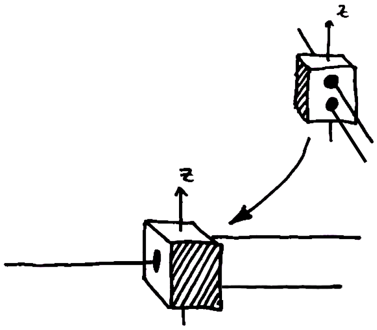
Contents

Chapter 1

Introduction

1.1 What is quantum mechanics?

Let us start by performing a thought experiment. Well, not so much of a thought experiment of the kind involving Schrödinger's cat or Wigner's friend and all that, to which we will come later in this course. But rather a mild abstraction of an experiment that can actually be done in the laboratory, nowadays in a quite simple fashion. And which is at the same time one that was one of the key experiments that were performed before the advent of quantum mechanics as a physical theory, and which profoundly influenced the way people thought about the physical world at the small scale.



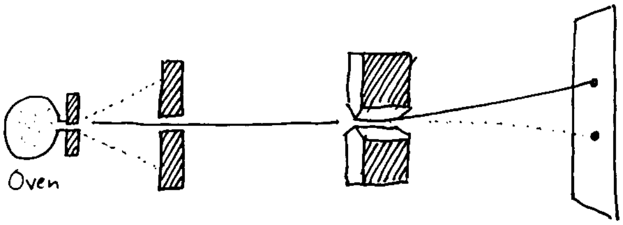
1.1.1 Boxes and Stern-Gerlach devices

For our purposes, let us think of this experiment as being performed with boxes. The box has one hole at one end and two holes at the other: One at the top and one at the bottom. Into this box one can send single electrons, so particles following the rules of quantum mechanics, one by one. At the other end, the particle can fly out either at the top or at the bottom, and it will surely leave the box at the other end through one of the holes.

What is going to happen?

Needless to say, this is exactly what any physical theory should deliver: It should make predictions of what is going to happen. It is the predictive power that makes physics so powerful.

Before we turn to what quantum theory says about our thought experiment, let us briefly discuss how one would do this experiment in the laboratory. Or how Otto Stern[1] and Walther Gerlach[2] actually did it, with polarized light, a simpler experiment can nowadays be done. What we described above is hence a *model* of the actual physical situation. The electrons were not quite single electrons, but silver atoms that had been heated up in an oven. Silver atoms consist of the nucleus, naturally, as well as a number of electrons, 47 in fact. 46 of them can be thought of as forming a spherically symmetric electron cloud with no resulting angular momentum. The nuclear spin also hardly contributes to our discussion here. At the end of the day, the remaining "angular momentum" that the atom can have is due to the spin of the 47th electron alone. Now, we have not yet clarified what the spin of an electron is – we will come to that. For the moment being, we state that it is a kind of angular momentum that is intrinsic to the electron, a kind of "inner angular momentum". Even if this sounds confusing, we swallow this impression for a moment and go on. For practical purposes, this 47th electron hence behaves like a free electron. Due to its spin, it has a magnetic moment μ.



This electron is now sent through a strongly inhomogeneous magnetic field B. Let us assume that the $\mathbf{B}$ field is homogeneous in the B_x and B_y directions and inhomogeneous along the z axis, so in B_z. We know that the energy of a particle with a magnetic moment in a magnetic field is given by $-\mu B$. Hence, the force the particle "feels" is

$$F_z = \mu_z \frac{dB_z}{dz}. \tag{1.1}$$

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[1] Otto Stern ($*$ 17 February 1888 in Sohrau, $+$ 17 August 1969 in Berkeley) was a German physicist of German-Jewish ancestry and a Nobel laureate in physics. He contributed to physics in many ways; in particular with the discovery of spin quantization in the Stern-Gerlach experiment discussed here – in work he did together with Walther Gerlach in Frankfurt – but also with work measuring the proton's magnetic moment.

[2] Walther Gerlach ($*$ 1 August 1889 in Biebrich, $+$ 10 August 1979 in Munich) was a German physicist who, together with Otto Stern, discovered the spin quantization in a magnetic field. Unlike Stern, he did not receive the Nobel price in physics. This is possibly related to the fact that unlike his colleague, who had left Germany in 1933, moving to Berkeley, he remained there, and made a career as the head of the physics section of the Reichsforschungsrat and was even involved in the nuclear physics programme of Nazi Germany.

