N-point function space-time mapping for dissipative quantum systems

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September 21, 2020

Introduction: spacetime transformation history

Space-time transformations for exact solution

► **Celestial mechanics** (singular Newton potential in 3D Kepler problem → 4D harmonic oscillator, improve the numerical stability of perturbative calculations)

Kustaanheimo and Stiefel (1965)

Stiefel & Scheifele, Springer, Berlin, (1971)

► **Markov processes** (relate different processes together)

Pelster & Kleinert, PRL 78, 565 (1997)

► **Quantum physics**: Complex → simple dynamics in special cases. E.g.

- 1D harmonic oscillator → free particle

  Cai, Inomata, and Wang (1982)

  Pelster and Wunderlin (1992)

- 3D Coulomb potential → 4D harmonic oscillator

  Duru and H. Kleinert (1979, 1982)

  Solov'ev, Sov. J. Nucl. Phys. 35, 136 (1982) – Hydrogen

- Atom-atom or atom-light collisions (adiabatic approximation for slow collisions)

  Grozdanov & Solov'ev, Phys Rev A 88, 022707 (2013)

Introduction: our motivation

Space-time transformations in quantum many-body physics:

► Exactly solvable quantum many-body problems? Not that many!

- o A few results in special cases of: inter-particle interaction, trapping, dimension, special regimes (mean-field, hydrodynamic,...)

Castin and R. Dum (1996); Castin (2004)

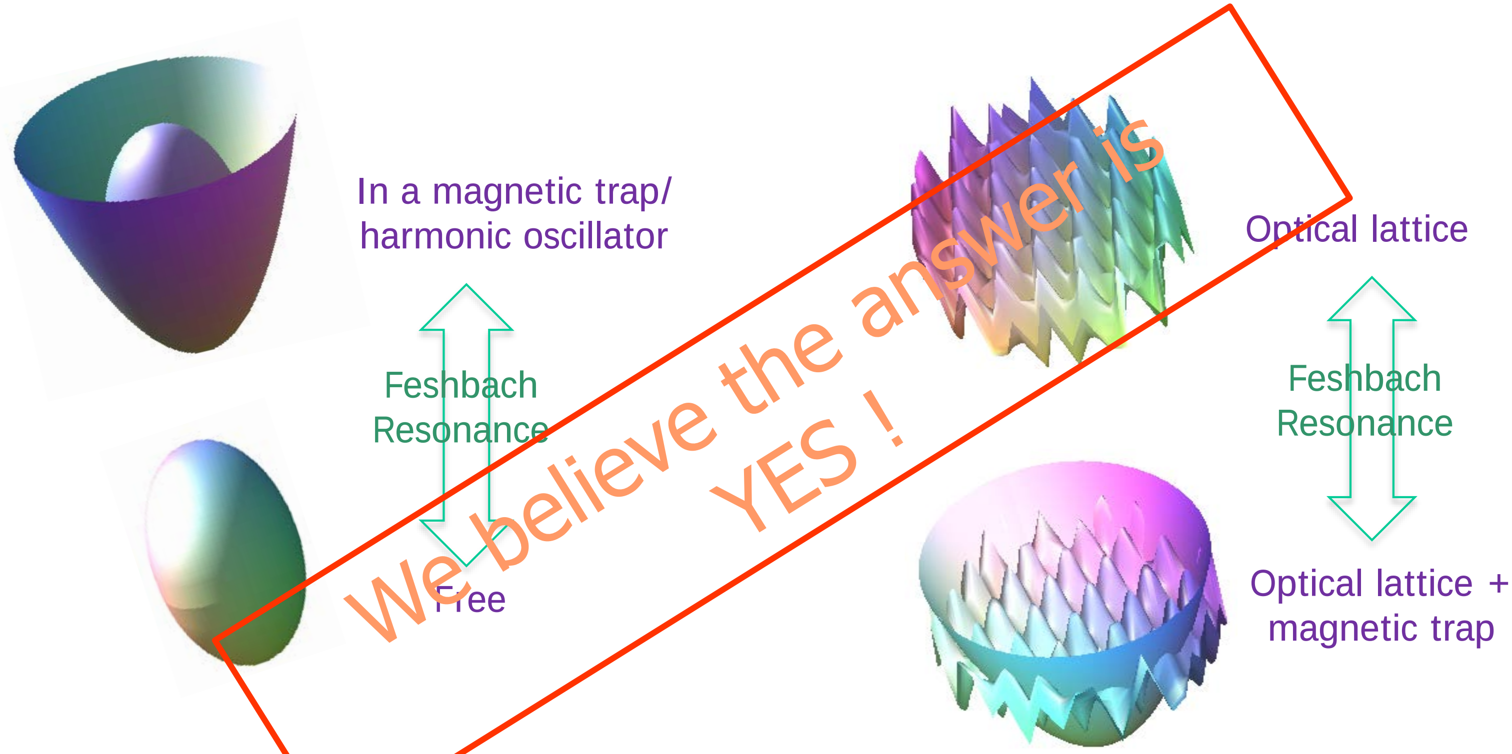
Wamba et al. (2008 - 2014)

► Experiments on trapped quantum gases can probe some challenging regimes of quantum many-body dynamics that cannot be exactly solved.

► BUT some experiments may be harder to achieve (inappropriate technique, low imaging resolution, high error, ...)

Introduction: our motivation

Q. Could we achieve a simpler experiment to mimick a more complex one?



Aim:

- Extend the use of exact space-time mappings to dissipative systems;
- Propose a way of deriving the observables of a dissipative system from another, yet very different, even when both are not exactly solvable.

Wamba, Pelster & Anglin, PRA 94, 043628 (2016)

Outline

- The quantum fields mapping (for closed systems)
  - ✓ Heisenberg equation and mapping identity
  - ✓ Features of the mapping
  - ✓ Illustration: Mapping the two specific evolutions onto each other

- Interesting dynamics in open systems
  - ✓ An experiment with controlled dissipator
  - ✓ Dynamics in presence of a dark soliton

- Some results on the N-point function mapping of lossy quantum systems
  - ✓ Lindblad evolution of the function
  - ✓ Mapping of two evolutions

- Conclusion

Quantum fields mapping: Heisenberg picture tools

- (Anti)Commutation relations (for particles of type *n* and *m*): +/- Fermions/Bosons

$$\left[\hat{\Psi}_m(\vec{r},t),\hat{\Psi}_n^{\dagger}(\vec{r}',t)\right]_{\pm}=\boldsymbol{\delta}_{mn}\,\boldsymbol{\delta}^{D}(\vec{r}-\vec{r}')$$

- Heisenberg equation (Evolution of the quantum gas):

$$i\hbar\frac{\partial}{\partial t}\hat{\Psi}_n(\vec{r},t)=\left(-\frac{\hbar^2}{2\,M_n}\,\nabla^2+V_n(\vec{r},t)\right)\hat{\Psi}_n(\vec{r},t)+\varphi_{int}$$

$$\varphi_{int}=\sum_{klm}\int d^D\vec{r}'\;U_{klmn}(\vec{r},\vec{r}',t)\,\hat{\Psi}_k^{\dagger}(\vec{r}',t)\hat{\Psi}_l(\vec{r}',t)\hat{\Psi}_m(\vec{r},t)$$

This is **NOT mean field**!

It gives the **exact evolution** of any observables in **any** quantum state.

Werner Heisenberg

Quantum fields mapping: Heisenberg picture tools

Any experimental measurement

can be expressed in terms of an N-point function:

$$F_{\mathbf{n},\mathbf{m}}(R, R', t, t') = \left\langle \left[\prod_{j=1}^{N} \widehat{\Psi}^{\dagger}_{n_{j'}}(\vec{r}_{j'}', t)\right] \left[\prod_{j=1}^{N} \widehat{\Psi}_{m_j}(\vec{r}_j, t')\right] \right\rangle$$

j'=N+1-j, $\mathbf{n} = \{n_1, \ldots, n_N\}, \mathbf{R} = \{\vec{r}_1, \ldots, \vec{r}_N\}; N\ depends\ on\ the\ quantity\ measured.$

Quantum fields mapping: Identity

Suppose the two-body interaction potential satisfies the homogeneity condition.

