Mapping between a Floquet problem and a slow evolution of a Bose gas in the mean-field regime

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Outline

- Motivation
- Mapping scheme and driving setup
- Heating estimation and avoidance
- Summary

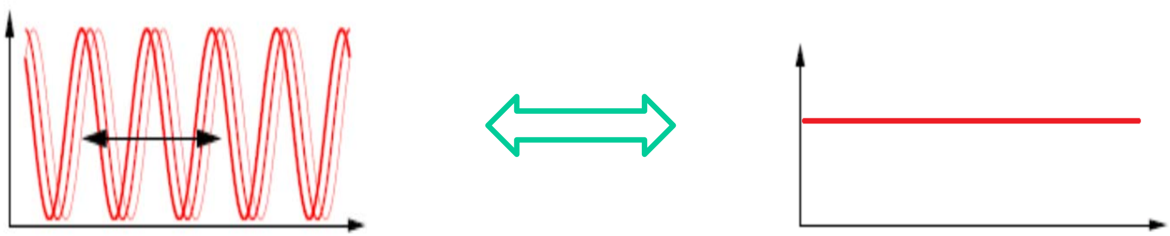
Motivation

❑ **Exactly solvable problems are rare in quantum many-body theory**, especially in nonequilibrium scenarios in which the system Hamiltonian is explicitly time-dependent.

Approximate methods are mostly used, **a few exact results have been obtained by means of space-time mappings**.

❑ **An exact mapping for periodically driven system**

The mapping relates a periodically time-dependent many-body Hamiltonian that can be treated with **Floquet theory**, onto a time-independent Hamiltonian which can be analysed with standard mean-field and perturbative methods.



Phys. Rev. A 87, 040701(R) (2016);
Rev. Mod. Phys. 82, 2731(2010);
Rev. Mod. Phys. 88, 039904 (2016)
Nature Commun. 8, 15085 (2017);
Scientific Reports 8, 11435 (2018)

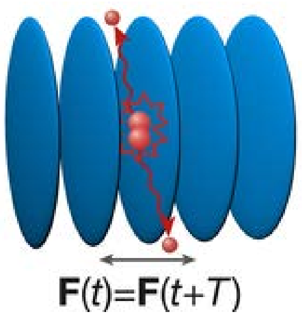
❑ **Why Floquet engineering is great**

- Local manipulation of magnetization in condensed matter systems
- Explore the parameter regimes inaccessible in solid-state experiments and non-equilibrium many-body physics, etc.
- Periodic modulation induces novel effects and structures in quantum systems

Motivation

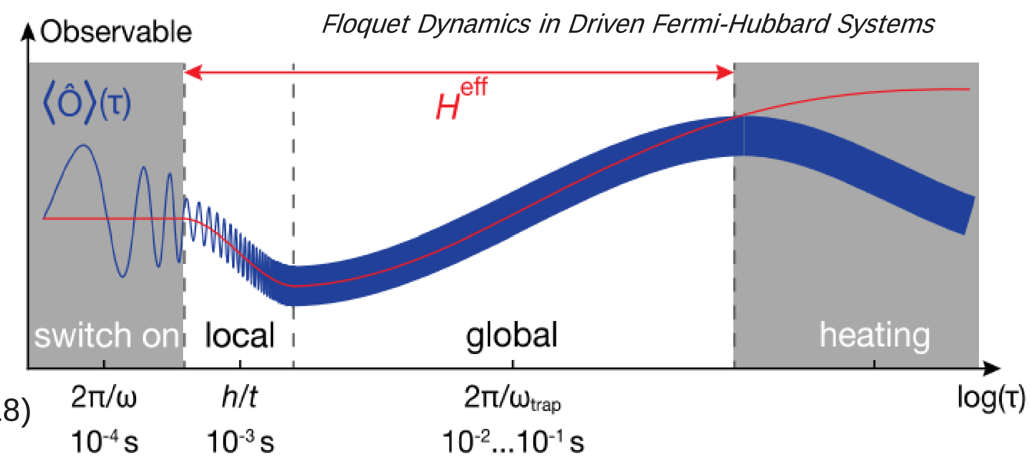
❑ Heating problem: Breakdown of the effective Hamiltonian picture

- Periodic modulation limits the implementation of interesting many-body states because of integrability breaking terms such as **interactions lead to heating** the system to infinite temperature for long timescales
- **Altering the effective Hamiltonian** results in a change of the measured observable within different timescales of the system.



One-dimensional lattice of "pancakes" formed with two laser beams. The lattice is shaken by periodically modulating the frequency of one of the laser beams.

