Advanced Atomic and Molecular Physics 2017-05-08 Prof. Heyne

2. assignment

E2 Deadline : lecture next Monday (2017-05-15)

1. For the H.O. it is
   $a^+ a\Psi_n = -2n\Psi_n$ for n = 0,1,2,…

   a) (2 points) Show that for quadratically integrable functions Ψ and φ
   $$\int (a^+\varphi)^*\psi \, dy = -\int \varphi^* a\psi \, dy$$
   by partial integration and
   $$\int (a^+\psi_n)^*(a^+\psi_n) \, dy = 2(n+1)\int \psi_n^*\psi_n \, dy$$

   b) (2 points) Use the results of b) for normalized Ψ_n and show
   $$\psi_{n+1} = \frac{1}{\sqrt{2(n+1)}} a^+\psi_n$$
   and
   $$a\psi_n = -\sqrt{2n}\psi_{n-1}$$

   c) (3 points) Show the transition probability from $\Psi_n \rightarrow \Psi_m$ is proportional to the square of the matrix element
   $$\int \psi_m^* x\psi_n \, dy = \sqrt{\frac{\hbar}{m\omega}}\int \psi_m^* y\psi_n dy = \sqrt{\frac{\hbar}{m\omega}}\int \psi_m^*\left(\frac{a-a^+}{2}\right)\psi_n dy$$
   use c) to prove that the transition probability is only not zero for m=n±1 (selection rule) and that
   $$\left|\int \psi_{n+1}^* x\psi_n dy\right|^2 = \frac{(n+1)\hbar}{2m\omega}$$
   $$\left|\int \psi_{n-1}^* x\psi_n dy\right|^2 = \frac{n\hbar}{2m\omega}$$

2. a) (2 points) Calculate the 5 sperical harmonics of Y_{lm} using the L_- operator starting at Y_{22} and determine the normalization constants.

   b) (2 points) Calculate the expectation value <r> in the state n=3 and l=2 of the hydrogen atom.

   c) (3 points) Use the variational method and the test function $e^{-\alpha r}$ to estimate the energy E_0 of the hydrogen atom. Show for the solution $\langle T\rangle = -1/2\langle V\rangle$.