

# Harbin



6 million people, a middle size city in China

# Harbin Institute of Technology



One of the top nine universities of China

Famous for robot and space craft

Therefore, the physics department represents more the applied side of physics.

# Harbin Robot Restaurant



Based on the robot technology of Harbin Institute of Technology

# Multi-Components Effective Action Method On the Bosonic Superlattice

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TECHNISCHE UNIVERSITÄT  
KAISERSLAUTERN

# Outline

1. Effective Action Method
2. Multi-Components Effective Action Theory
3. The Application on the Bonsonic Superlattice
4. Future Plan on This Subject
5. Conclusion

# Effective Action Method(1)

Effective action theory is based on Landau's phase transition theory and field theory.

1. Add spatially constant source terms:

$$\begin{aligned}\hat{H}_{\text{BH}}(J, J^*) = & -t \sum_{i,j} \hat{a}_i^\dagger \hat{a}_j + U/2 \sum_i \hat{n}_i (\hat{n}_i - 1) \\ & - \mu \sum_i \hat{n}_i + \sum_i (J^* \hat{a}_i + J \hat{a}_i^\dagger)\end{aligned}$$

2. Expand free energy in power series of the source:

$$F(J, J^*, t) = N_s \left( F_0(t) + \sum_{p=1}^{\infty} c_{2p}(t) |J|^{2p} \right)$$

# Effective Action Method(2)

3. Perform Legendre transformation to get effective potential:

$$\Gamma(\psi, \psi^*, t) = F/N_s - \psi^* J - \psi J^*.$$

4. Expand effective potential in power series of order parameter:

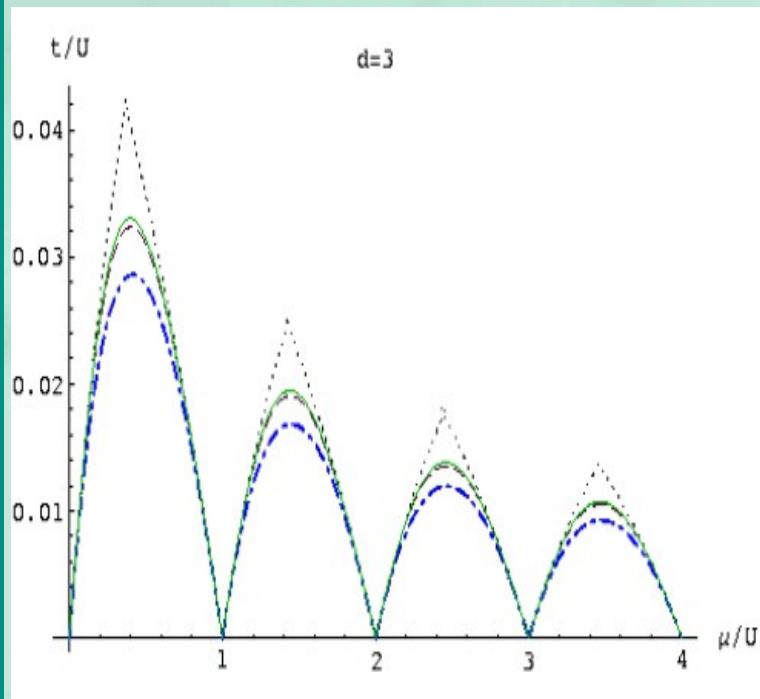
$$\Gamma(\psi, \psi^*, t) = F_0(t) - \frac{1}{c_2(t)} |\psi|^2 + \frac{c_4(t)}{c_2(t)^4} |\psi|^4 + \dots$$

