1) To describe particle <u>decay</u> we use an effective Hamiltonian that is not hermitian, which we split in hermitian and anti-hermitian parts:

$$H = \frac{H + H^{+}}{2} + \frac{(H - H^{+})}{2} = M - \frac{i}{2}\Gamma$$

with

$$M = \frac{H+H^{+}}{2}\, and\ \ \Gamma = \mathrm{i}(\mathrm{H} - \mathrm{H}^{+})$$

Both M and Γ are hermitian. Let the decay be described by

$$i\hbar\,\frac{\partial}{\partial t}\psi(t) = H\psi(t)$$

Show that

$$\frac{\partial}{\partial t}\langle\psi(t)|\psi(t)\rangle = -\frac{1}{\hbar}\langle\psi(t)|\Gamma|\psi(t)\rangle$$

Decay requires $\langle\psi|\Gamma|\psi\rangle \geq 0$, i.e. the operator Γ must have positive eigenvalues. Since Γ is hermitian its eigenvalues are real. Let ψ_1 be an eigenstate of Γ with eigenvalue Γ_1>0, then ψ_1 decays exponentially with $\tau = \hbar/\Gamma_1$.

2) <u>Rotations/particle mixing.</u> Consider the set of matrices

$$U(\theta) = \begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}$$

between which the operation of matrix multiplication is defined. Show that this set forms a group. Is the group abelian?

3) <u>Recoil energy</u>. An electron ($E, \vec{p}$, m) scatters from a particle (proton, nucleus, mass M) that is at rest before the collision. After the elastic collision, the electron scattered at an angle θ has the energy $E^{/}$. Show that for ultra-relativistic electrons

$$E^{/} = \frac{E}{1 + \frac{E(1-\cos\theta)}{Mc^2}}$$

The energy transferred to the target particle ($E - E^{/}$), the recoil energy, thus depends critically on the ratio E/Mc² and θ. Electrons with E = 10 GeV are scattered from protons. Calculate $E^{/}$ for θ = o° and θ = 180°. Sketch $E^{/}/_{E}$ as a function of θ for

$E/_{Mc^2}$ = ½ and $E/_{Mc^2}$ = 10.