Then $\{\hat{\Phi}_n(\vec{r},t), U(\vec{r},t), V(\vec{r},t)\} \leftrightarrow \{\hat{\Psi}_n(\vec{r},t), \tilde{U}(\vec{r},t), \tilde{V}(\vec{r},t)\}$ where:

$$\tilde{U}(\vec{r},\vec{r}',t) = \lambda^{2-s}\, U(\vec{r},\vec{r}',\tau)$$

$$\tilde{V}_n(\vec{r},t) = \lambda^2 \left[V_n\left(\lambda\vec{r}, \int_0^t \lambda(t')^2\, dt'\right) + \frac{1}{2} M_n \vec{r}^{\,2} \lambda\, \hat{\mathcal{O}}^2 \lambda \right]; \;\; \hat{\mathcal{O}} = \left(\frac{1}{\lambda^2}\frac{d}{dt}\right)$$

$$\boxed{\hat{\Psi}_n(\vec{r},t) = \lambda^{D/2}\, e^{-i\frac{1}{2\hbar} M_n \vec{r}^{\,2} \lambda \hat{\mathcal{O}} \lambda}\;\; \hat{\Phi}_n\left(\lambda\vec{r}, \int_0^t \lambda(t')^2\, dt'\right)}$$

λ is free parameter (chosen depending on the expt to perform/mimic). s depends on the type of particle interaction: $s = D$ for contact int; $s = 3$ for dipole-diplole.

Quantum fields mapping: Key features

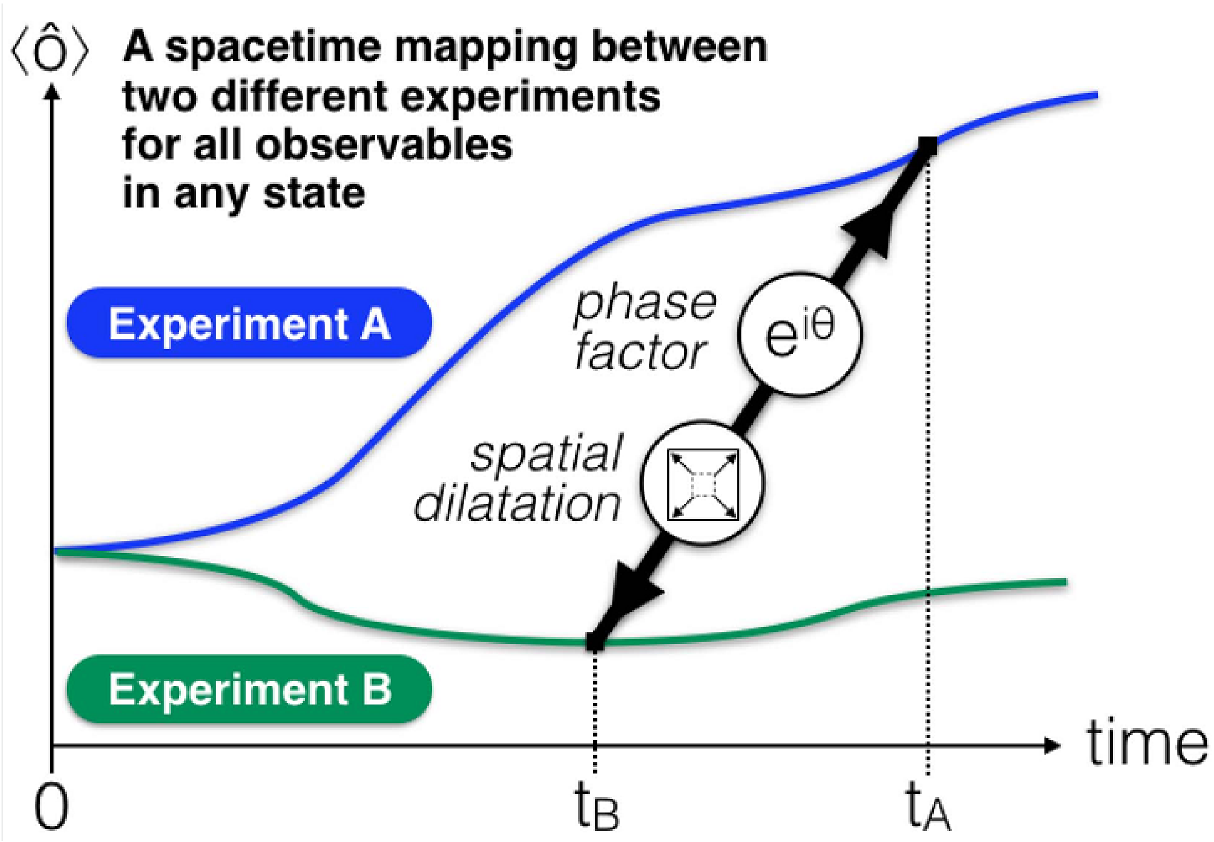
Our (spacetime) mapping consists of:

Space dilatation **+** **Non-trivial time transformation**

(nonstationary scaling of length)

$$\vec{r} \to \lambda(t)\,\vec{r}$$

$$t \to \int_0^t \lambda(t')^2\, dt'$$



Quantum fields mapping: Key features

Like the Heisenberg equation, our spacetime mapping is:

Exact !

No approximation made, it is **not about Gross-Pitaevskii equation** but Heisenberg equation.

General !

- use quantum fields not c-number fields
- valid for bosons, fermions, any mixture (species, hyperfine structures, spins)
- most real interactions
- arbitrary initial state
- all space dimensions,
- arbitrary traps
- all possible measurements, ...

An Example of the mapping: free expans. to ramped int.

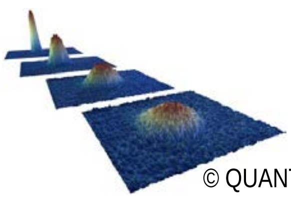
We apply the mapping to two achievable experiments to support our predictions.

A

Free expansion of a cigar-shaped quantum gas:

$$g = g_0$$

$$V(x) = \quad 0$$



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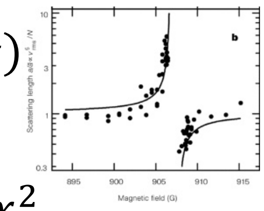
B

Ramped interactions of a trapped cigar-shaped quantum gas:

$$g(t) = g_0\,\lambda(t)$$

$$V(x) = \frac{M}{2}\,\omega^2 x^2$$



© Stamper-Kurn group

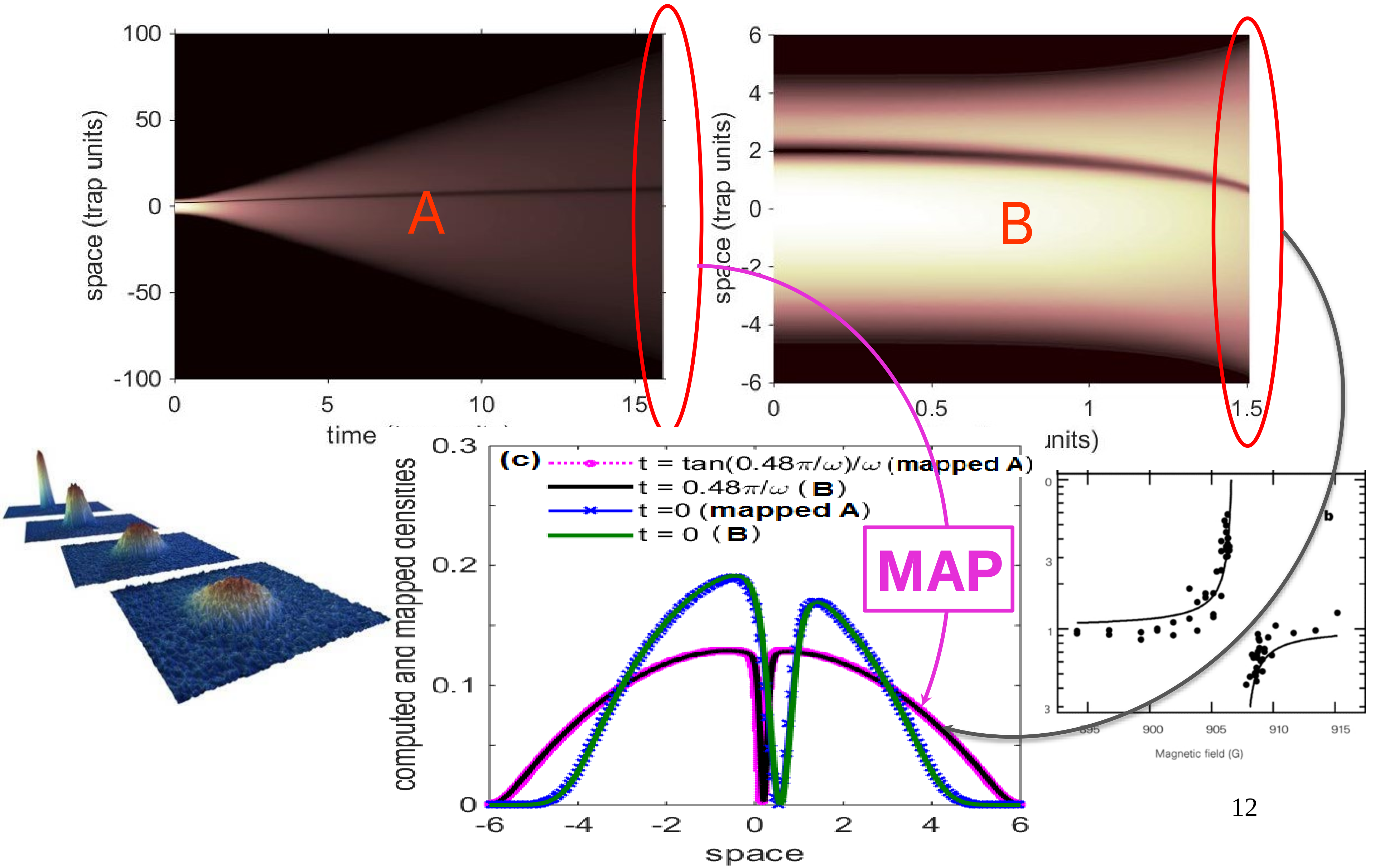
$$t_A = \frac{\tan(\omega\, t_B)}{\omega}\,;\; x_A = \lambda(t_B)\, x_B \qquad \lambda(t) = \frac{1}{\cos(\omega\, t)}$$

$$\boxed{\hat{\Psi}_B(x_B, t_B) = \lambda(t_B)^{1/2}\, e^{-i\,\frac{M\omega}{2\hbar}\, x_B^2 \tan(\omega\, t_B)}\;\; \hat{\Psi}_A(x_A, t_A)}$$

We cannot plot quantum fields. But the mapping also works for mean fields

An Example of the mapping: free expans. to ramped int.

Mean-field density evolution in the two experiments (illustration of mapping).



An Example of the mapping: free expans. to ramped int.

We applied the mapping to two achievable experiments to support our predictions.

For longer times, mean field breaks down in both cases, but our mapping does not.

This is just an example!

Infinitely many pairs of experiments

can be exactly mapped with closed quantum gases .

More information in E. Wamba et al., Phys. Rev. A 94, 043628 (2016)

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  - ✓ Illustration : Mapping the two specific evolutions onto each other

- Interesting dynamics in open systems
  - ✓ An experiment with controlled dissipator
  - ✓ Dynamics in presence of a dark soliton

- Some results on the N-point function mapping of lossy quantum systems
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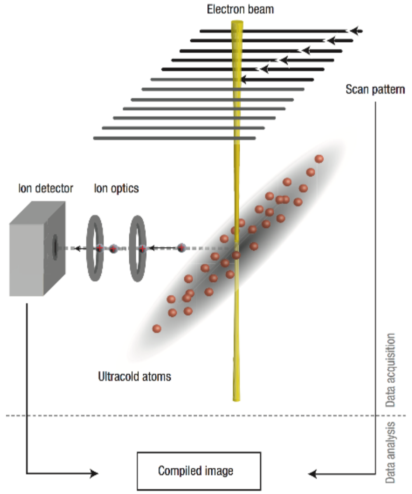
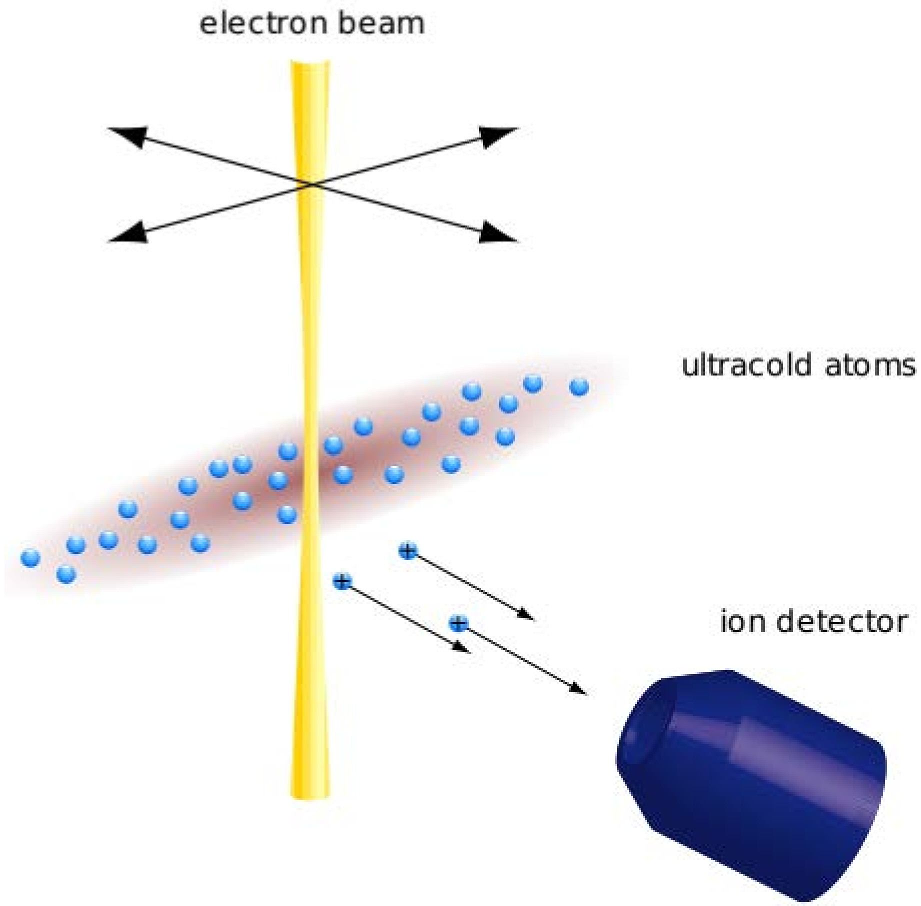
Controlled dissipation: Scanning electron microscopy

A focused electron beam is scanned over the atom cloud and ionizes single atoms, which are subsequently detected by an ion detector





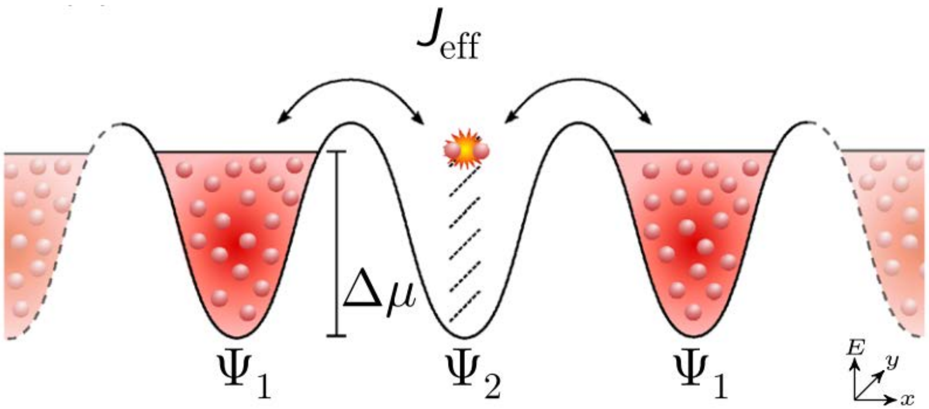
Working principle of **scanning electron microscopy** applied to ultracold quantum gases.