5. Perturbative calculation of second order coefficients:

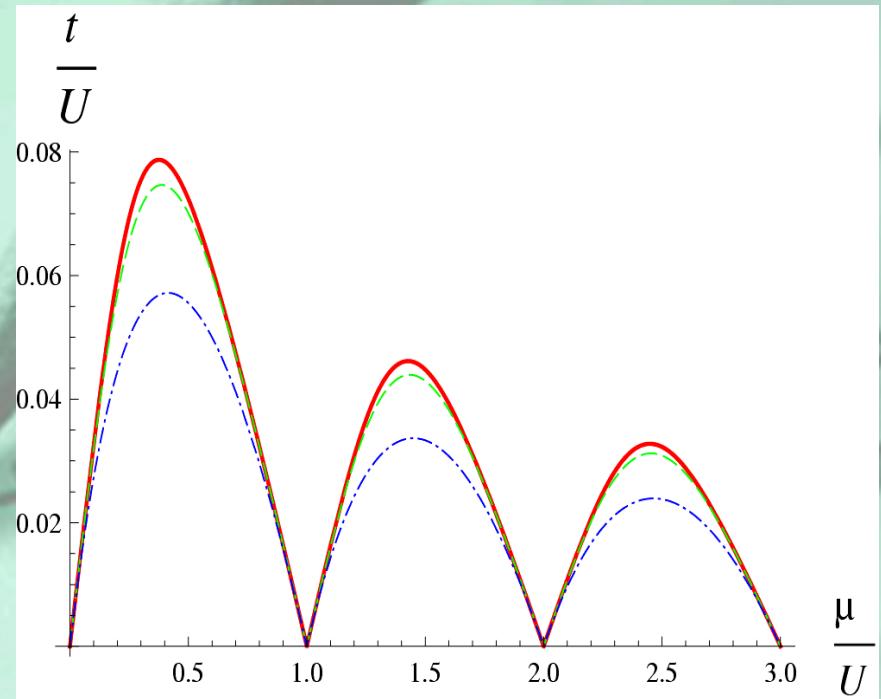
$$\frac{1}{c_2(t)} = \frac{1}{\alpha_2^{(0)}} \left( 1 + \frac{\alpha_2^{(1)}}{\alpha_2^{(0)}} t + \left[ \left( \frac{\alpha_2^{(1)}}{\alpha_2^{(0)}} \right)^2 - \frac{\alpha_2^{(2)}}{\alpha_2^{(0)}} \right] t^2 + \dots \right)$$

# Effective Action Method(3)

Effective action theory achieved great success both in quadratic and frustrated lattice system



Quadratic Lattice



Triangular lattice

1. F. E. A. dos Santos and A. Pelster, Phys. Rev. A **79**, 013614 (2009)
2. Z. Lin, J. Zhang and Y. Jiang, Phys. Rev. A **85**, 023619 (2012)

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# Multi-Components Effective Action Theory(1)

Muti-Components Effective Action Theory is in fact a more generalized form of effective action theory.

**Purpose:** to solve the second-order phase transition problem for a system that can be divided into several subsystem.

1. Add independent source terms for every subsystem:

$$\begin{aligned}\hat{H} = & - \sum_{i,j=1}^m \sum_{\langle k,l \rangle} t_{i,j} \left( \hat{a}_{i_{(k)}}^\dagger \hat{a}_{j_{(l)}} + \hat{a}_{i_{(k)}} \hat{a}_{j_{(l)}}^\dagger \right) \\ & + \sum_{i=1}^m \frac{U_i}{2} \sum_k \hat{n}_{i_{(k)}} (\hat{n}_{i_{(k)}} - 1) - \sum_{i=1}^m \mu_i \sum_k \hat{n}_{i_{(k)}} \\ & - \sum_{i=1}^m \sum_k \left( J_i \hat{a}_{i_{(k)}}^\dagger + J_i^* \hat{a}_{i_{(k)}} \right).\end{aligned}$$