PRL 119, 200402 (2017)



Phys. Rev. Lett. 121, 33603 (2018)

Blue: full time evolution of the observable; **Red**: time evolution under $\hat{H}_{\mathrm{eff}}$.

❑ **Using exact space-time mappings to learn** how periodically driven quantum many-body systems can **avoid being heated**.

Mapping scheme and driving setup

Mapping identities

- $\{\hat{\Psi}_A(\vec{r},t), U_A(\vec{r},\vec{r}',t), V_A(\vec{r},t)\}$ and $\{\hat{\Psi}_B(\vec{r},t), U_B(\vec{r},\vec{r}',t), V_B(\vec{r},t)\}$ are two evolutions of the Heisenberg equations.

- The fields operators map as:

$$\hat{\Psi}_A(\vec{r},t) \mapsto \hat{\Psi}_B(\vec{r},t) = \lambda^{D/2}\, e^{-i\frac{M}{2\hbar}\frac{\dot{\lambda}}{\lambda}r^2}\; \hat{\Psi}_A\left(\lambda\,\vec{r}, \int_0^t \lambda(t')^2\, dt'\right) \quad \text{(1a)}$$

- The interaction potentials map as

$$U_A(\vec{r},\vec{r}',t) \mapsto U_B(\vec{r},\vec{r}',t) = \lambda^{2-D} U_A(\vec{r},\vec{r}',t) \quad \text{(1b)}$$

- For the trapping potentials:

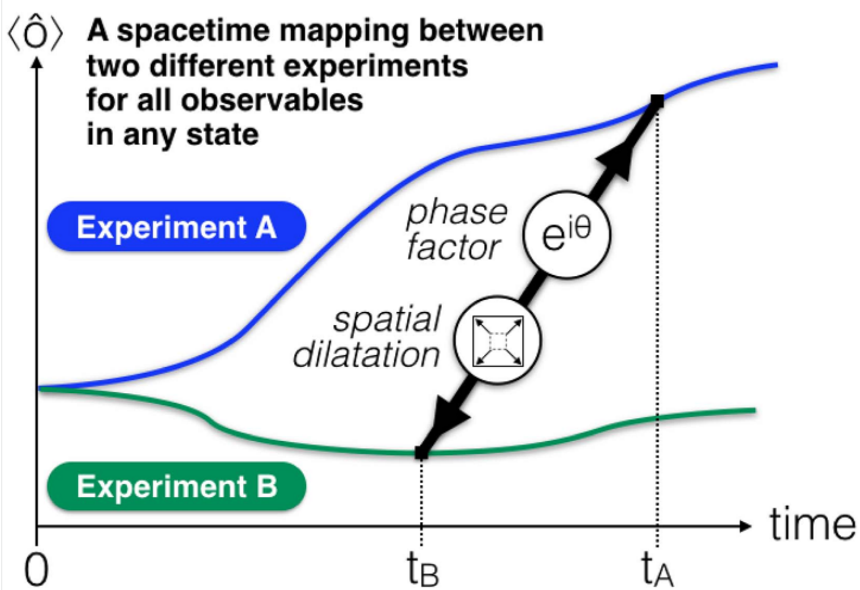
$$V_A(\vec{r},t) \mapsto V_B(\vec{r},t) = \lambda^2\left[V_A\left(\lambda\,\vec{r}, \int_0^t \lambda(t')^2\, dt'\right) + \frac{1}{2}M f(t)\, r^2\right] \quad \text{(1c)}$$

where $f(t) = \lambda(\lambda^{-2}\,\partial_t)^2\lambda$.

E. Wamba, A. Pelster, and J. R. Anglin, Phys. Rev. A **94**, 043628 (2016)

Mapping scheme and driving setup

Key features of the mapping



Space modulation

$$\vec{r} \to \lambda(t)\,\vec{r}$$

\+ **Non-trivial time transformation**

$$t \to \int_0^t \lambda(t')^2\,dt'$$

Exact !

No approximation made, it is **not** about **Gross-Pitaevskii** equation but **Heisenberg** equation.

General !

