Deadline: lecture Thursday (2017-05-25)

1. (4 points) Since the kinetic energies in the Schrödinger equation are not relativistic, there is a relativistic correction for energies of the hydrogen atom. The relativistic term for kinetic energy of the electron is
$$\sqrt{p^2c^2 + m^2c^4} - mc^2$$
For a $10\,\mathrm{eV}$ electron is $(v/c)^2 \approx 10^{-5}$ and $\frac{p}{mc} \ll 1$. Hence, it is enough to expand the square root into a series and only account for the first two remaining terms. The derived correction on $p^2/2m$ can be seen as perturbation H' on H_0:
$$H = H_0 + H'$$
Calculate the first order energy correction of the ground state of the hydrogen atom.
$$\Delta E = \int \psi^*_{100} H' \psi_{100} \, \mathrm{d}\mathbf{r}$$
Note: rewrite the factors $\mathbf{p}^2$ in H' with the help of
$$H_0 = \frac{\mathbf{r}^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r}$$
since ψ_{100} is a Eigenfunction of H_0. Estimate ΔE and compare its value to E_1.

2. (4 points) The radius of the nucleus with the atomic mass number A is well approximated by
$$R = 1.3 \times 10^{-13} A^{\frac{1}{3}} \mathrm{cm}.$$
We assume that the proton-charge inside the sphere is distributed homogeneously. Now the potential energy of the electron inside the hydrogen atom is not given $-e^2/4\pi\epsilon_0 r$ anywhere, instead it has been modified inside the sphere.

   First, calculate the potential energy with help of the Electrostatics (Gauss). The perturbation due to the finite size of the proton is only different from zero inside the sphere. Using first order perturbation theory, calculate the energy shift of the $1s$ ground state of hydrogen. Use $R/a_0 \ll 1$ for solving integrals.