Tatjana Gericke, PhD thesis (2010)

Controlled dissipation: Electron beam on an optical lattice

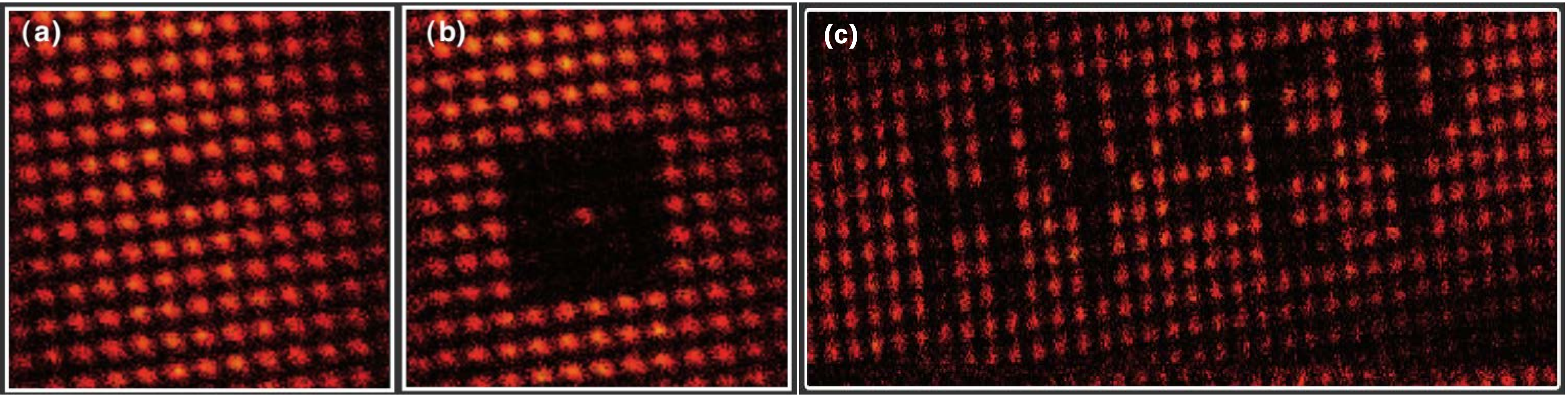
High controllability of every parameter => system is a promising candidate for engineering fully governable open quantum systems.

- Prepare a quantum transport device for neutral atoms



- Prepare any arbitrary lattice

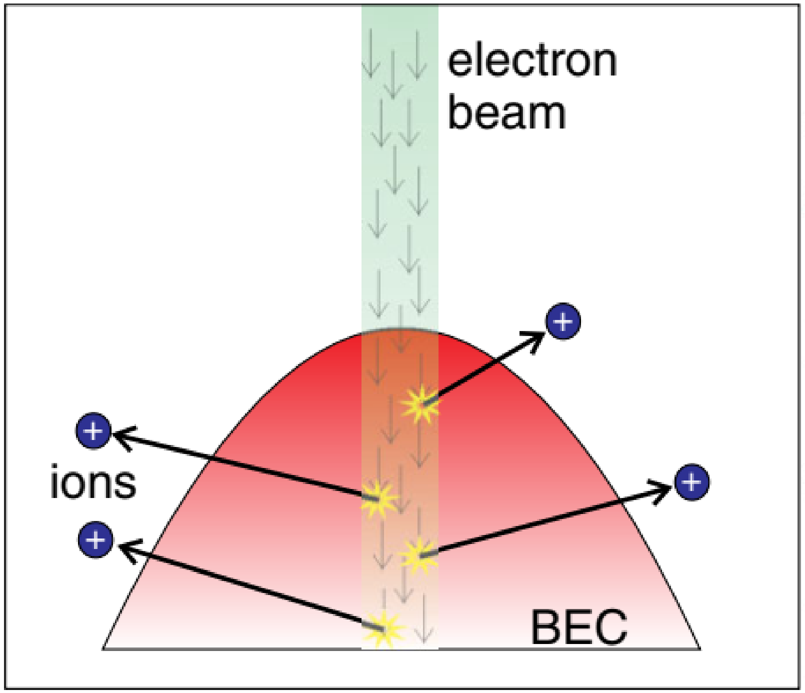
(a) single empty site, (b) isolated occupied site, (c) any well-controlled distribution of sites.



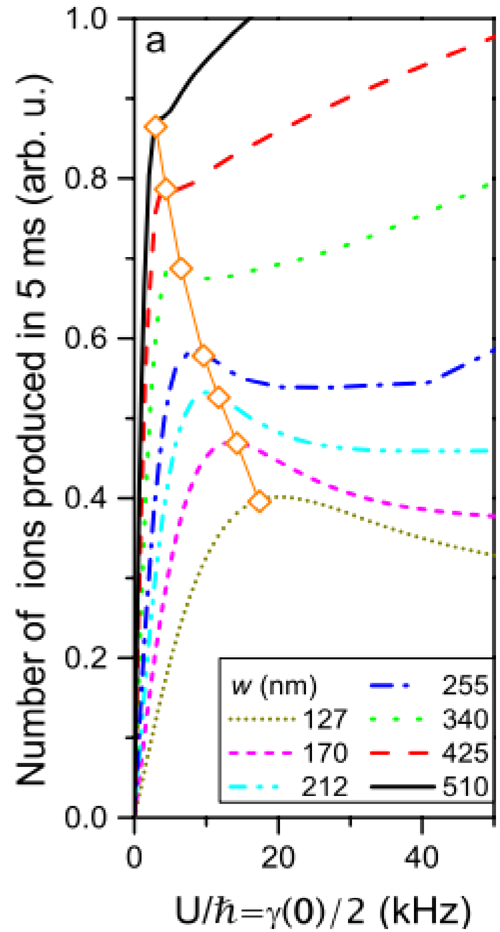
T. Gericke, PhD thesis (2010); R. Labouvie, PhD Thesis (2015), R. Labouvie, et al PRL 115, 050601 (2015)

Controlled dissipation: Electron beam on a BEC

Fully controllable, environmentally induced imaginary potential acting on a quantum system



$$\gamma(r) = \gamma(0)\, e^{-\frac{r^2}{2\,w^2}},\ \gamma(0) \propto \frac{I}{w^2}$$



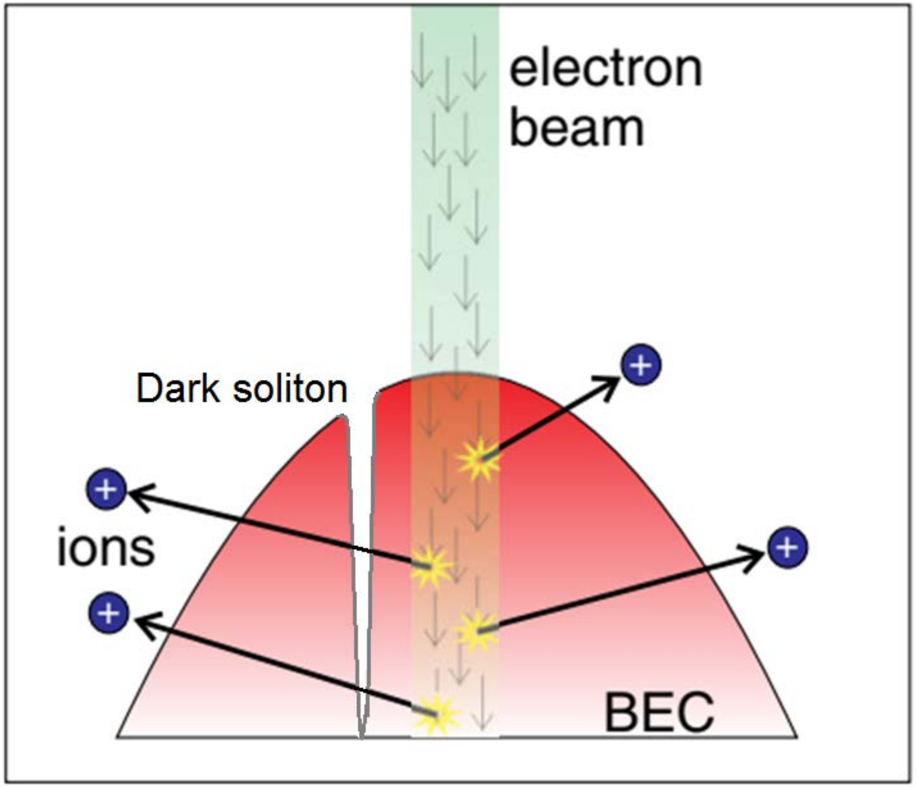
The action of a localized dissipative potential on a macroscopic matter wave probes