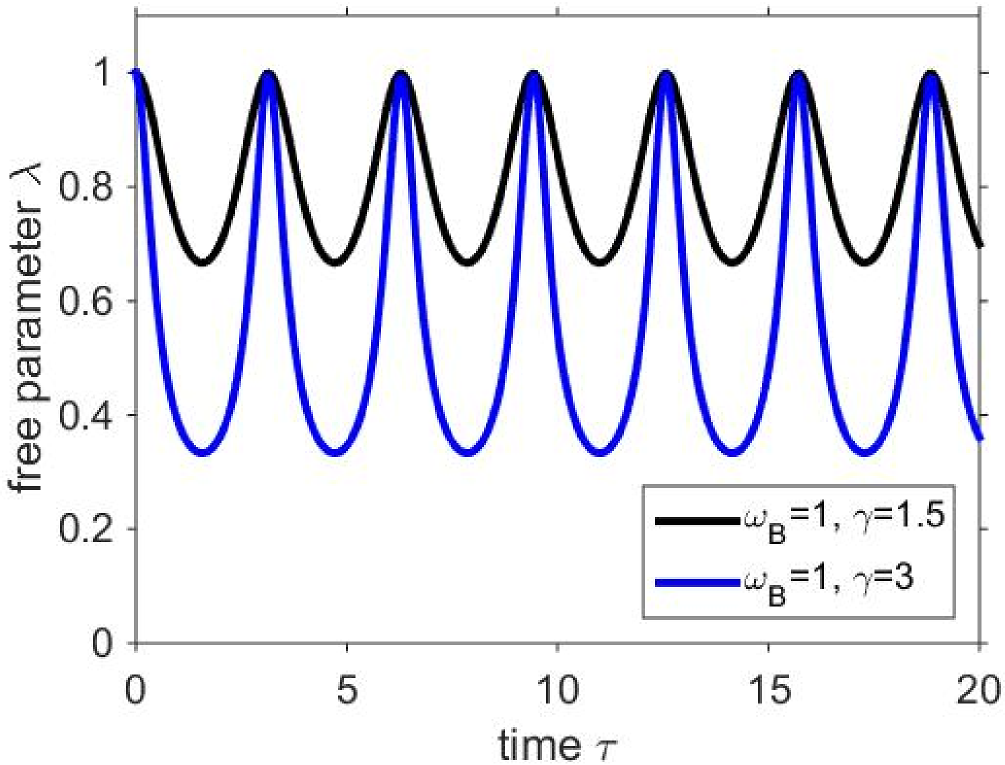
- use quantum fields not c-number fields
- valid for bosons, fermions, any mixture (species, hyperfine structures, spins)
- most real interactions and arbitrary traps
- arbitrary initial state and space dimensions,
- all possible measurements, ...

Mapping scheme and driving setup

Driving protocol and mean-field model equation

- Based on our mapping we construct a model many-body system with rapid driving but no heating.

Choose $$\boxed{\lambda(t) = \frac{1}{\sqrt{a\cos(2\omega_B t)+b}}}$$ and $U(\vec{r},\vec{r}',t) = g(t)\,\delta(\vec{r}-\vec{r}')$ (2)



$$a = \frac{1-\gamma^2}{2}$$
$$b = \frac{1+\gamma^2}{2}$$

$$\gamma = \omega_A/\omega_B$$

Mapping scheme and driving setup

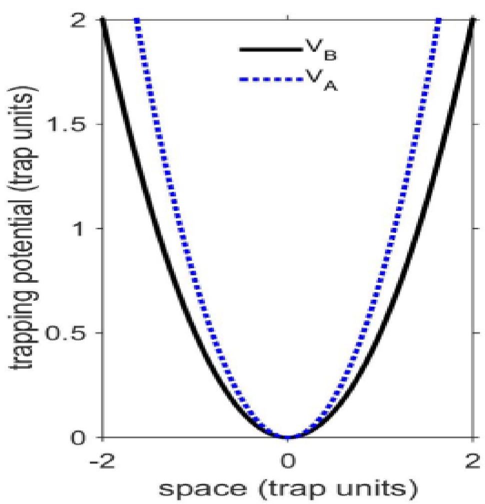
Driving protocol and mean-field model equation

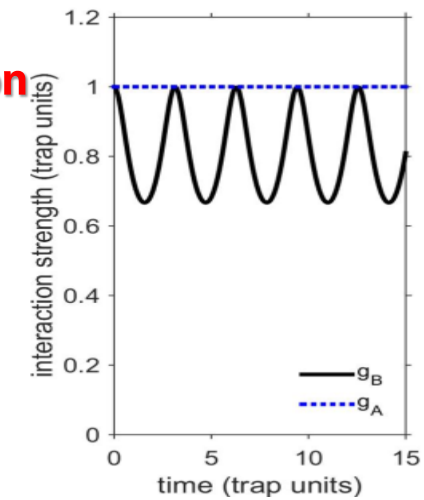
- With the setting above, we get:

$$\begin{cases} g_A = \text{const.} \\ V_A = \frac{1}{2}\gamma^2 M\omega_B^2\, r^2 \end{cases} \Longrightarrow \begin{cases} g_B = g_A \lambda^{2-D} \\ V_B = \frac{1}{2} M\omega_B^2\, r^2 \end{cases} \qquad (3)$$