Since the particle is heavy, a motion along the x axis can be described in a classical fashion. This is true, but for quite subtle reasons, and again we will come to that later. So as a net effect, the particle will fly through the inhomogeneous magnetic field, and it will "feel" an upward force if $\mu_z < 0$ and a downward force if $\mu_z > 0$. Since the magnetic moment is proportional to the spin, in effect this device "measures" the spin of the particle.

Now one could argue that the particle could take continuous values of the spin. So on a plate, suitable placed far away along the x axis, one should observe a big black region, reflecting the continuum of possible positions to which the atoms are bent to by the magnetic field. This is, however, not what one observes. One finds two very distinct narrow spots, as if the spin could take only two different values. This was seen as the first observation of the "quantization of spin", with the semantics of the word "quantum" meaning here something like "coming in packets" or in distinct units, rather than in continuum values. Indeed, one often observes quantities taking discrete values in quantum theory, which gave rise to the term "quantum mechanics" in the first place. One should not think, however, that all properties in quantum mechanics take discrete values: this is a common misunderstanding. So let us take the term quantum mechanics rather as a historical term. In any case, we can close the lid and take our experimental apparatus again as a black box, with particles flying in, and flying out at exactly one of the two possible holes at the other end of the box.

So to come back to it: Where does the particles fly out? The point is that quantum mechanics does not make a claim about that at all. It will merely say with what *probability* the particle will fly out. In the preparation chosen here, quantum mechanics will say that

- with probability $\frac{1}{2}$ the particle appear at the upper hole ("spin up") and
- with probability $\frac{1}{2}$ the particle appear at take the lower hole ("spin down").

This is remarkable! So unlike the situation in classical physics, where of course one can in all instances predict where a particle will fly along, once one precisely knows the initial condition, quantum mechanics remains utterly silent about this. It will only talk about probabilities. This is not a detail at all, but one of the key structural elements in quantum theory. There is an element of randomness. This is so important that this statement will get a box:

Statistical character of quantum theory: Quantum mechanics is a statistical theory. It makes predictions about probabilities of outcomes of measurements.

Of course, this insight begs for an answer to the question: But how can one then verify whether a prediction has been correct? If for single outcomes of an experiment, the theory usually remains silent, how can one find out whether the prediction has been correct?

The answer is: In a single experiment, this is impossible. One has to make many experiments and look at relative frequencies. If one obtains k times spin up in n experiments, and $n - k$ times spin down, the relative frequencies are

$$f_1 = \frac{k}{n},\ f_2 = \frac{n-k}{n}. \tag{1.2}$$

For a large number of experimental outcomes, one will find

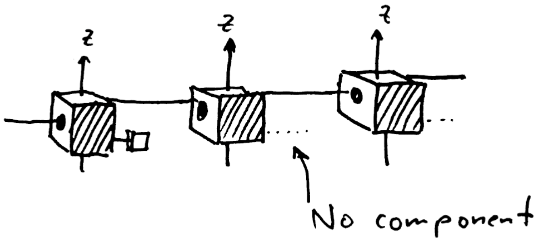
$$f_1 \to \frac{1}{2},\ f_2 \to \frac{1}{2}\ (n \to \infty). \tag{1.3}$$

One has hence always has to estimate probabilities from relative frequencies in experiments. From a single run of an experiment, one learns very little. One might take this as a shortcoming of the theory: How can it be that the theory does not speak about what the "actual property of the particle really is"? Surely it must have been determined beforehand, we merely do not have precise knowledge about it? We will see that this is a statement that can under extremely mild conditions be falsified.

1.1.2 More boxes and Stern-Gerlach devices

How does the situation look like for more boxes that we apply in a consecutive fashion? For example, we take one box, and apply the same box afterwards again. So if the particle went through the upper hole – so we had "spin up" – what will happen in the subsequent round?

It turns out that it will, once it has taken the upper arm, will *always* take the upper arm again in the next round. There is no randomness involved now, and the theory says that it will take with certainty the upper arm again. Of course, to certify this in an experiment with high statistical significance, we still have to do many runs, as from a single experiment we cannot infer that the particle has always shown "spin up". But if we do n experiments, we will indeed get n times a "spin up".



