Deadline: lecture Thursday (2017-06-01)

1. (2 points) Calculate the length of the transition dipole moment of the $2p_z \rightarrow 2s$ transition between two hydrogen like states with the nuclear charge $Z = 2$. Assuming that $\left|\underline{\vec{\mu}}\right|$ can be interpreted as the product of the electrical charge and an effective distance over which the electron is moved by the state transition, how long is the distance?

2. (6 points) Interaction of atoms with electromagnetic radiation

   (a) Prove that

   $$(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}})(\hat{\epsilon} \cdot \vec{\mathbf{p}}) = \frac{1}{2}\left[(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}})(\hat{\epsilon} \cdot \vec{\mathbf{p}}) + (\hat{\epsilon} \cdot \vec{\mathbf{r}})(\vec{\mathbf{k}} \cdot \vec{\mathbf{p}})\right] + \frac{1}{2}\left[(\vec{\mathbf{k}} \times \hat{\epsilon})(\vec{\mathbf{r}} \times \vec{\mathbf{p}})\right]$$

   (b) Show that

   $$\frac{1}{2}\left[(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}})(\hat{\epsilon} \cdot \vec{\mathbf{p}}) + (\hat{\epsilon} \cdot \vec{\mathbf{r}})(\vec{\mathbf{k}} \cdot \vec{\mathbf{p}})\right]$$

   evaluates to the electronic quadrupole transition dipole moment Q_{ij}.
   Note: Use $\vec{\mathbf{p}} = \frac{im}{\hbar}[H_0, \vec{\mathbf{r}}]$ and that Ψ_m and Ψ_k are eigenfunctions of H_0. Additionally note that: $\vec{\mathbf{k}} \cdot \hat{\epsilon} = 0$.

   (c) Starting with Q_{ij}, derive the transition rules for electronic quadrupole transitions (as in the lecture, calculate Δm, apply the parity operator and use the result to get $\Delta\ell$).

   (d) The derivation of the transition rules for magnetic transition is simpler, since the initial and the final state are eigenfunctions of L^2 and L_z. For L_x and L_y it is advisable to use L_+ and L_- . What are the transition rules for magnetic dipole transitions?

3. (1 point) Show that the squared absolute value $\kappa = |\hat{\varepsilon} \cdot \underline{\vec{\mu}}|^2$, with the linear polarisation $\hat{\varepsilon} = (0, 0, 1)$ and isotropic distribution of the $\underline{\vec{\mu}}$ vectors in space, evaluates to:

   $$\overline{\kappa} = \frac{1}{3}.$$

   Here, $\overline{\kappa}$ is the average over all directions of ϑ and φ. Don't forget the normalisation factor.