## Multi-Components Effective Action Theory(2)

2. Expand free energy in power series of source terms:

Define:  $\mathbb{J} = \{J_1, \dots, J_m\}$

$$F(\mathbb{J}, t) = N_s (F_0(t) + \mathbb{J} C_2 \mathbb{J}^\dagger + \dots)$$

3. Perform Legendre transformation to get effective potential:

Define:  $\Psi = \frac{1}{N_s} \frac{\partial F}{\partial \mathbb{J}^\dagger} = \{\langle \hat{a}_1 \rangle, \dots, \langle \hat{a}_m \rangle\}$

$$\Gamma(\Psi, t) = F/N_s - \Psi \mathbb{J}^\dagger - \mathbb{J} \Psi^\dagger$$

4. Expand effective potential in power series of order parameter :

$$\Gamma(\Psi, t) = \Gamma_0(t) + \Psi A_2 \Psi^\dagger + \dots$$

## Multi-Components Effective Action Theory(3)

5. Resulting relation between coefficients C and A:

$$-1 = A_2 \frac{d\Psi}{d\mathbb{J}} = A_2 \frac{1}{N_s} \frac{\partial^2 F}{\partial \mathbb{J}^\dagger \partial \mathbb{J}} = A_2 C_2$$

The second-order phase transition boundary condition is:

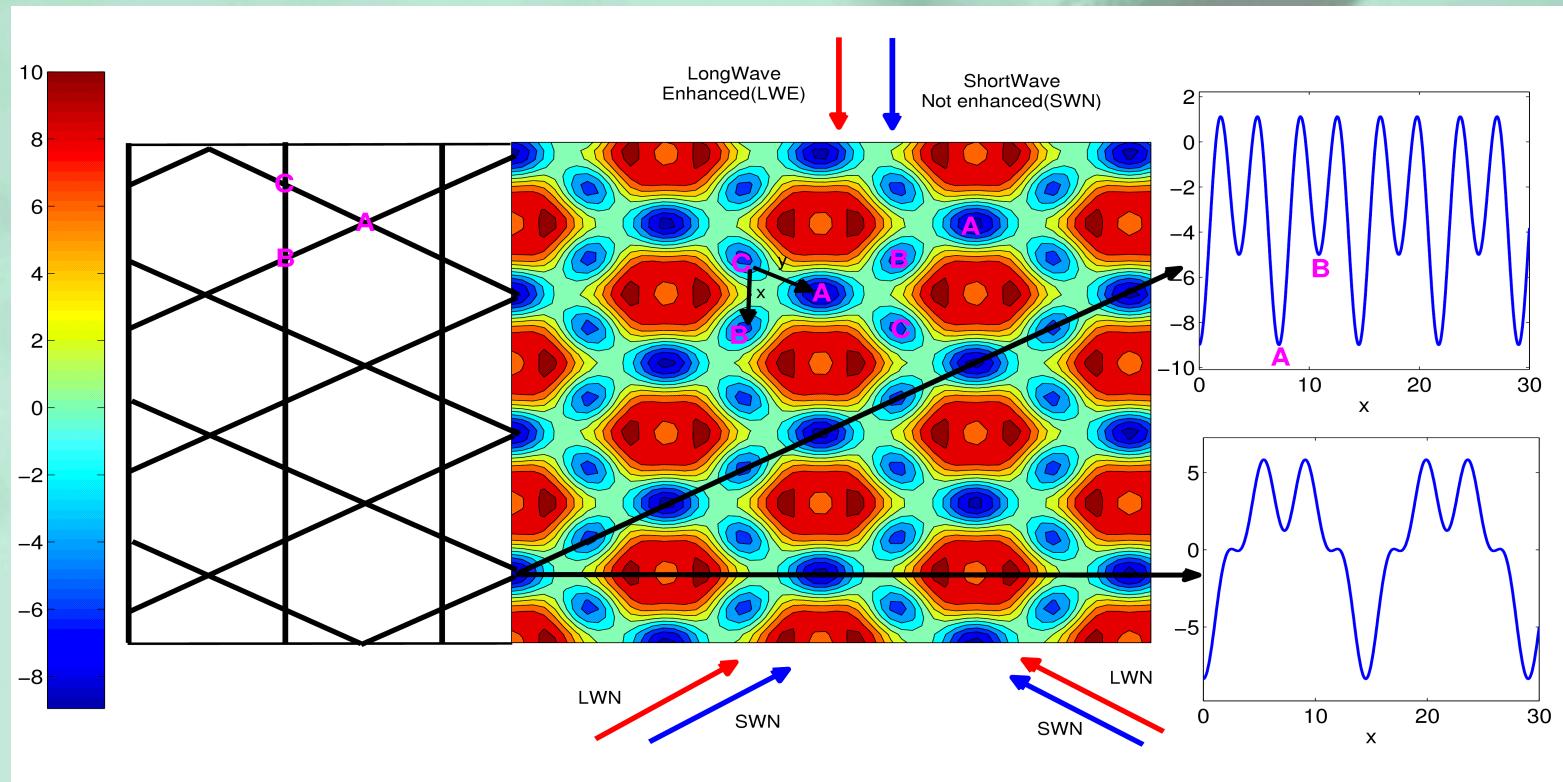
$$\frac{-1}{\begin{vmatrix} c_{2A11} & \dots & c_{2A1m} \\ \dots & \dots & \dots \\ c_{2Am1} & \dots & c_{2Amm} \end{vmatrix}} = 0$$