- The backflow paradox (when the strength of the dissipation exceeds a critical limit)
- The generalized Zeno effect (a system cannot change while being observed).

G. Barontini et al. , Phys Rev Lett 110, (2013)

Dark soliton in a dissipative BEC:

BEC + electron beam + dark soliton => Enriched dynamics

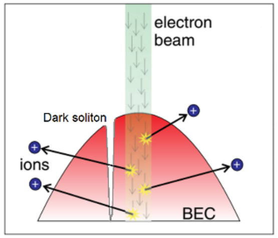


Mean-field dynamics described by Gross-Pitaevskii + imaginary potential

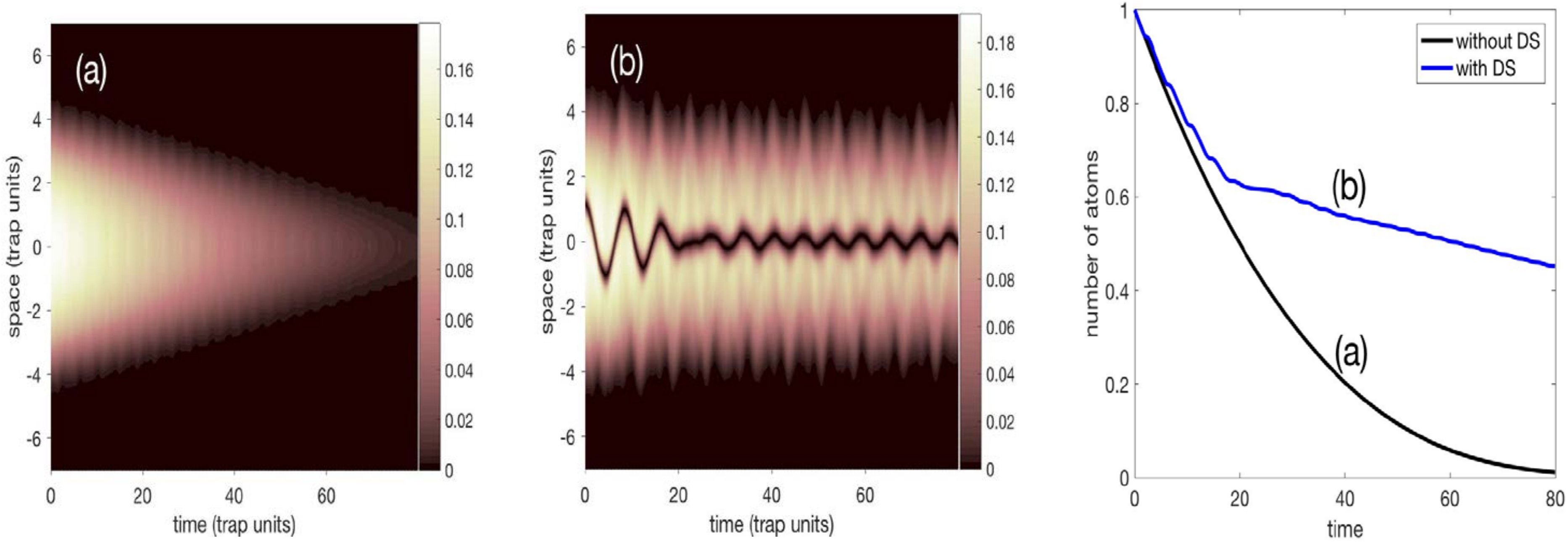
$$\gamma(x) = \gamma(0)\, e^{-\frac{x^2}{2\,w^2}},\ \gamma(0) \propto \frac{I}{w^2}$$

Dark soliton in a dissipative BEC: Decays

BEC + electron beam + dark soliton => decays



Preliminary results!

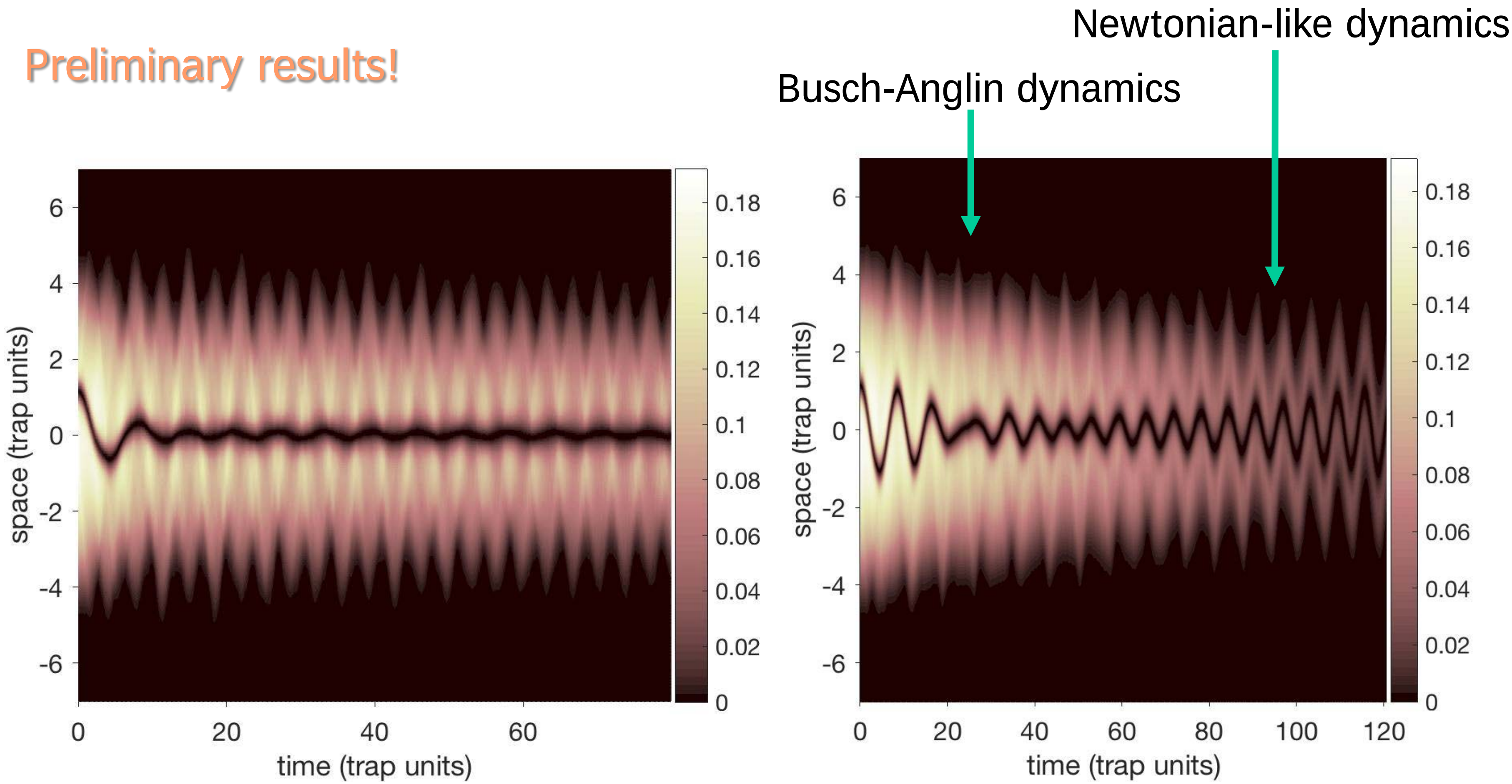


- Condensate radius, density, atom number, and soliton motion all decay.
- Dark soliton drastically changes the decay rates

Dark soliton in a dissipative BEC: Capture and release

BEC + electron beam + dark soliton => capture/ release

Preliminary results!



