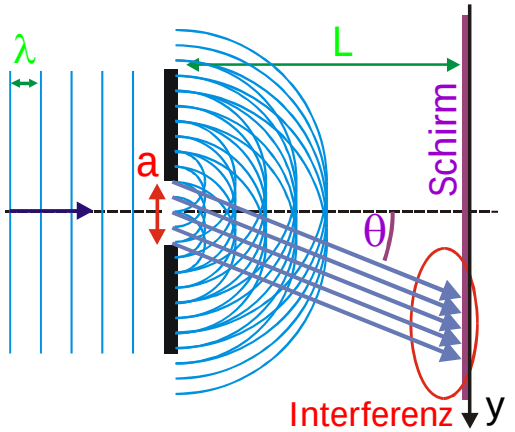
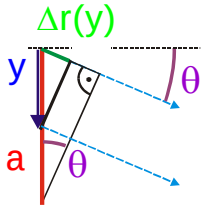
Zusammenfassung vom 16.07.2009

XI Beugung am endlich breiten Spalt

Partialwellen: $$d\Phi(y') = \mathrm{Re}\left\{\frac{1}{a}\frac{\vec{E}_0}{r} e^{i[kr-\omega t+\delta(y')]} dy'\right\}$$



Phasendifferenz: $$\delta(y') = k\,\Delta r \cong \frac{2\pi}{\lambda} y' \sin\theta$$



Gesamtamplitude: $$\vec{\Phi}(\sin\theta) = \mathrm{Re}\left\{\frac{\vec{E}_0}{a\,L}\int_{-a/2}^{+a/2} e^{i[kL-\omega t+\delta(y')]} dy'\right\} = \mathrm{Re}\left\{\frac{\vec{E}_0}{a\,L} e^{i(kL-\omega t)}\int_{-a/2}^{+a/2} e^{i\frac{2\pi}{\lambda}\sin\theta\, y'} dy'\right\}$$

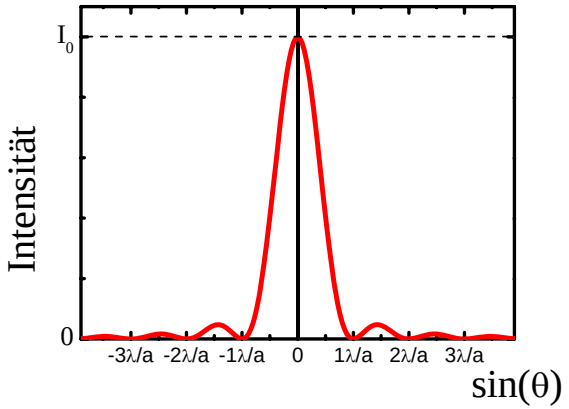
($r \cong L$)

$$\rightarrow \vec{\Phi}(\sin\theta) = \mathrm{Re}\left\{\frac{\vec{E}_0}{a\,L}\frac{e^{i(kL-\omega t)}}{i\frac{2\pi}{\lambda}\sin\theta}\left(e^{i\frac{\pi a}{\lambda}\sin\theta} - e^{-i\frac{\pi a}{\lambda}\sin\theta}\right)\right\} = \frac{\vec{E}_0}{L}\cos(kL-\omega t)\frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\frac{\pi a}{\lambda}\sin\theta}$$

Intensität: $$\bar{I}(\theta) = \left\langle \varepsilon_0 c \vec{\Phi}^2 \right\rangle_t = I_0 \frac{\sin^2\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2}$$

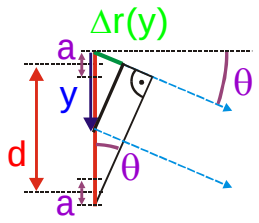
$$I_0 = \frac{\varepsilon_0 c}{2L^2}\vec{E}_0^2$$

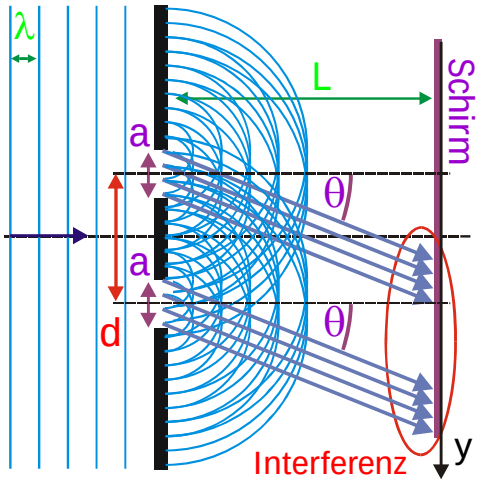
I_0 = Intensität der Welle ohne Spalt im Abstand L



XI Beugung am endlich breiten Doppelspalt

Phasendifferenz: $\delta(y) = k\,\Delta r \cong \frac{2\pi}{\lambda}\, y \sin\theta$





Gesamtamplitude: $\vec{\Phi}(\sin\theta) = \mathrm{Re}\left\{ \frac{\vec{E}_0}{a\,L} \int_{\text{Spalte}} e^{i(kL-\omega t+\delta(y'))} dy' \right\}$

$$\rightarrow \quad \vec{\Phi}(\sin\theta) = \frac{\vec{E}_0}{a\,L} \frac{e^{i(kL-\omega t)}}{\frac{i2\pi}{\lambda}\sin\theta} \left[\left(e^{i\frac{\pi d}{\lambda}\sin\theta} + e^{-i\frac{\pi d}{\lambda}\sin\theta} \right) \left(e^{i\frac{\pi a}{\lambda}\sin\theta} - e^{-i\frac{\pi a}{\lambda}\sin\theta} \right) \right]$$

$$= \frac{2\vec{E}_0}{L} e^{i(kL-\omega t)} \cos\left(\frac{\pi d}{\lambda}\sin\theta\right) \frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\frac{\pi a}{\lambda}\sin\theta}$$

Intensität: $\bar{I}(\theta) = 4 I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right) \frac{\sin^2\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2}$

$$I_0 = \frac{\varepsilon_0 c}{2L^2} \vec{E}_0^{\,2}$$

I_0 = Intensität der Welle ohne Doppelspalt im Abstand L