Harmonic trap $\Longrightarrow$ Harmonic trap

Static interaction $\Longrightarrow$ Modulated interaction



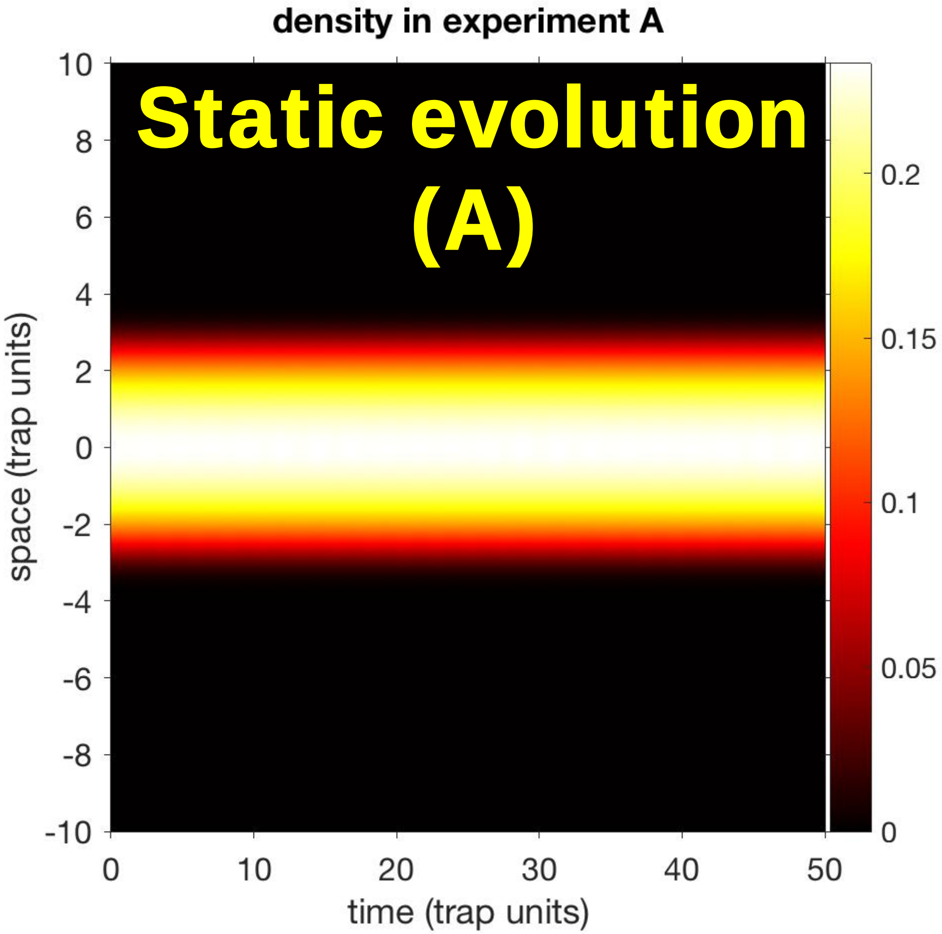


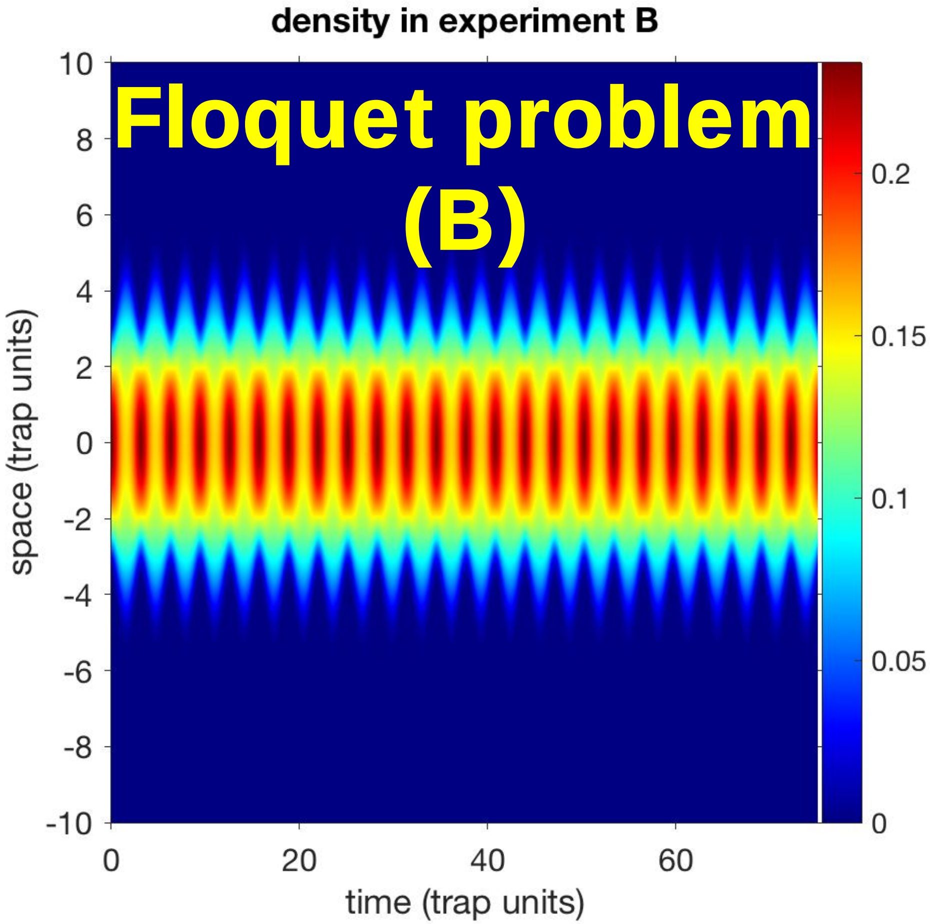
- Suppose the Bose gas is a cigar-shaped BEC. We numerically solve