- ❑ Capture and release of dark soliton (by the dissipator) can happen

Hint from V.N. Serkin, Optik - International Journal for Light and Electron Optics 173, 1-12 (2018)

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  - ✓ Schrödinger evolution of the function
  - ✓ Mapping of two different evolutions

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Schrödinger evolution: The Lindblad equation

Realistic evolution of the gas (in the Schrödinger picture):

$$i\hbar \frac{\partial}{\partial t}\hat{\rho} = \left[\hat{H}, \hat{\rho}\right] + i\hbar \mathcal{L}\hat{\rho}$$

Hamiltonian:

$$\hat{H} = \int d^D\vec{r} \left[\hat{\Psi}^\dagger(\vec{r})\left(-\frac{\hbar^2}{2M}\nabla^2 + V(\vec{r},t)\right)\hat{\Psi}(\vec{r}) + \varphi_{int}\right]$$

$$\varphi_{int} = \frac{1}{2}\int d^D\vec{r}'\ \hat{\Psi}^\dagger(\vec{r})\hat{\Psi}^\dagger(\vec{r}')\, U(\vec{r},\vec{r}',t)\hat{\Psi}(\vec{r}')\hat{\Psi}(\vec{r})$$

Lindbladian:

$$\mathcal{L}\hat{\rho} = -\int d^D\vec{r}\ \left(\hat{Q}^\dagger\hat{Q}\,\hat{\rho} + \hat{\rho}\hat{Q}^\dagger\hat{Q} - 2\hat{Q}^\dagger\hat{\rho}\hat{Q}\right)$$

Lindblad generators (loss/gain channel):

$$\hat{Q} = \hat{\Psi}(\vec{r})\sqrt{\frac{\gamma(\vec{r})}{2}}$$

arXiv:2009.04270v2 (PRA in press)

Schrödinger evolution: The Lindblad equation

Lindblad equation in the Heisenberg picture?

Replace the density operator by the evolving field operator (Naive solution):

- The density matrix is a sort of hybrid operator
- The equation fails to evolve the product of 2 operators properly

- Adding a Langevin force may solve the problem
- Consider only expectation values (easier, experiment directed)

Schrödinger evolution: Evolution of Schro. N-point function

The N-point function in terms of Schrödinger's operators:

$$F_S = \left\langle \left[\prod_{j=1}^{N} \hat{\Psi}^\dagger(\vec{r}_{j'}{}')\right] \left[\prod_{j=1}^{N} \hat{\Psi}\,(\vec{r}_j)\right] \right\rangle \qquad j'=N+1-j$$

Use the following evolution rule and apply the Tr properties:

$$\partial_t \langle \hat{A}(\vec{r}) \rangle = \mathrm{Tr}\big(\hat{A}(\vec{r})\, \partial_t \hat{\rho}\big)$$

The N-point function satisfies:

$$\boxed{\begin{aligned} i\hbar\, \partial_t F_S = & \sum_{j=0}^{N-1} \left\langle \left(\prod_{i=1}^{N} \hat{\Psi}_{i'}'^{\dagger}\right) \left(\prod_{i=1}^{j} \hat{\Psi}_i\right) \overrightarrow{h_{j+1}} \left(\prod_{i=j+1}^{N} \hat{\Psi}_i\right) \right\rangle \\ & - \sum_{j=0}^{N-1} \left\langle \left(\prod_{i=j+1}^{N} \hat{\Psi}_{i'}'^{\dagger}\right) \overleftarrow{h'_{j+1}} \left(\prod_{i=1}^{j} \hat{\Psi}_i'^{\dagger}\right) \left(\prod_{i=1}^{N} \hat{\Psi}_i\right) \right\rangle \\ & - i\hbar\, F_S \sum_{i=1}^{N} \frac{\gamma(\vec{r}_i) + \gamma(\vec{r}_i{}')}{2} \qquad i'=N+1-i \end{aligned}}$$

$$h_i = -\frac{\hbar^2}{2M} \nabla_{r_i}^2 + V(\vec{r}_i) + \int d^D \vec{r}_i{}'\, \hat{\Psi}^\dagger(\vec{r}_i{}') U(\vec{r}_i, \vec{r}_i{}') \hat{\Psi}(\vec{r}_i{}')$$

‘Heisenberg’ evolution: Evol. of Heis. N-point function

Invoke picture independence of expectation values:

$$F = \left\langle \left[\prod_{j=1}^{N} \hat{\psi}^{\dagger}(\vec{r}_{j'}', t)\right] \left[\prod_{j=1}^{N} \hat{\psi}\,(\vec{r}_j, t)\right] \right\rangle \equiv F_S$$

Introduce a unitary operation:

$$\hat{\Psi}\,(\vec{r}_j) \to \hat{\psi}\,(\vec{r}_j, t) = \hat{U}^{\dagger}\,\hat{\Psi}\,(\vec{r}_j)\hat{U}$$

The ‘Heisenberg’ N-point function satisfies:

$$\boxed{\begin{aligned} i\hbar\,\partial_t F = {} & \sum_{j=0}^{N-1} \left\langle \left(\prod_{i=1}^{N} \hat{\psi}_{i'}^{\prime\dagger}\right)\left(\prod_{i=1}^{j} \hat{\psi}_i\right) \overrightarrow{\mathbf{h}_{j+1}} \left(\prod_{i=j+1}^{N} \hat{\psi}_i\right) \right\rangle \\ & - \sum_{j=0}^{N-1} \left\langle \left(\prod_{i=j+1}^{N} \hat{\psi}_{i'}^{\prime\dagger}\right) \overleftarrow{\mathbf{h}'_{j+1}} \left(\prod_{i=1}^{j} \hat{\psi}_i^{\prime\dagger}\right)\left(\prod_{i=1}^{N} \hat{\psi}_i\right) \right\rangle \\ & - i\hbar\, F \sum_{i=1}^{N} \frac{\gamma(\vec{r}_i) + \gamma(\vec{r}_i')}{2} \end{aligned}}$$

$$\mathbf{h_i} = -\frac{\hbar^2}{2M}\,\nabla_{r_i}^2 + V(\vec{r}_i, t) + \int d^D\vec{r}_i{}'\,\hat{\psi}^{\dagger}(\vec{r}_i', t)U(\vec{r}_i, \vec{r}_i{}', t)\hat{\psi}(\vec{r}_i', t)$$

Mapping of two evolutions: The mapped evolution

Define the rescaled N-point function as

$$\tilde{F} = \left\langle \left[\prod_{j=1}^{N} \widehat{\boldsymbol{\Phi}}^{\dagger}\left(\vec{x}'_{j'}, \tau\right)\right] \left[\prod_{j=1}^{N} \widehat{\boldsymbol{\Phi}}\left(\vec{x}_{j}, \tau\right)\right] \right\rangle$$