6. Perturbatively calculated second-order coefficient

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# Superlattice Model



Hamiltonian of the system

$$\hat{H}_{BH} = -t \sum_{\langle ij \rangle} \hat{a}_i^\dagger \hat{a}_j + U/2 \sum_i \hat{n}_i (\hat{n}_i - 1) - \mu \sum_i \hat{n}_i - \Delta\mu \sum_{A1(i)} \hat{n}_{A1(i)}$$

G. -B. Jo, et al., Phys. Rev. Lett. 108, 045305 (2012)

# Quadratic Superlattice(1)

For quadratic superlattice system, we decouple our system into two subsystems.

Follow the process introduced in last chapter.

Phase boundary condition:

$$\frac{-1}{\begin{vmatrix} c_{2A11} & c_{2A12} \\ c_{2A21} & c_{2A22} \end{vmatrix}} = 0.$$

Expand in hopping series

$$c_{2Aij}(t) = \sum_{n=0}^{\infty} (-t)^n \alpha_{2Aij}^{(n)}$$

# Quadratic Superlattice(2)

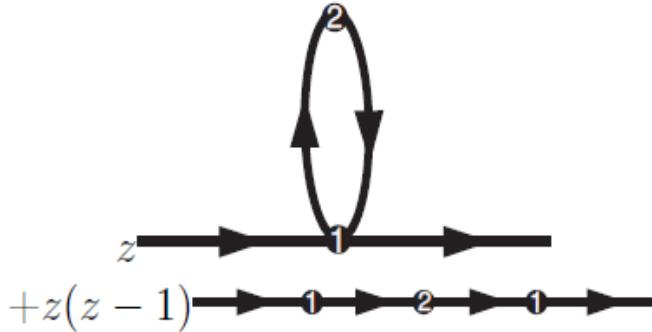
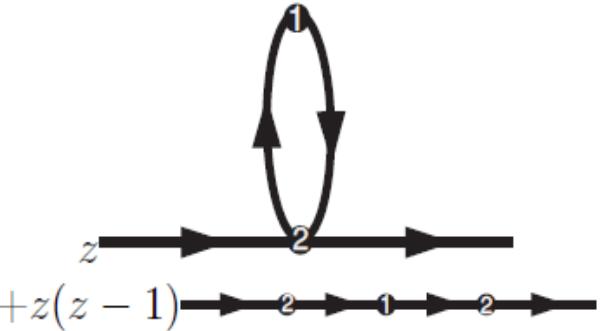
Make a truncation at second hopping order:

$$1 + \frac{1}{\sqrt{\alpha_{2A11}^{(0)} \alpha_{2A22}^{(0)}}} t + \frac{-\alpha_{2A11}^{(2)} \alpha_{2A22}^{(0)} - \alpha_{2A11}^{(0)} \alpha_{2A22}^{(2)} + 2(\alpha_{2A12}^{(1)})^2}{2\sqrt{\alpha_{2A11}^{(0)} \alpha_{2A22}^{(0)}}} t^2 = 0$$

Considering the smallest root:

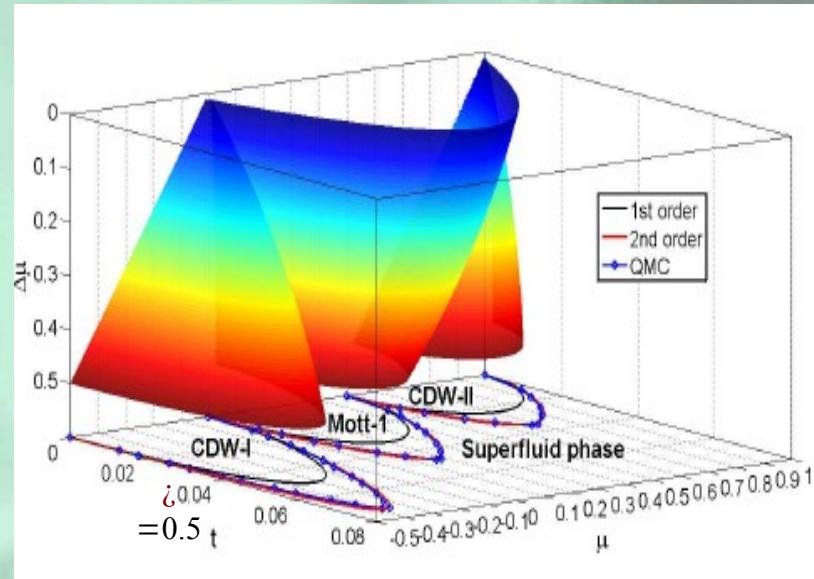
$$t_c = \frac{-\alpha_{2A11}^{(0)} \alpha_{2A22}^{(0)} \alpha_{2A12}^{(1)} + \sqrt{2 \left( \alpha_{2A11}^{(0)} \alpha_{2A22}^{(0)} \right)^2 \left( \alpha_{2A11}^{(2)} + \alpha_{2A22}^{(2)} - \alpha_{2A12}^{(1)} \right)}}{\left( \sqrt{\alpha_{2A11}^{(0)} \alpha_{2A22}^{(0)}} \right) \left( \alpha_{2A11}^{(2)} \alpha_{2A22}^{(0)} + \alpha_{2A11}^{(0)} \alpha_{2A22}^{(2)} + 2 \left( \alpha_{2A12}^{(1)} \right)^2 \right)}.$$

# Quadratic Superlattice(3)

	zeroth order	first order	second order
$\alpha_{2A11}$		0	 $z$ $+z(z-1)$
$\alpha_{2A12}$	0		0
$\alpha_{2A22}$		0	 $z$ $+z(z-1)$

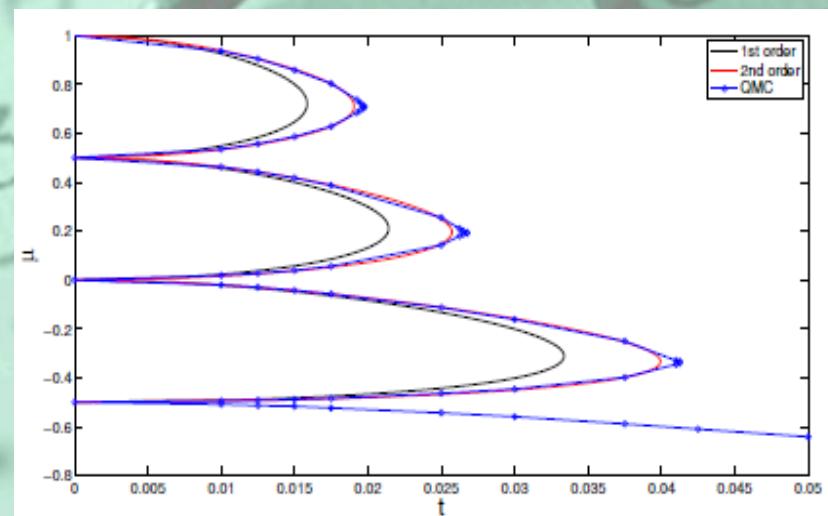
# Quadratic Superlattice(4)

