Freie Universität Berlin
Tutorials for Advanced Quantum Mechanics
Wintersemester 2018/19
Sheet 10 (Holiday Revision Sheet)

Due date: 10:15 11/01/2019 J. Eisert

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1. **Bosonic Field Operators**(4 points)

   A general symmetrized state vector of N bosonic particles can be written in terms of field operators and the symmetrized wave function as

$$|\psi_N\rangle = \frac{1}{\sqrt{N}} \int d\xi_1 \ldots d\xi_N \psi_N(\xi_1, \ldots, \xi_N) \Psi^\dagger(\xi_N) \ldots \Psi^\dagger(\xi_1) |\phi\rangle. \tag{1}$$

   Prove explicitly that application of a bosonic field operator to such a state gives the following:

$$\Psi(\xi)|\psi_N\rangle = \sqrt{N} \int d\xi_1 \ldots d\xi_{N-1} \psi_N(\xi_1, \ldots, \xi_{N-1}, \xi) \Psi^\dagger(\xi_{N-1}) \ldots \Psi^\dagger(\xi_1) |\phi\rangle. \tag{2}$$

2. **Fermionic communication and causality**(2+2+2 points)

   Consider a two mode fermionic system with a state vector written in second quantised form as,

$$|\psi\rangle = (|n_1, 0\rangle + |n_1, 1\rangle)/\sqrt{2} \tag{3}$$

   shared between two parties who might be arbitrarily far apart.

   Now imagine that the second party measures the Hermitian operator $m_2 = (f_2 + f_2^\dagger)/\sqrt{2}$.

   (a) Calculate the expectation value $\langle\psi|m_2|\psi\rangle$.

   (b) What would this measurement tell us about the absence or presence of a particle in the first mode? Why would this be a violation of causality?

   (c) What is the resolution to this seeming paradox? Why do such causality violations not arise in real fermionic systems?