Using our quantum-field mapping, the evolution of the mapped N-point function is

$$\boxed{\begin{aligned} i\hbar\, \partial_t \tilde{F} = &\sum_{j=0}^{N-1} \left\langle \left(\prod_{i=1}^{N} \widehat{\boldsymbol{\Phi}}'^{\dagger}_{i'}\right) \left(\prod_{i=1}^{j} \widehat{\boldsymbol{\Phi}}_{i}\right) \overrightarrow{\mathbf{h}_{j+1}} \left(\prod_{i=j+1}^{N} \widehat{\boldsymbol{\Phi}}_{i}\right) \right\rangle \\ &- \sum_{j=0}^{N-1} \left\langle \left(\prod_{i=j+1}^{N} \widehat{\boldsymbol{\Phi}}'^{\dagger}_{i'}\right) \overleftarrow{\mathbf{h}'_{j+1}} \left(\prod_{i=1}^{j} \widehat{\boldsymbol{\Phi}}'^{\dagger}_{i}\right) \left(\prod_{i=1}^{N} \widehat{\boldsymbol{\Phi}}_{i}\right) \right\rangle \\ &- i\hbar\, \tilde{F} \sum_{i=1}^{N} \frac{\tilde{\gamma}(\vec{x}_i, \tau) + \tilde{\gamma}(\vec{x}'_i, \tau)}{2} \end{aligned}}$$

$$\mathbf{h}_\mathbf{i} = -\frac{\hbar^2}{2M} \nabla^2_{x_i} + V(\vec{x}_i, \tau) + \int d^D \vec{x}_i{}' \, \widehat{\boldsymbol{\Phi}}^{\dagger}(\vec{x}'_i, \tau) U(\vec{x}_i, \vec{x}_i{}', \tau) \widehat{\boldsymbol{\Phi}}(\vec{x}'_i, \tau)$$

Mapping of two evolutions: The new identity

If the two-body interaction potential satisfies the homogeneity condition:

Then $\{\hat{\Phi}_n(\vec{r},t), U(\vec{r},t), V_n(\vec{r},t), \gamma(\vec{r},t)\} \leftrightarrow \{\hat{\Psi}_n(\vec{r},t), \tilde{U}(\vec{r},t), \tilde{V}_n(\vec{r},t), \tilde{\gamma}(\vec{r},t)\}$

where:

$$\tilde{U}(\vec{r},\vec{r}',t) = \lambda^{2-s}\, U(\vec{r},\vec{r}',\tau)$$

$$\tilde{V}_n(\vec{r},t) = \lambda^2 \left[V_n\left(\lambda\vec{r}, \int_0^t \lambda(t')^2\, dt'\right) + \frac{1}{2} M_n \vec{r}^{\,2} \lambda\, \hat{\mathcal{O}}^2 \lambda \right];\ \hat{\mathcal{O}} = \left(\frac{1}{\lambda^2}\frac{d}{dt}\right)$$

$$\boxed{\hat{\Psi}_n(\vec{r},t) = \lambda^{D/2}\, e^{-i\frac{1}{2\hbar} M_n \vec{r}^{\,2} \lambda \hat{\mathcal{O}} \lambda}\ \hat{\Phi}_n\left(\lambda\vec{r}, \int_0^t \lambda(t')^2\, dt'\right)}$$

$$\boxed{F \to \tilde{F};\quad \tilde{\gamma}(\vec{r},t) = \lambda^{-2}\, \gamma\left(\lambda\vec{r}, \int_0^t \lambda(t')^2\, dt'\right)}$$

Example with a dissipated cigar-shaped BEC

A

Modulation of electron beam current:

$$g = g_0$$

$$V(x) = \frac{M}{2}\,{\omega_A}^2 x^2$$

$$\gamma(x,t) = \frac{\sigma_0}{\lambda(t)^2}\,\exp\left(-\frac{x^2}{2{W_0}^2}\right)$$

B

Ramping of interactions and modulation of electron beam waist:

$$g(t) = g_0\,\lambda(t)$$

$$V(x) = \frac{M}{2}\,{\omega_B}^2 x^2$$

$$\gamma(x,t) = \sigma_0\,\exp\left(-\frac{x^2}{2\left(W_0/\lambda(t)\right)^2}\right)$$

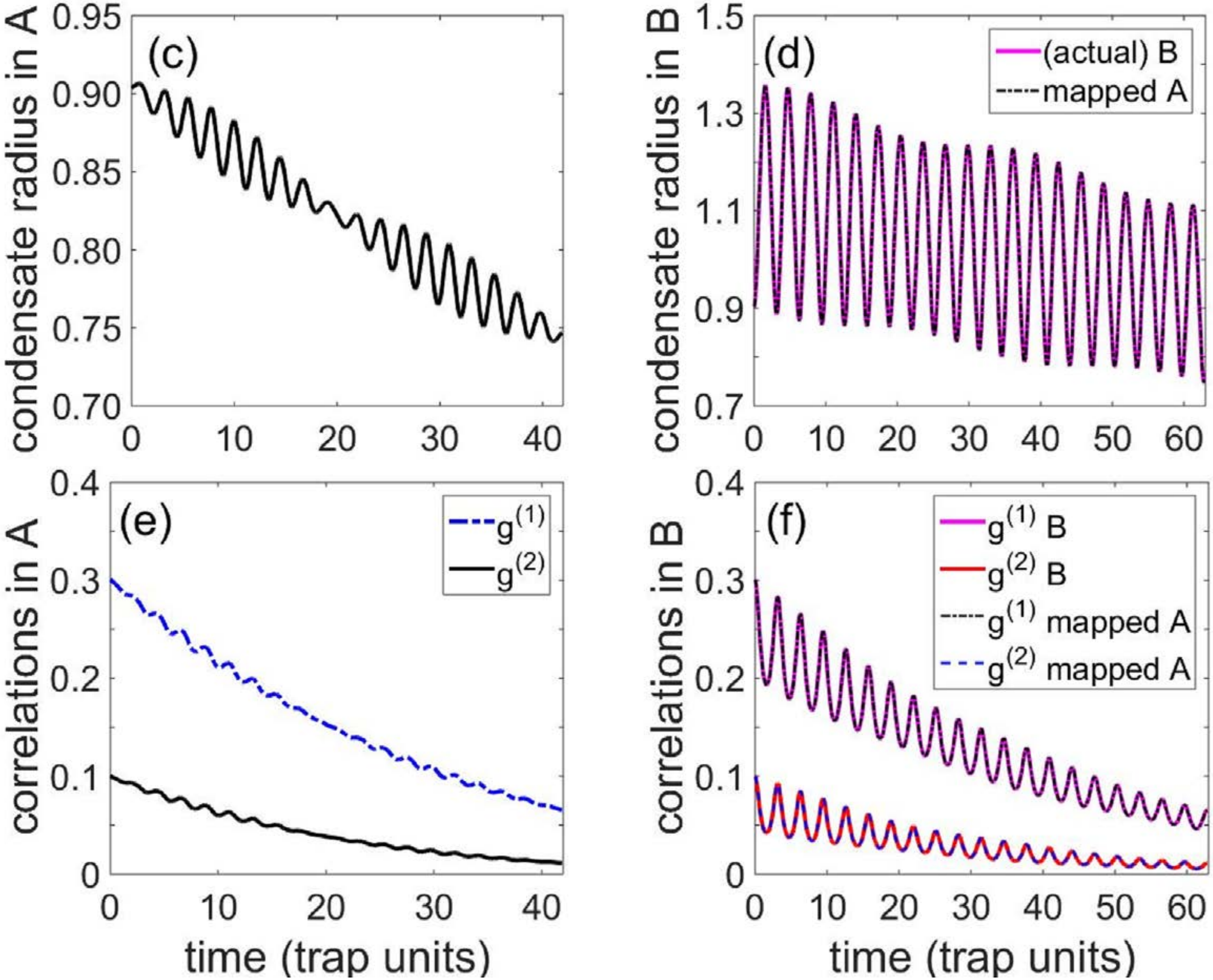
$$\lambda(t) = \frac{\omega_B}{\sqrt{a_-\cos(\omega_B t) + a_+}}, \qquad a_\pm = \frac{{\omega_B}^2 \pm {\omega_A}^2}{2}$$

Define the ν–body correlation function:

$$g^{(\nu)}(t) \propto \int |\phi(x,t)|^{2(\nu+1)}\,dx$$

Example with a dissipated cigar-shaped BEC

Mean-field density evolution in the two experiments (illustration of mapping).