the mean-field Gross-Pitaevskii equation with the above parameters.

Heating estimation and avoidance

Result of direct numerical experiments (with GPE)

- With $\gamma = 1.5, \omega_B$=1, g_A=1, we get:





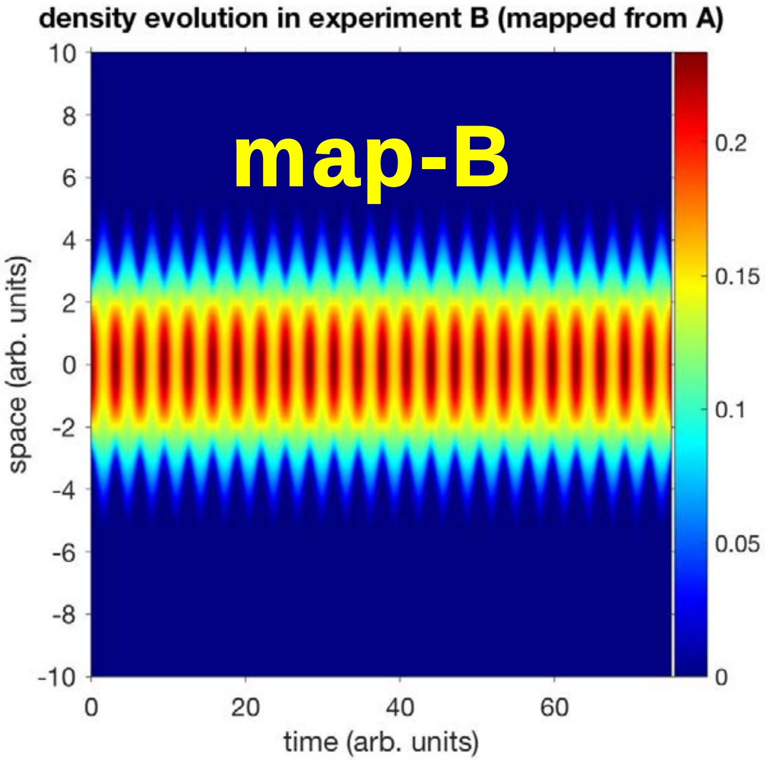
F(t)=F(0)