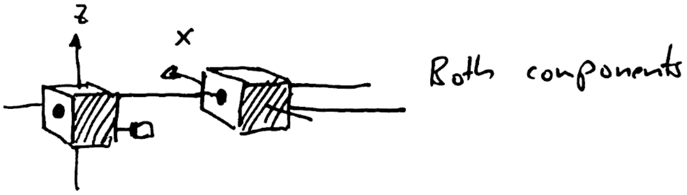
The same will be true if we do a third round, and a fourth and so on. So in a way, the random aspect seems to have disappeared in this experiment. Only in the first instance, the outcomes are random, then no longer.

We now perform a new experiment. We turn the second box and now make a measurement along the x axis – and no longer along the z direction.

What will happen now? Will it fly always left? Always right? Will it be random? We find that again

- with probability $\frac{1}{2}$ the particle will take the left hole ("spin left") and
- with probability $\frac{1}{2}$ the particle will take the right hole ("spin right").

So randomness is back. What if we repeat the experiments along the x axis? Then we will, once the particle had taken the right branch *always* get right, and once the particle had taken the left branch in the first round, it will always go left.



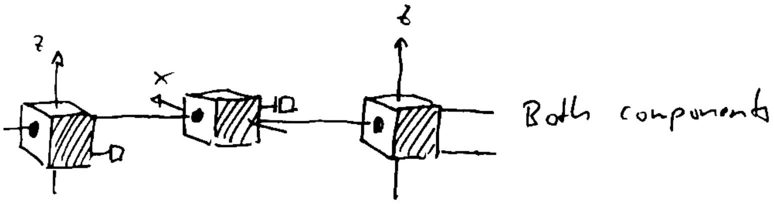
A final experiment will do the following: We first measure along the z axis. In a second round, we measure along the x axis. And again in a final round, we will measure along the z axis again.

Now, again, what will happen? We had measured along the z before, so one could argue that we already know the spin is pointing up, so we should get always "spin up". Or could it be any different?

In fact,

- with probability $\frac{1}{2}$ the particle will be in "spin up" and
- with probability $\frac{1}{2}$ the particle will be in "spin down".

So the measurement along the x axis seems to have destroyed the alignment of the spin along the z axis.



If you find this confusing, do not worry: It is confusing, since we are faced with a situation very much unknown in classical physics – and presumably with anything you have seen before. There are a couple of structural elements that are new:

- Again, outcomes are random.
- A measurement seems to be more than determining a state that was predetermined, but it seems to alter the state of the quantum particle in a way.

These are structural elements of quantum mechanics that will not go away any more. But once we have developed our theoretical machinery in the subsequent chapter, it will no longer look so spooky. Even if the clash with everyday intuition that is shaped by notions of classical physics will remain.

1.1.3 Randomness in quantum physics

At the risk of becoming slightly redundant at this point, we emphasize again that the outcomes of measurements in quantum mechanics are random. Of course, this feature may be taken as a shortcoming of the theory: One might argue that the theory does not "provide the full story". Surely the state of the system must have been determined beforehand, merely revealed by the measurement. After all, we encounter loads of random processes in physics and in everyday life.

Say, if we meet a friend on the street, we might be tempted to say: "Oh what a coincidence, what a nice random event!" But of course we know that there is nothing truly random about this event. We could have known that the friend is in the city, and we could have known that we meet at a specific instance in time. We merely would have had to call beforehand, or send an SMS or write a Facebook entry. In other words: This apparent "randomness" is entirely due to our lack of classical knowledge. We could have known, and adding knowledge easily explains the apparently random process.

We can go further than that. Events that are happening on Roulette tables are presumably random: This is the entire point of the game. But really, again it is not really random. If we knew exactly the velocity and direction with which the ball is thrown into the wheel by the croupier, we knew about the wind inside the room and had knowledge about the precise movement of the wheel. Well, then we could surely determine, using the rules of classical mechanics, at what number the ball will finally come to rest. Of course, in practice this is difficult to do. But not that difficult: I have read that people have used cameras in mobile phones to record part of the motion and used supercomputers to calculate the final position. Of course, this calculation does not have to be always right, but one can rob the bank by already having a slight bias towards the correct outcome. As far as I know, this is one of the reasons for mobile phones not being allowed in casinos (apart from them being annoying in this context).

