Particles scatter and absorb electromagnetic radiation. One often needs to compare the amount of scattering/absorption/extinction for particles of different shapes, composition, sizes and incident light properties (polarization, frequency and angle). In this regard, the concept of cross-sections comes into picture. There are three types of cross-sections, 1) scattering 2) absorption and 3) extinction. All of them have units of area, m^2, (which I will show soon) and provide a measure to quantify scattering/absorption process.

To understand the concept of cross-section, one needs to understand how to quantify the transfer of electromagnetic energy over the surface. Consider a surface A which encloses a volume V. We can also choose a normal vector,$\hat{n}$, on every point of A, such that it has postive magnitude as it faces outwards. The rate at which electromagnetic energy transfers from this surface is given by $\boldsymbol{W} = -\oint \boldsymbol{S} \cdot \hat{n} dA$, where $\boldsymbol{S}$ indicates the time-averaged poynting vector. Time-averaged Poynting vector indicates the average rate of transfer of electromagnetic energy per unit area and is given by, $\boldsymbol{S} = \frac{1}{2} Re\{\boldsymbol{E} \times \boldsymbol{H}^*\}$. It has the units of W/m^2. A negative $\boldsymbol{W}$ indicates energy transferred out of the surface and positive $\boldsymbol{W}$ indicates transfer of energy into the surface.

Now lets imagine a particle of arbitrary geometry enclosed by A and light of some frequency and polarization hits this particle. At any point belonging to A, the time-averaged Poynting vector at that point is given by $\boldsymbol{S} = \frac{1}{2} Re\{\boldsymbol{E} \times \boldsymbol{H}^*\}$. $\boldsymbol{S}$ is a sum of three terms, $\boldsymbol{S} = \boldsymbol{S_i} + \boldsymbol{S_s} + \boldsymbol{S_{ext}}$. S_i represents the time-averaged Poynting vector due to incident light, S_s represents the time-averaged Poynting vector of scattered light and S_{ext} represents the time-averaged Poynting vector of interaction due to scattered light and incident light. They can expressed interms of scattered and incident electric and magnetic fields by following relations:

$$\boldsymbol{S_i} = \frac{1}{2} Re\{\boldsymbol{E_i} \times \boldsymbol{H_i^*}\},$$
$$\boldsymbol{S_s} = \frac{1}{2} Re\{\boldsymbol{E_s} \times \boldsymbol{H_s^*}\},$$
$$\boldsymbol{S_{ext}} = \frac{1}{2} Re\{\boldsymbol{E_i} \times \boldsymbol{H_s^*} + \boldsymbol{E_s} \times \boldsymbol{H_i^*}\}$$

The rate at which energy comes into the surface A is given by $\boldsymbol{W_i} = -\oint \boldsymbol{S_i} \cdot \hat{n} dA$, where $\boldsymbol{S_i} = \frac{1}{2} Re\{\boldsymbol{E_i} \times \boldsymbol{H_i^*}\}$. The rate at which energy gets scattered and transfers out of A is given $\boldsymbol{W_s} = -\oint \boldsymbol{S_s} \cdot \hat{n} dA$, where $\boldsymbol{S_s} = \frac{1}{2} Re\{\boldsymbol{E_s} \times \boldsymbol{H_s^*}\}$.

A part of incident lights gets scattered and the rate at which scattered light is transferred across A is given $\mathrm{W_{scat}} = -\oint \boldsymbol{S_{scat}} \cdot dA$. A part of it gets also absorbed

and a part of light gets absorbed by the particle. Lets start with quantification of scattering process, if we assume a surface (A) that completely surrounds the particle, there is rate at which scattered energy (W_{scat}) is transferred across this surface, this is given by the integeral of poynting vector (which is electromagnetic energy/unit area, W/m^2) over the whole surface.

In other words, $P_{scat}(\omega) = Re\left[\hat{n} \cdot \oint_A \boldsymbol{E}_{scat}(\omega) \times \boldsymbol{H}^*_{scat}(\omega).d^2x\right]$,

$\boldsymbol{E}_{scat}(\omega) = \boldsymbol{E}(\omega) - \boldsymbol{E}_{inc}(\omega)$

$P_{abs}(\omega) = Re\left[\hat{n} \cdot \oint_{Monitors} \mathtt{E}(\omega) \times \mathtt{H}^*(\omega).d^2x\right]$

$\sigma_{scat}(\omega) = \frac{P_{scat}(\omega)}{I_{inc}(\omega)}$

$\sigma_{abs}(\omega) = \frac{P_{abs}(\omega)}{I_{inc}(\omega)}$