

Freie Universität Berlin
Tutorials for Advanced Quantum Mechanics
Wintersemester 2018/19
Sheet 10 (Holiday Revision, Bonus Sheet)

Due date: 10:15 10/01/2020

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Points gained in this tutorial sheet serves as extra, bonus points that will be added on top of homework grades.

1. Bosonic Field Operators (4 points)

A general symmetrized state vector of N bosonic particles can be written in terms of field operators and the symmetrized wave function as

$$|\psi_N\rangle = \frac{1}{\sqrt{N!}} \int d\xi_1 \dots d\xi_N \psi_N(\xi_1, \dots, \xi_N) \Psi^\dagger(\xi_N) \dots \Psi^\dagger(\xi_1) |\phi\rangle. \quad (1)$$

Prove explicitly that application of a bosonic field operator to such a state gives the following:

$$\Psi(\xi) |\psi_N\rangle = \sqrt{N} \int d\xi_1 \dots d\xi_{N-1} \psi_N(\xi_1, \dots, \xi_{N-1}, \xi) \Psi^\dagger(\xi_{N-1}) \dots \Psi^\dagger(\xi_1) |\phi\rangle. \quad (2)$$

2. Fermionic communication and causality (2+2+2 = 6 points)

Consider a two mode fermionic system with a state vector written in second quantised form as,

$$|\psi\rangle = (|n_1, 0\rangle + |n_1, 1\rangle) / \sqrt{2} \quad (3)$$

shared between two parties who might be arbitrarily far apart.

Now imagine that the second party measures the Hermitian operator $m_2 = (f_2 + f_2^\dagger) / \sqrt{2}$.

- (a) Calculate the expectation value $\langle \psi | m_2 | \psi \rangle$.
- (b) What would this measurement tell us about the absence or presence of a particle in the first mode? Why would this be a violation of causality?
- (c) What is the resolution to this seeming paradox? Why do such causality violations not arise in real fermionic systems?

3. Reading exercises (20 points)

In the lectures you have had a first encounter with the BCS theory for superconductivity. Choose one of the following papers to read:

- (a) "From Cooper Pairs to Luttinger Liquids with Bosonic Atoms in Optical Lattices", B. Parades and J.I. Cirac, PRL 90, 150402 (2003)
- (b) "Theory of dirty superconductors", P.W. Anderson, Journal of Physics and Chemistry of Solids 11.1-2 (1959): 26-30

After reading, try to formulate good questions, especially in the context of extending these works for further research. For each well formulated question, you will be awarded up to 4 points, and a maximum of 5 questions will be considered.