But coming back to the issue of randomness: Again, the apparent randomness is entirely due to our ignorance about the initial conditions. If we knew them, we could perfectly determine the outcome, and the randomness disappears. In fact, classical mechanics has no random component whatsoever, it is a deterministic theory. And once the initial conditions are fixed, it just moves as if on tracks, with no random element.

Quantum theory is different. It is in a way truly random. At this point one is tempted to say: How do we know? It could be that there are further properties, possibly hidden, that completely determine the outcome in every measurement. Then again, the randomness of the theory would again be just of the same kind, an "apparent randomness due to a lack of classical information". This seems a perfectly metaphysical question, and not one of physics. We will see later, however, that it is a question of physics, and that under very mild conditions, we can rule out the existence of such "hidden parameters". In a sense yet made to be precise, nature at the quantum level is truly and genuinely random. But enough of this for now.

1.2 Historical overview

Before we get going with the framework, let us have a brief look at the historical events that led physicists to abandon classical mechanics at the level of atoms, ions, electrons, photons. They did so in favor of the new theory called quantum theory. This history is remarkable in many ways. The most notably thing is that starting from the early years of the 20th century, a body of experimental findings accumulated was at odds with what classical mechanics predicted. This was unexpected in a sense, as the sentiment of the times was that classical physics could explain everything, that nature had already been explained and that physics had already achieved its task: to describe nature at the fundamental level in a mathematical form. All that was seen and observed in nature seemed compatible with classical physics. In those days it was not uncommon at all to think that the study of physics was essentially a dead end, as everything substantial had already been said and done. Then, suddenly, this body of data accumulated which confused researchers in a very refreshing way. Theories were put up, steps were made, until in a very short period of time, quantum mechanics emerged as a physical theory. This finally happened in fewer than three years, in an explosion of ideas. Notably, this took place in times when there was no preprint server, no internet communication, no Facebook, and travel was less convenient than it was today. Still, within a period of less than two years, quantum theory was formulated from scratch.

1.2.1 Spectrum of black body radiation

The historically first glimpse into the world of quantum physics was the observation of the spectrum of black body radiation. Let us consider an empty box of volume $V = L^3$ of temperature T, in equilibrium. The energy density $e(\omega)$ as a function of the frequency $\omega > 0$ is given by the familiar *Rayleigh-Jeans law*[3]. It is

$$e(\omega) = \frac{k_B T}{\pi^2 c^3} \omega^2. \tag{1.4}$$

This law can easily be obtained by considering reflecting metal boundaries and counting the number of waves of a certain wave length. This is how the energy

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[3] John William Strutt 3rd Baron Rayleigh ($*$ 12 November 1842, $+$ 30 June 1919) was an English physicist who first described the phenomenon now called Rayleigh scattering, which is at the basis of the understanding of why the sky is blue. He was awarded the Nobel price in physics in 1904, on the basis of his discovery of the element argon, together with William Ramsay.

density should scale with the frequency in classical physics. But this formula cannot be valid for all frequencies: It clearly implies that the total energy diverges

$$E = \int_0^\infty e(\omega) d\omega = \infty. \tag{1.5}$$

This is sometimes called the "ultraviolet catastrophe". And indeed, while measurements of the energy density nicely confirmed the validity of the Rayleigh-Jeans law, for large frequencies one observed a quite different behavior. For large ω the function e was found to be nicely approximated by

$$e(\omega) \approx A\omega^3 e^{-c\omega/T}, \tag{1.6}$$

with $A, c > 0$ being constants. As a consequence of this, Max Planck[4] did something outrageous in 1900: He simply interpolated the two formulae, based on thermodynamic considerations and a good deal of physical intuition to the formula

$$e(\omega) = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{e^{\hbar\omega/(k_B T)} - 1}. \tag{1.7}$$

Originally, this was rather a trick of interpolation, and not seen as reflecting fundamental physics. Still, a new number $\hbar$ emerged, taking the value

$$\hbar = 1.054571726(47) \times 10^{-34} Js. \tag{1.8}$$

This number is now known as (the reduced) *Planck's constant*. Max Planck was also able to give evidence to the hypothesis that the energy of the plates to the radiation field is merely emitted in multiples of $\hbar\omega$. These may be regarded as small units of energy, so little "quanta" of energy. Clearly, this interpolation was a consequence of a streak of genius. Still at the time, people were still far from a complete quantum theory. It was a clear indication, however, that something was fundamentally flawed in the classical picture of physics.