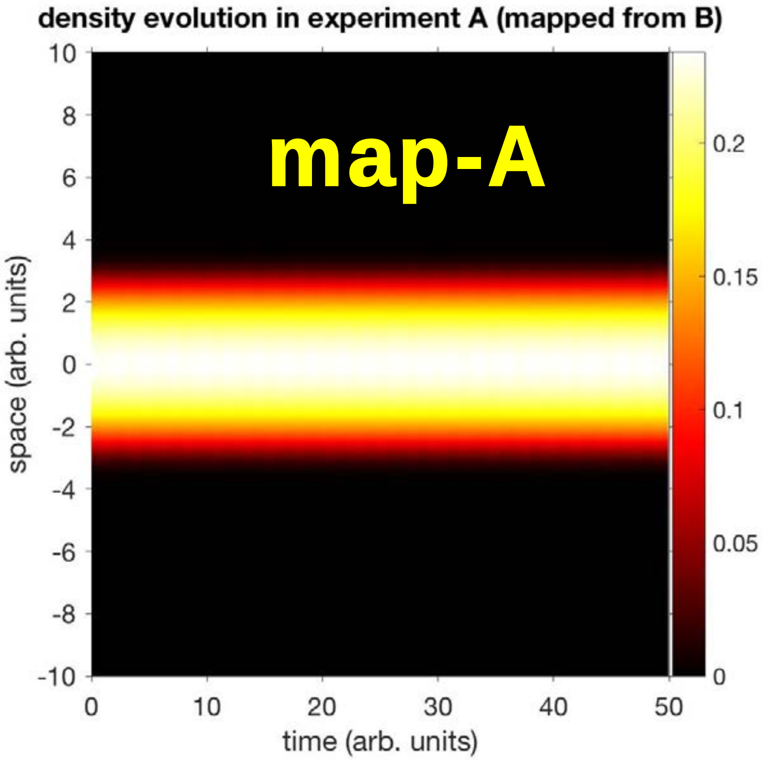
F(t)=F(t+T)

Heating estimation and avoidance

Mapping of the results and comparison

- With $\gamma = 1.5, \omega_B$=1, g_A=1, we get:





- **map B** = **B**, and **map A** = **A** ; so the mapping exactly reproduces the computed data.
- Static evolution A has no heating $\Longrightarrow$ no heating in A (periodically driven system)
- Validity of the result guaranteed even if mean-field breaks down.

