1. **Bosonic Field Operators** (4 points)

A general symmetrized state vector of \( N \) bosonic particles can be written in terms of field operators and the symmetrized wave function as

\[
|\psi_N\rangle = \frac{1}{\sqrt{N}} \int d\xi_1 \cdots d\xi_N \psi_N(\xi_1, \ldots, \xi_N) \Psi^\dagger(\xi_N) \cdots \Psi^\dagger(\xi_1) |\phi\rangle.
\]  

(1)

Prove explicitly that application of a bosonic field operator to such a state gives the following:

\[
\Psi(\xi)|\psi_N\rangle = \sqrt{N} \int d\xi_1 \cdots d\xi_{N-1} \psi_N(\xi_1, \ldots, \xi_{N-1}, \xi) \Psi^\dagger(\xi_{N-1}) \cdots \Psi^\dagger(\xi_1) |\phi\rangle.
\]  

(2)

2. **Fermionic communication and causality** (2+2+2 points)

Consider a two mode fermionic system with a state vector written in second quantised form as,

\[
|\psi\rangle = (|n_1, 0\rangle + |n_1, 1\rangle)/\sqrt{2}.
\]  

(3)

shared between two parties who might be arbitrarily far apart.

Now imagine that the second party measures the Hermitian operator \( m_2 = (f_2 + f_2^\dagger)/\sqrt{2} \).

(a) Calculate the expectation value \( \langle \psi|m_2|\psi\rangle \).

(b) What would this measurement tell us about the absence or presence of a particle in the first mode? Why would this be a violation of causality?

(c) What is the resolution to this seeming paradox? Why do such causality violations not arise in real fermionic systems?