1.2.2 Photoelectric effect

From then on, evidence that the classical picture had to be massively revised accumulated rapidly. For example, if one shines light (or usually rather ultraviolet light) with frequency ω onto a metal surface, one finds that the minimum kinetic energy of the emitted electrons with mass m_e takes the value

$$\frac{1}{2} m_e v_e^2 = \hbar\omega - W, \tag{1.9}$$

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[4] Max Planck ($*$ April 23 1858, $+$ October 4, 1947) is widely seen as one of the father figures of quantum theory. His work dating back to 1900 on the spectrum of black body radiation was the first paper mentioning the Planck constant $\hbar$. He was awarded the Nobel Price in physics in 1918. His personal life was actually very much tragic: He had two daughters, both of which died due to the same complication when giving birth to the respective first child. His son was accused of taking part in Stauffenberg's July 20 plot and was executed on 23 January 1945. Apart from his manifold exceptional contributions to physics, this fact was one of the motivations of renaming the Kaiser-Wilhelm-Gesellschaft to Max-Planck society in 1948.

where W is the minimum energy required to remove a delocalised electron from the surface of the metal. This effect had already been observed by Heinrich Hertz[5] in 1887, but it took the genius of Albert Einstein[6] to conjecture that this could have to do with light merely being absorbed in units of $\hbar\omega$, again giving rise to the evidence of light coming in "quanta" of $\hbar\omega$.

1.2.3 Double slit experiments

The *double slit experiment* is so famous that we do not have to spend much time with it. First performed with light by Thomas Young[7] in 1802 where light is shone onto a barrier having two narrow slits, it can also be performed with massive particles following the laws of quantum mechanics. It turns out that the notion of a particle "passing exactly one of the slits" is meaningless. The alternatives of a particle taking either of the two slits is incompatible with the situation that one observes, namely an interference pattern of a particle with itself. A variant of this experiment was first done with electrons in 1927, when Clinton Davisson[8] and Lester Germer[9] showed the wave character of electrons when shining them into a Nickel cristal.

There was a lot of other evidence collected as well. The *Compton effect* considered the scattering of electrons with Röntgen radiation, with observations deviating from what one would have expected classically. Atoms showed *discrete energy levels*. An impressive manifestation of this effect was visible in the Franck-Hertz-experiment by James Franck[10] and Gustav Hertz[11] in 1913,

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[5] Heinrich Rudolf Hertz ($*$ 22 February 1857 in Hamburg, + 1 January 1894 in Bonn, only aged 36) was a German physicist who significantly extended Maxwell's electromagnetic theory of light. One of his key contributions was the prove the existence of electromagnetic waves by engineering instruments to transmit and receive radio pulses.

[6] Albert Einstein ($*$ 14 March 1879 in Ulm, + 18 April 1955 in Princeton) is for good reasons by far the most famous physicist of all times. His contributions to physics are numerous, starting from the description of the photoelectric effect (for which he primarily was awarded the Nobel price in physics, not the theory of relativity), to a first meaningful theory of Brownian motion, to his key contributions to quantum theory. Needless to say, he is also the inventor of both the special and the general theory of relativity. His contributions to the foundations of quantum theory in the form of the EPR paradox can also hardly be overestimated: Even though Einstein is sometimes claimed to have been "wrong" in this debate, he was the first to realize that the question of randomness in quantum theory could be subject to experimental studies.

[7] Thomas Young ($*$ 13 June 1773, + 10 May 1829) was an English scientist. He was one of the last scientists that truly fulfilled the Renaissance ideal of a universal scientist. His performance of the double slit experiment is by far not his only important contribution. Originally, he was actually a medical doctor, and made important contributions to medicine. He is most famous actually for having partly deciphered Egyptian hieroglyphics before Jean-Franois Champollion eventually expanded on his work and provided a complete solution.

[8] Clinton Joseph Davisson ($*$ 22 October 1881, + 1 February 1958), was an American physicist. He was awarded the Nobel prize in physics in 1937 for his discovery of electron diffraction, together with Lester Germer.

[9] Lester Halbert Germer ($*$ 10 October 1896, + 3 October 1971) together with Clinton Davisson experimentally found the phenomenon of electron diffraction.

