Name: ___________________________

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Advanced Solid State Physics  Prof. Dr. W. Kuch
Winter semester 2014/2015
7th exercise sheet

Submission: Tuesday, 02. December 2014, before the lecture
(or drop until 10 o'clock on the same day in mailbox between rooms 1.2.38 and 1.2.40)

**19. Standing waves on Cu(111)** (*) (4 points)

The energy dispersion of the surface state of Cu(111) can be approximated by a parabola with $E = \frac{\hbar^2 k_{\parallel}^2}{2m^*} - E_0$, where E is measured relative to the Fermi energy, and $m^* = 0.45\ m_e$, $E_0 = 0.4$ eV.

What is the distance between maxima of the standing waves that are observed next to step edges in scanning tunneling experiments of a Cu(111) surface if the bias voltage between tip and sample approaches zero?

**20. Quantum well states in laterally confined structures** (**) (4 points)

Scanning tunneling spectroscopy in the dI/dV mode can be used to measure the local density of states $\rho(E,\vec{r})$ at the surface.

a) Consider first a one-dimensional problem and determine the energies E at which $\rho(E,\vec{r})$ of electrons with parabolic dispersion and an effective mass m* is maximum in the center of a one-dimensional lateral constriction of length L, which we assume to represent an infinitely high quantum well. Calculate the energies of the seven lowest-lying states that show an antinode of the wave function in the center for the surface state of Cu(111), using the parabolic approximation of exercise 19, and a width of the well L = 142.6 Å.

b) In a circular two-dimensional quantum well of radius R and infinite barrier height the wave functions are given by $\ell$-th order Bessel functions $J_\ell(\vec{k}\cdot\vec{r})$ (see also eigenmodes of a circular membrane in advanced classical mechanics textbooks, and the Bessel differential equation in mathematics textbooks) with energies $E_{n,\ell} = \frac{\hbar^2 k_{n,\ell}^2}{2m^*}$, and $n = 1,2,\ldots$ Here $k_{n,\ell} = \frac{z_{n,\ell}}{R}$, where $z_{n,\ell}$ is the n-th zero crossing of $J_\ell(z)$. Antinodes in the center of the quantum well, and hence maxima of $\rho(E,\vec{r})$, are found only for Bessel functions of 0-th order. $J_0(z)$ can be approximated by $J_0(z) \approx \sqrt{\frac{2}{\pi z}}\cos(z - \frac{\pi}{4})$ (asymptotic approximation). Calculate, analogously to a), the energy values of the seven lowest-lying eigenstates with maxima of $\rho(E,\vec{r})$ in the center for the surface state of Cu(111) in a circular quantum well with 2R = 142.6 Å, and compare to the result of a).

**21. Graphene** (**) (4 points)

The distance between nearest-neighbor carbon atoms in graphene is 1.42 Å. Calculate the areal density (mass per area) of graphene. (For the structure of graphene, refer to textbooks or the lecture notes.)