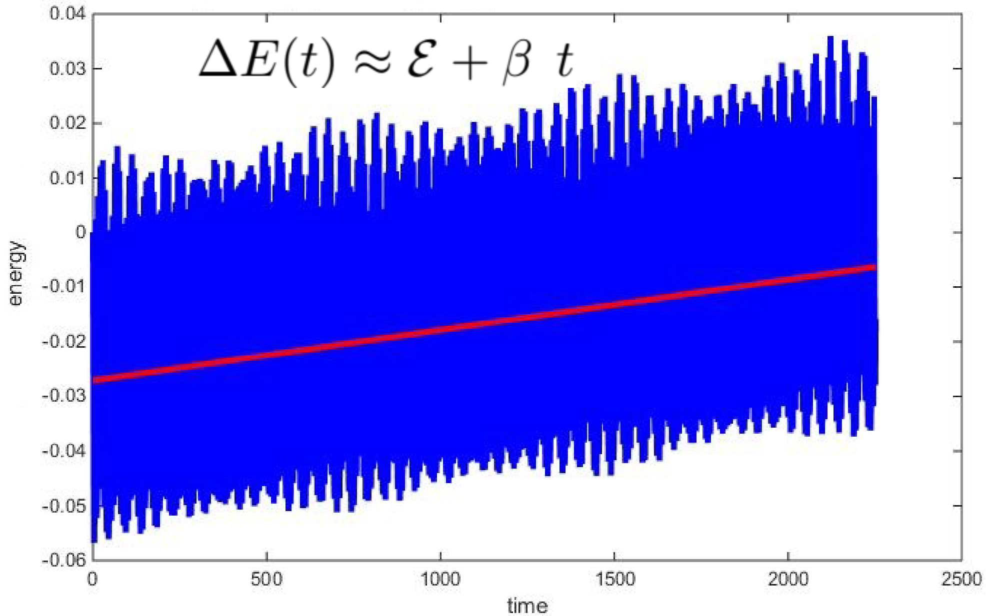
Estimation of the heating

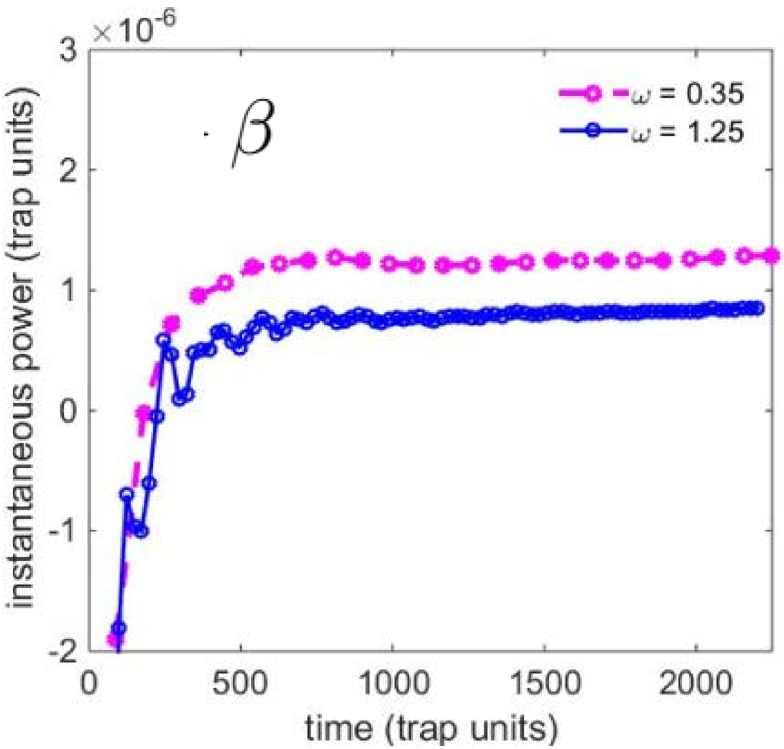
- Run the numerics and compute the energy difference

$$\Delta E(t) = E_{\mathrm{GP}}(t) - E_{\mathrm{GP}}(0),$$

with

$$E_{\mathrm{GP}}(t) = \int d^3r \left[\frac{\hbar^2}{2M} |\nabla\psi|^2 + V(\vec{r})|\psi|^2 + \frac{g(t)}{2}|\psi|^4 \right]$$