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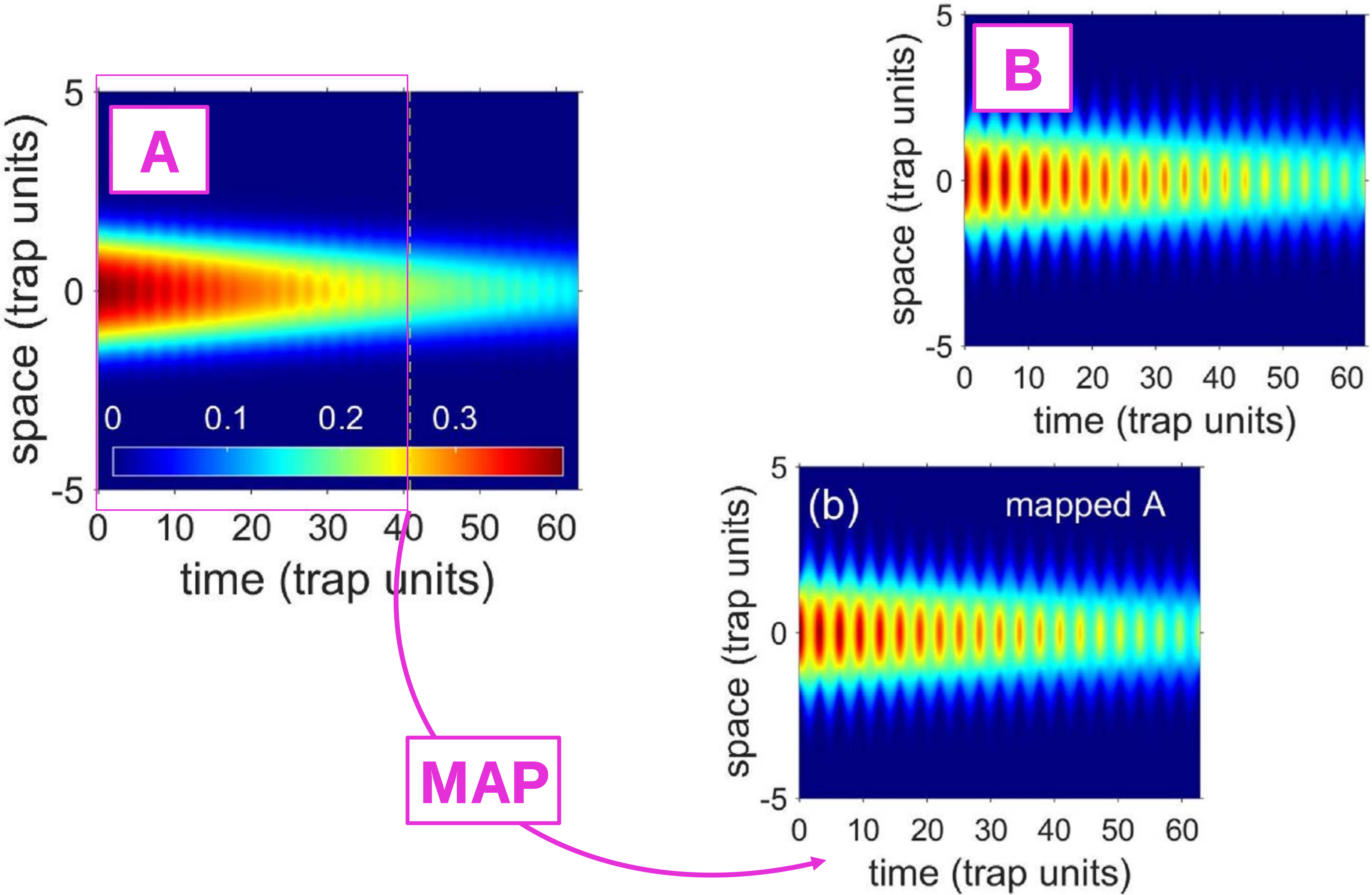


Conclusion

Given an experiment A with a quantum system, it exists a corresponding experiment B, such that the N-point functions of both A & B are related together, whether the system is closed or open!

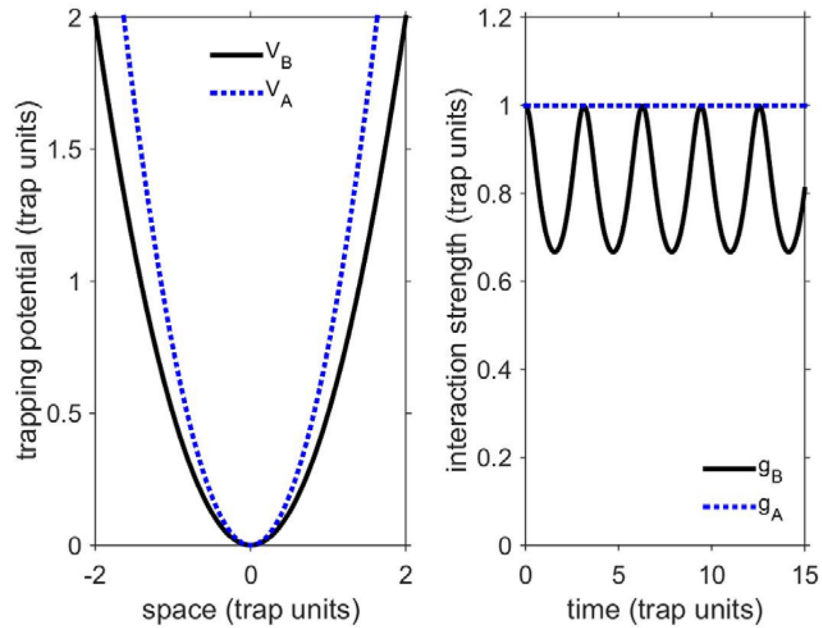
The mapping is

- **Realistic and general (bosons, fermions, all real interactions, arb. initial state, mixtures, arb. dimensions, arb. traps, all possible measurements, …)**
- Suitable for testing for experimental errors
- A tool to expand experimental techniques, e.g. by allowing time-dependent traps to mimic time-dependent interactions, or vice-versa
- A tool to provide long-run outcome of experiments in a shorter time
- A possible microscope (tool to solve imaging resolution issues)

Outlook

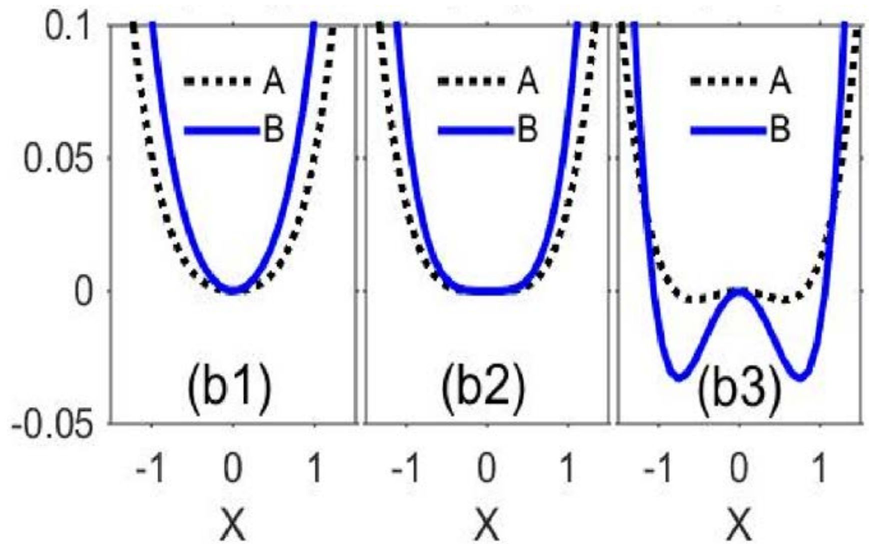
- **Probe heating suppression in periodically driven many-body quantum systems**

  Floquet ↔ static evolutions



- **Explore quench-like changes in many-body quantum systems in search for hidden adiabaticity**

  Fast ↔ slow evolution of a «quenched» system



Thank you!