2d system result

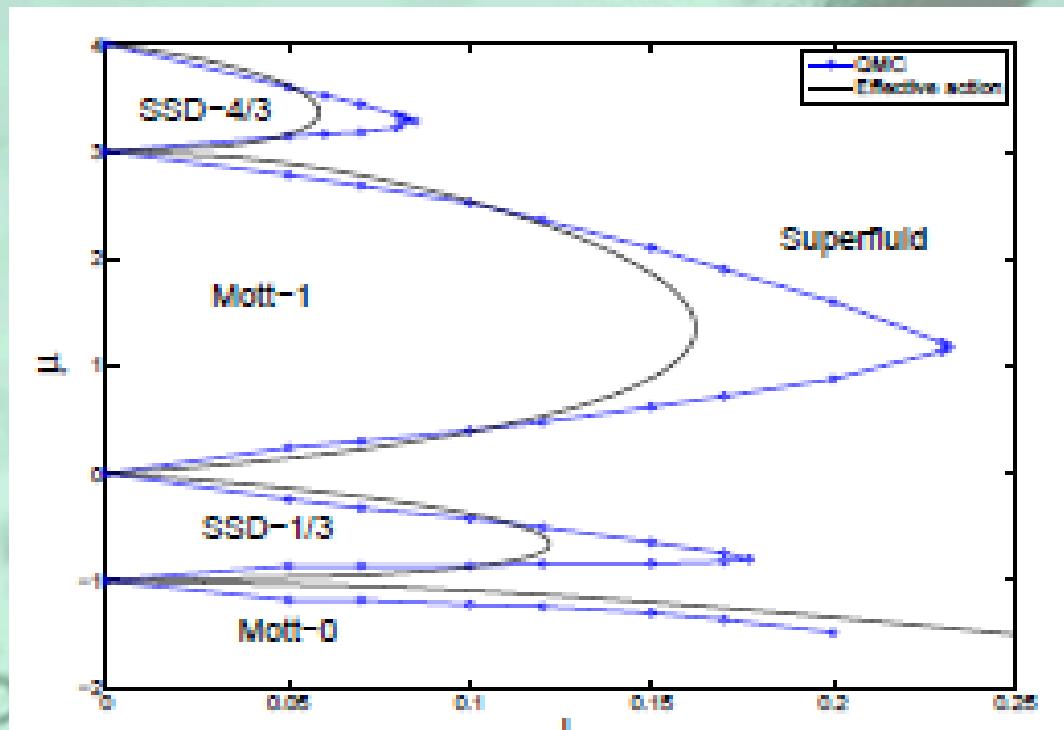


3d system result

$$\Delta\mu=0.5$$



# Frustrated Superlattice



Kagome Superlattice result (1st order)

# Advantage comparing to decoupled mean field theory

1. Our method has much higher accuracy:

method	DMF	MCEA (2nd order)
accuracy	About 30%	Less than 3%

2. Using DMF method the free energy is at the local maximal point in the Mott lobe, our method does not have this problem.

3. Our method gets very good result in Kagome superlattice system, while DMF gets totally wrong result comparing to QMC.

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# Future Plan on This Subject

Using process-chain method to calculate higher-order coefficients

Purpose:

1. get more accurate phase boundary.
2. find the universality class of the phase transition by calculating critical exponents

N. Teichmann, D. Hinrichs, M. Holthaus and A. Eckardt,  
Phys. Rev. B **79**, 224515 (2009).

Developing the method to calculate the phase transition from DSW-SS for hard-core bosons system

Because there is so far no practical method for this system when hopping term becomes negative.

A. Eckardt, C. Weiss, and M. Holthaus,  
Phys. Rev. Lett. **95**, 260404 (2005)

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# Conclusion

1. we developed a multi-components effective action method based on effective action theory.
2. our method achieved a great success in calculating superlattice systems both for the quadratic and the frustrated case.
3. we found that decoupled mean- field theory, which people have used a lot, has local-minimum problem.
- 4.our next step is to develop process-chain method for multi-components system in order to calculate higher hopping orders

*Thank you for your attention!*