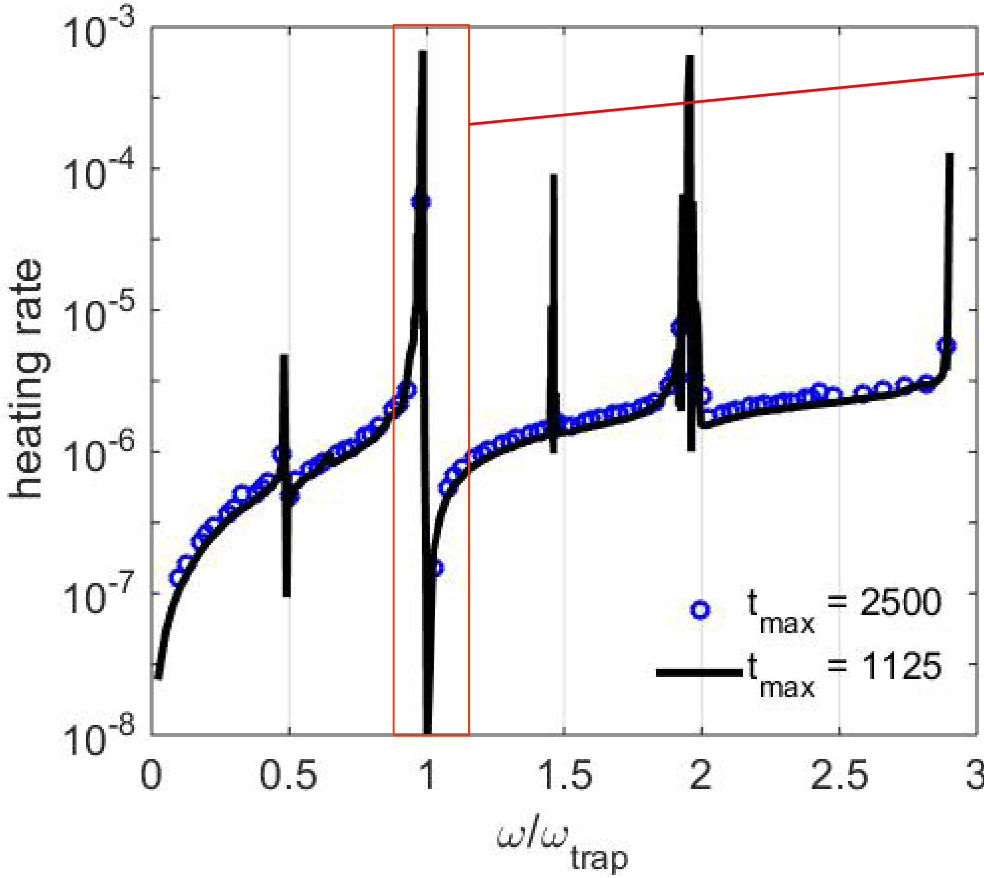


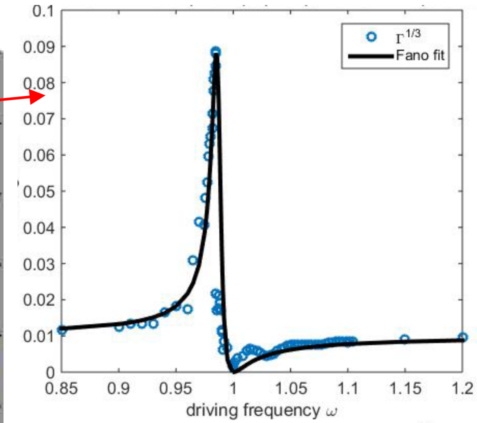
heating rate $\propto \beta\ \omega$

Heating estimation and avoidance

Heating rate versus driving frequency and interaction

- Run the experiment for many driving frequencies and compute the heating rate

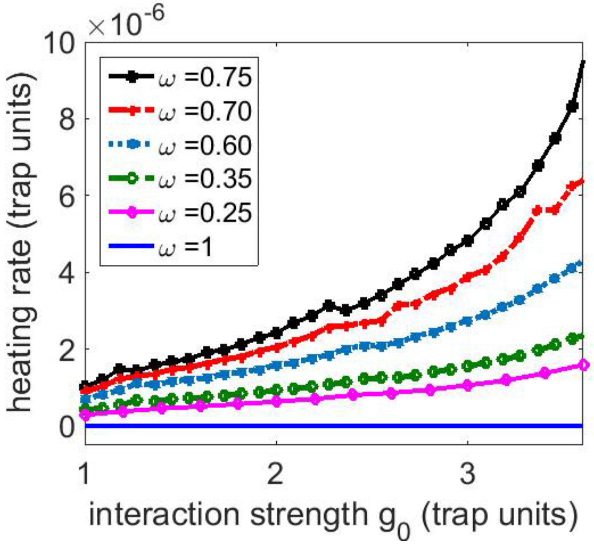




Fano resonance?

$$\Gamma^{1/3} = 0.01\frac{(-2.8+\epsilon)^2}{1+\epsilon^2}$$

$$\epsilon = 210\,(\omega\; - 0.987)$$

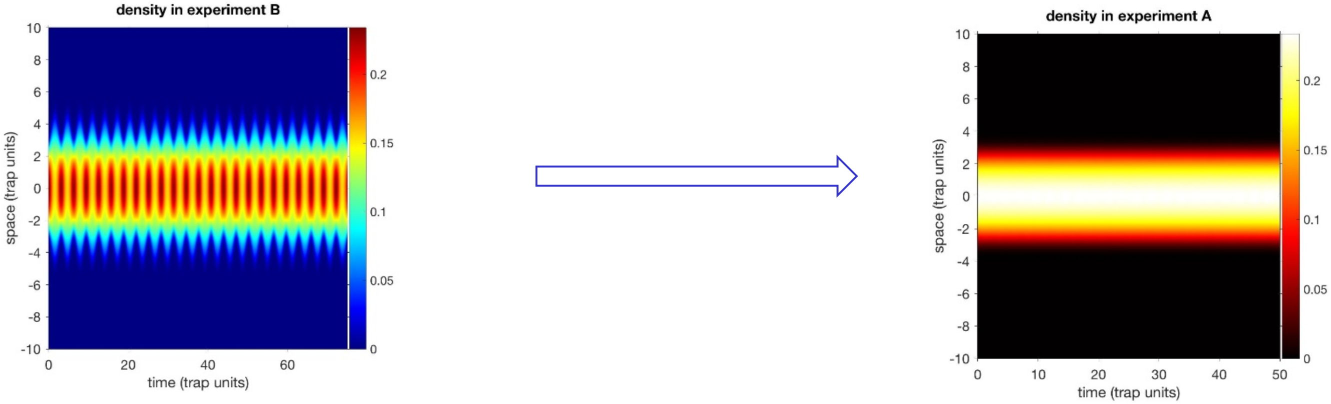


- A heating trough appears in the frequency spectrum!

Summary

Learning about heating avoidance in a Floquet system using a mapping!

Using our mapping, we have revealed that among experiments with periodically driven systems**, it exists a special class with rapid periodic driving which nevertheless do not suffer from heating**, because its time evolution has a kind of hidden adiabaticity, inasmuch as it can be mapped exactly onto that of a slowly driven or undriven system.



Beyond mean-field effects ... ?

People involved in this project:

Etienne Wamba, Axel Pelster & James R. Anglin

Thank you!