[10] James Franck ($*$ 26 August 1882, + 21 May 1964) was a German physicist and a Nobel laureate.

[11] Gustav Ludwig Hertz ($*$ 22 July 1887, + 30 October 1975) was a German physicist and a Nobel Prize winner. He was actually the nephew of the above mentioned Heinrich Hertz.

which depicted a clear level structure in the absorption spectrum of Hg atoms. This merely a small selection of experimental evidence that classical physics had to be abandoned, in favor of a radically new theory. Except from the view that energy of radiation fields would come in units and that the view of how atoms are constituted changed significantly, theoretical progress – putting all this experimental evidence into the perspective of a coherent theory – was initially slow.

1.2.4 An explosion of ideas

There is no way to give a detailed account on the history of quantum mechanics. Here we merely sketch a few basic milestones in the history of quantum theory as we know it today. The field was largely driven by young, extraordinarily talented researchers, culminating in an explosion of ideas in the few years from January 1925 to January 1928.

- In 1913 Niels Bohr[12] entered stage by the proposition that electrons in atoms would only occupy certain stationary states, including the ground state. They could only change their energy by "jumping" between such stationary states emitting light the wavelength of which would depend on the energy difference between the distinct energy levels. This was a cunning proposition at the time. One should not forget that the idea of atoms even existing was just a decade or so old at the time. These jumps were referred to as "quantum jumps", a slightly anachronistic but still quite accurate description, and one – albeit in a totally wrong context – that is still present in urban language.

- In 1923 Louis de Broglie[13] proposed that the particle behavior of light should have its counterpart in the wave behavior of particles. He already did so in his PhD thesis, quite an impressive achievement.

- In 1925, the Indian physicist Satyendra Bose[14] proposed new way to explain the Planck radiation law, taking the indistinguishable nature of photons (nowadays identified as being bosons) seriously. So in a way, some aspects of quantum statistical mechanics were proposed even before the advent of quantum mechanics as a coherent theory.

From January 1925 on, the development became extraordinarily rapid. In the period of the following three years, all of the following happened:

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[12] Niels Henrik David Bohr ($*$ 7 October 1885, $+$18 November 1962) was a Danish physicist. He made seminal contributions in particular to the understanding of the atomic structure, as well as to the interpretation of quantum mechanics as a whole. He was awarded the Nobel prize in physics in 1922.

[13] Louis-Victor-Pierre-Raymond 7th duc de Broglie, ($*$ 15 August 1892 , $+$ 19 March 1987) was a French physicist and a Nobel laureate in 1929, for work done in his PhD thesis.

[14] Satyendra Nath Bose ($*$ 1 January 1894, $+$ 4 February 1974) was an Indian mathematician and physicist, famous in particular for his collaboration with Einstein leading to the discovery of the Bose-Einstein distribution.

- Wolfgang Pauli[15] proposed the exclusion principle. This idea had several profound consequences, among which was the foundation of the theoretical basis for the periodic table.

- Werner Heisenberg[16], together with with Max Born[17] and Pascual Jordan[18], discovered matrix mechanics, the first version of quantum mechanics. We will in fact come to matrix mechanics in the subsequent chapter. In fact, we already saw a hint of this when we discussed boxes: Heisenberg was driven by the idea that properties that would in a sense not commute – reflecting the idea that the order of measurements does matter – would have to correspond to other objects than scalar quantities. He suggested that properties could be associated with matrices – a view that we will see nicely fleshed out very soon.

- Almost at the same time, Erwin Schroedinger[19] invented the theory of wave mechanics. At the time, this was seen as a second, alternative formulation of quantum theory. Very soon, however (and for us again in the subsequent chapter), the two pictures were shown to be equivalent. Werner Heisenberg and Erwin Schroedinger are widely seen as the two main figures in the development of quantum theory.

- Heisenberg put forth the famous uncertainty principle. We will again come to that in the second chapter.

- Electrons were found to obey a new type of statistical law, the Fermi-Dirac statistics. It was realized that indistinguishable particles would either satisfy Fermi-Dirac or Bose-Einstein statistics,

- Paul Dirac[20] proposed a relativistic wave equation for the electron, in a

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[15] Wolfgang Ernst Pauli (∗ 25 April 1900, + 15 December 1958) was an Austrian physicist. As one of the pioneers of quantum physics, he was awarded the Nobel prize in physics in 1945.

