

Freie Universität Berlin
Tutorials for Advanced Quantum Mechanics
Wintersemester 2019/20
Sheet 2

Due date: 10:15 01.11.2019

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1. **Basic Quantum Mechanics Recap Continued** (2+2+2+6+2+2 = 16 points)
Consider the two-particle Hilbert space $\mathcal{H} = \mathbb{C}^2 \otimes \mathbb{C}^2$, and the Hamiltonian $H = X \otimes Z$, for Pauli operators X and Z . Suppose we prepare the states

$$\begin{aligned} |\psi_1\rangle &= |00\rangle \\ |\psi_2\rangle &= |11\rangle \\ |\psi_3\rangle &= (1/\sqrt{3})|01\rangle + (\sqrt{2/3})|10\rangle, \end{aligned}$$

with probabilities $p_1 = 1/2$, $p_2 = 1/8$ and $p_3 = 3/8$ respectively.

- (a) Write down the density operator ρ for this ensemble, both in bra-ket notation and explicitly a matrix, with respect to the basis for \mathcal{H} built from the computational basis for a single particle, i.e. the basis $\{|0\rangle = (1, 0)^T, |1\rangle = (0, 1)^T\}$ for \mathbb{C}^2 .
 - (b) Write down the unitary operator, both in bra-ket notation, and explicitly as a matrix, for a basis transformation from the computational basis for \mathcal{H} to the basis built from the single particle basis $\{|+\rangle, |-\rangle\}$, where $|\pm\rangle = (1/\sqrt{2})(|0\rangle \pm |1\rangle)$.
 - (c) Write down the density operator ρ for the above ensemble, again both in bra-ket notation and explicitly as a matrix, but with respect to the $\{|+\rangle, |-\rangle\}$ basis.
 - (d) If $\rho(0) = \rho$ from (a), calculate $\rho(t)$ in the Schrödinger picture, using any basis.
 - (e) Define $Y_1 = Y \otimes \mathbf{1}$. Working in the Heisenberg picture, what is $Y_1(t)$ if $Y_1(0) = Y_1$.
 - (f) Calculate $\langle Y_1(t) \rangle$ in both the Heisenberg and Schrödinger picture.
2. **Identical Particles and Permutations 1** (4+4 = 8 points)
Consider the single particle Hilbert space $\mathcal{H}_s = \mathbb{C}^2$.

For both the (a) two particle and (b) three particle Hilbert spaces - i.e. $\mathcal{H}_s^{\otimes 2}$ and $\mathcal{H}_s^{\otimes 3}$ respectively - Write down an explicit basis for both the symmetric and anti-symmetric subspaces, if such a subspace exists.

3. Identical Particles and Permutations 2 (3+3=6 points)

Consider a system formed by three identical particles, with wave functions $\psi(ijk)$, where $i \neq j \neq k$ take the values 1, 2, 3. We denote the permutation group on 3 elements as S_3 . Now consider the following three states:

$$\psi_a(123) = \frac{1}{\sqrt{6}}[\psi(123) + \psi(132) + \psi(231) + \psi(213) + \psi(312) + \psi(321)]$$

$$\psi_b(123) = \frac{1}{\sqrt{6}}[\psi(123) - \psi(132) + \psi(231) - \psi(213) + \psi(312) - \psi(321)]$$

$$\psi_c(123) = \frac{1}{2\sqrt{3}}[2\psi(123) + \psi(132) + 2\psi(213) - \psi(231) - \psi(312) - \psi(321)]$$

- (a) Identify which of the above states are (i) fully symmetric, (ii) fully anti-symmetric (iii) of mixed symmetry, under the action of S_3 .
- (b) Show explicitly that for the state of mixed symmetry, which we denote as ψ_m , there exists some permutation $P \in S_3$ such that $P\psi_m \neq \lambda\psi_m$. What are the physical implications of this?