dependence)



EXPERIMENT

- Fermi surface follows rotation of $\vec{B}$
- Major axis is always parallel to $\vec{B}$

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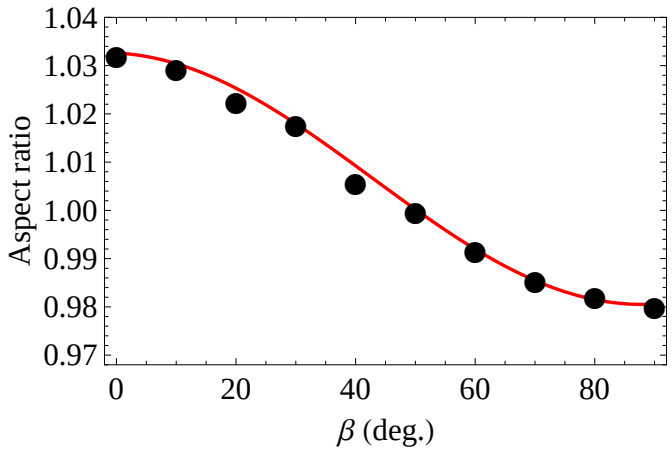
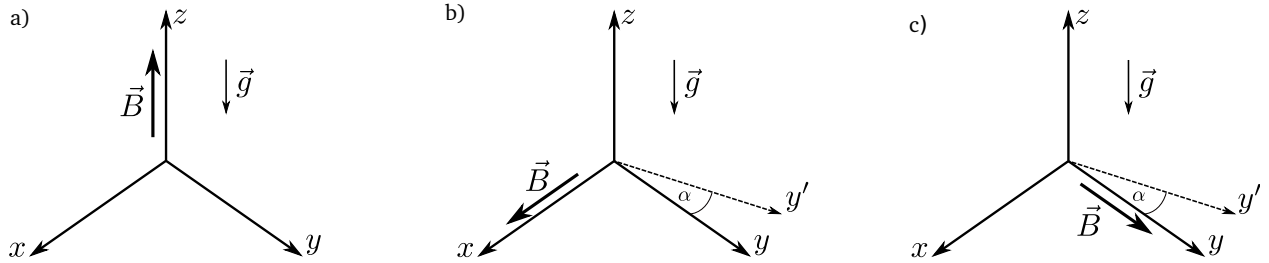


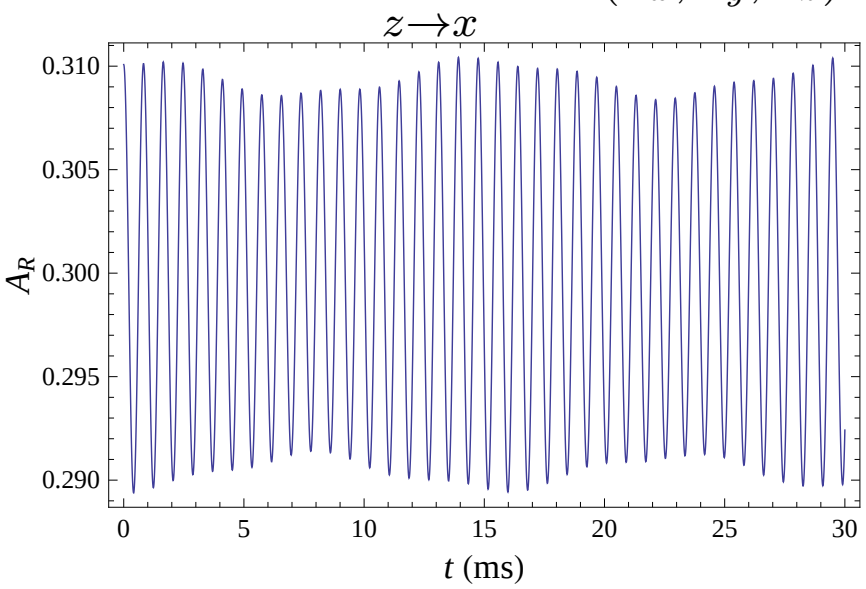
THEORY

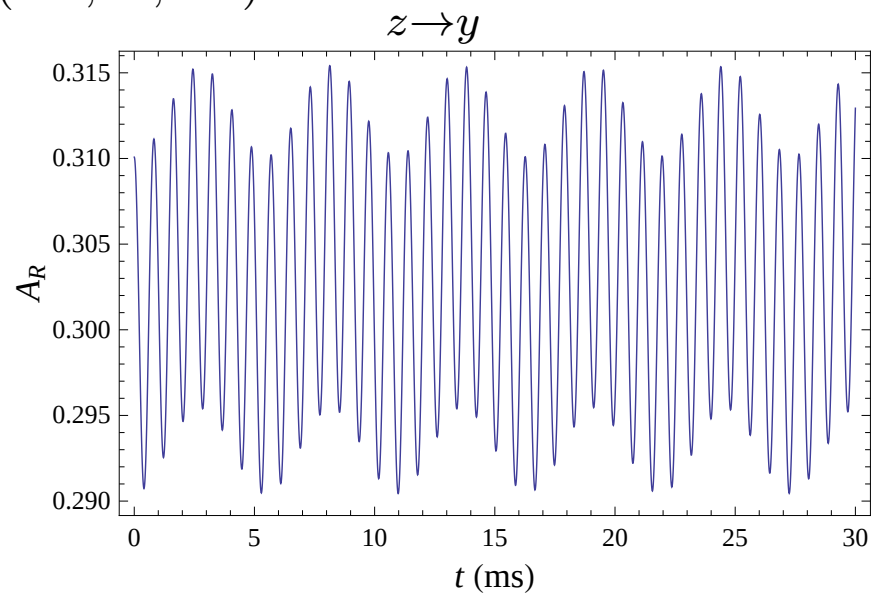
- Extended Hartree-Fock MF theory
- Agreement with experiment

Quench ($z \to x$ and $z \to y$)



$(\omega_x, \omega_y, \omega_z) = 2\pi(579, 91, 611)$ Hz





$(\Omega_x, \Omega_y, \Omega_z) = 2\pi(1154, 181, 1220)$ Hz

$(\Omega_x, \Omega_y, \Omega_z) = 2\pi(1165, 185, 1230)$ Hz

$\Omega_i \approx 2\omega_i \longrightarrow$ collisionless regime