[16] Werner Karl Heisenberg (∗ 5 December 1901, + 1 February 1976) was a German theoretical physicist. Together with Schroedinger, he is one of the two main protagonists of the development of quantum mechanics as a modern theory. The development of matrix mechanics is largely attributed to him, as well as the uncertainty principle. His later role in Nazi Germany – where he was still active as a physicist while most others had fled – is still subject to a debate among historians.

[17] Max Born (∗ 11 December 1882, + 5 January 1970) was a German-British physicist and mathematician and again one of the key figures in the development of quantum theory.

[18] Pascual Jordan (∗ 18 October 1902, + 31 July 1980) was a theoretical and mathematical physicist. He was one of the inventors of quantum theory and quantum field theory, to which he made seminal contributions. He was one of the few leading physicists of the time who joined the NSDAP, which isolated him internationally in the scientific community, although he was later rehabilitated and recovered the status of tenured professor in 1953. It is interesting to observe how much the development of quantum theory – being to such a large extent driven by German and Austrian scientists – is intertwined with the rise of Nazi Germany and the subsequent exodus of a complete intellectual generation.

[19] Erwin Rudolf Josef Alexander Schroedinger (∗ 12 August 1887, + 4 January 1961) was an Austrian physicist and theoretical biologist . Together with Werner Heisenberg he is one of the principal fathers of quantum mechanics.

[20] Paul Adrien Maurice Dirac (∗ 8 August 1902, + 20 October 1984) was an English theoretical physicist. He made important contributions to the development of quantum mechanics and quantum electrodynamics. He shared the Nobel prize in physics with Erwin Schroedinger in 1933.

theory that also laid the foundations of quantum field theory by his idea of how to incorporate the electromagnetic field into the theory.

- Bohr proposed the complementary principle, a principle related to the non-commutativity of observables.

The speed and creativity of these sometimes called "golden years" of quantum theory are remarkable in many ways, and there are a number of good historical accounts on that phase available. In some parts, work progressed by thoughts developed by protagonists in solitude. For example, in 1925 a fierce attack of hay fever forced Werner Heisenberg to leave his office and spend some time in Helgoland, where he developed matrix mechanics between walks and baths. In fact, he must have looked so awful that the landlord naturally assumed upon arrival that Heisenberg had been involved in a fight. When he came back and showed his work to Born, he was very much hesitant and referred to his work as a "crazy paper", but Born quickly realized what he was holding in his hands. In other parts coincidences helped progress, say, when Born met Pauli by accident on the train between Gottingen and Hanover. All this happened in times when internet communication was not available. In fact, the book written by the universal genius John von Neumann[21] in 1932 can be read in many ways as a modern textbook on quantum mechanics. It should be mentioned that quantum field theory emerged almost at the same time as quantum mechanics itself as an idea; although while basic quantum mechanics was basically complete when the 1930ies started, some conceptual issues with quantum field theory still remain unresolved. Still, the predictive power is enormous: The prediction of QED of the interaction strength between an electron and a magnetic field has been experimentally confirmed to a remarkable precision of two parts in 1,000,000,000,000.

1.2.5 A word on quantum mechanics in the modern world

Quantum theory is by no means only interesting as a event in the history of physics. Quantum mechanics, augmented in quantum field theory and then in the standard model, is still the best theory we have of nature today. What is more, quantum effects are ubiquitous in modern devices (since electronics is of course based on quantum effects). Today, quantum devices such as lasers and transistors make a very large contribution to modern economies. Quantum mechanics governs properties of materials, and is essentially responsible for bodies being solid in the first place. One of the large fields of physics it the field of condensed matter physics, being concerned with how properties of material bodies or fluids emerge from a quantum mechanical description. We

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[21] The contribution of John von Neumann ($*$ 28 December 1903, $+$ 8 February 1957) a Hungarian-American mathematician, to quantum theory is sometimes underappreciated. This is possibly due to the fact that he made so many other important contributions to science, largely inventing game theory and significantly developing operator theory. But he made key contributions in particular to statistical quantum theory. He also helped clarifying the mathematical structure of quantum theory.

will also hear about modern applications of quantum mechanics at the end of this course in the form of quantum simulators and quantum information, very much exploiting the quantum character of nature at the microscopic level.