# PROBLEM SESSION 1 // 14.10.2009 // TAKE-HOME LESSONS

- $E^2 = \vec{p}^2 c^2 + m^2 c^4$ LOW & HIGH RELATIVISTIC LIMITS
  MASS-SHELL, LIGHT-CONE.
- UNITS MASS & MOMENTUM: $GeV/c^2$, $GeV/c$
- KINETIC ENERGY : $E - mc^2$
- ULTRA RELATIVISTIC OR $m=0$ : $E = pc$
- PARTICLE DECAY

DESCRIBED BY NON-HERMITIAN HAMILTONIAN, LEADING TO COMPLEX ENERGY EV:

$$E = E_0 - i\frac{\Gamma}{2} \qquad \Gamma \text{ REAL } \& > 0$$

THEN FOR PARTICLE AT REST

$$|\psi(t)|^2 = |\psi(0)|^2 e^{-\Gamma t/\hbar} = |\psi(0)|^2 e^{-t/\tau} \; ; \; \boxed{\tau = \frac{\hbar}{\Gamma}}$$

SINCE $e^{-iEt/\hbar}$ NOW DAMPED IT CONTAINS NOT ONLY THE ENERGY $E_0$ (FREQ $\omega_0 = E_0/\hbar$) BUT MANY OTHERS.

WRITING

$$\psi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g(\omega)\, e^{-i\omega t}\, d\omega$$

WE CAN DETERMINE $g(\omega)$ BY FT & SUBSTITUTION OF $\psi(t) = \psi(0)\, e^{-iEt/\hbar}$. SINCE $\Gamma > 0$ THE ONLY CONTRIBUTION IN TIME INTEGRAL FOR $g(\omega)$ COMES FROM $t=0$ AND

$$g(\omega) \sim \frac{1}{\left[ (\hbar\omega - E_0) + \frac{i\Gamma}{2} \right]}$$

CONTRIBUTION, WEIGHT OF $\omega$ (AND $E = \hbar\omega$) : $|g(\omega)|^2$

$\Rightarrow$ PROBABILITY FOR E, P(E)

$$P(E) = \frac{\Gamma}{2\pi} \frac{1}{\left[(E-E_0)^2 + \left(\frac{\Gamma}{2}\right)^2\right]}$$

LORENTZ

BREIT–WIGNER

WITH

$$\int P(E)\, dE = 1$$

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DÉJÀ-VU FOR NATURAL LINEWIDTH OF SPECTRAL TRANSITION. FOR OPTICAL ALLOWED ELECTROMAGNETIC TRANSITION IN ATOMS, MOLECULES

$\tau \sim 10\,ns \Rightarrow \Gamma$ VERY SMALL AND NARROW LINES. FOR STRONG INTERACTION TRANSITIONS $\tau$ EXTREMELY SHORT ($10^{-24} - 10^{-22}\,s$), CANNOT BE MEASURED DIRECTLY IN TIME $\Rightarrow$ MEASURE $\Gamma$.

FOR $Z^0$, DATA SHOWN. FIT $\Rightarrow$ $\boxed{\Gamma = 2.5\ GEV}$

WITH

$\hbar c \approx 200\,MEV \cdot FM \Rightarrow$

$$\tau_{Z^0} \approx \frac{10^{-24}}{3}\,s$$